SAMPLING INSPECTION

Principles, Procedures, and Tables
for Single, Double, and Sequential Sampling
in Acceptance Inspection and Quality Control
Based on Percent Defective

BY THE

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SAMPLING INSPECTION

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PREFACE

SPONSORSHIP OF THE WORK

This volume was prepared by the Statistical Research Group, Division of War Research, Columbia University, for the Applied Mathematics Panel, National Defense Research Committee, Office of Scientific Research and Development. The work was initiated under contract OEMsr-618 and completed under contract OEMsr-1131, both between the trustees of Columbia University and the Office of Scientific Research and Development.

The Statistical Research Group was organized, and its initial staff selected, by Warren Weaver, chief of the Applied Mathematics Panel, on July 1, 1942. The staff was drawn from universities and research organizations throughout the country, many of which generously granted leaves for the work. The group disbanded Sept. 30, 1945, except for a few persons who have prepared this and another volume as monographs in connection with the Applied Mathematics Panel's summary technical report.

NATURE OF THE BOOK

Sampling inspection is the process of evaluating the quality of material by inspecting some but not all of it. Its purpose is the control of quality through disposition of materials already produced or through regulation of the processes of production.

The foundation of this book is the set of sampling tables presented in Part V. A standard procedure for using the tables is given in Part III. For those who want to develop their own procedures for using the tables, an exposition of the principles for doing so is given in Part II. Part I is a brief explanation of the nature and uses of sampling inspection, and Part IV is a technical supplement on the construction of the tables in Part V and the procedure in Part III.

Readers who require a complete, consistent, and workable sampling-inspection program ready for installation will regard Parts I, III, and V as the essential parts of the book. Readers interested in constructing their own programs will regard Parts I, II, and V as the essential material. Those interested primarily in the technical-mathematical aspects of sampling inspection will concentrate their attention on Part IV. For teaching, Parts II, III, and IV will be most useful.

While this organization of the book makes for flexibility, adaptability to a wide variety of uses, and ease of reference, it has necessitated a certain amount of repetition. Parts II and III, and to a considerable degree Part IV, have been made self-contained; in fact, even Part V, with its concise instructions for Tables 1, 2, and 3, could almost stand alone.

Attention should be called to the usefulness of the tables for purposes other than sampling inspection. They will be valuable in many cases where statistical tests of significance based on the binomial distribution are used, for example, in psychological testing, biological experimentation, engineering research and development, and certain problems in market analysis and public opinion measurement.

BACKGROUND

This book is intended as a systematic account of certain of the best current inspection practices, together with tables and detailed instructions for carrying out these practices. Sampling inspection has been developing rapidly for about two decades. Much of the information on the subject exists in scattered papers, in relatively inaccessible memoranda, as word-of-mouth knowledge in the profession, or even as actual practice not systematically described in writing or orally. Until now, no systematic treatment has been published.

Nearly all the basic ideas presented in this book have been developed elsewhere. To say exactly where they were developed would, however, require a major piece of research. From such inquiries as we have made into the origin of the important ideas in acceptance inspection, one notable fact stands out: nearly all roads seem to lead toward the Bell Telephone Laboratories. Acceptable quality level (AQL), lot tolerance percent defective (LTPD), producer's risk (α), consumer's risk (β), average outgoing quality (AOQ), average outgoing-quality limit (AOQL), reduced inspection, tightened inspection, double and multiple sampling—to mention only a few—are among the ideas at the core of modern sampling inspection: and all of these concepts were introduced by Bell Telephone Laboratories engineers and mathematicians.*

The Army Ordnance Department, with which several of the key people from the Bell Laboratories worked during the war, has also made heavy contributions to sampling inspection; and these have greatly influenced both industry and other branches of the armed forces, notably the Army Quartermaster Corps, the Army Signal Corps, the Navy Bureau of Ordnance, and the Navy Bureau of Ships. As Part IV brings out, the tables and procedures in this book are in large measure adapted from or based on Army Ordnance tables and procedures.†

ORIGIN OF THIS BOOK

Late in February, 1945, the Statistical Research Group, which had been working on sampling-inspection problems for the Navy Bureau of

* Including H. F. Dodge, G. D. Edwards, T. C. Fry, E. C. Molina, P. S. Olmstead, H. G. Romig, and W. A. Shewhart. Important work, particularly on multiple sampling, was done by W. Bartky in an affiliated organization, the Western Electric Company.

† The Army Ordnance personnel most directly concerned with the type of sampling covered in this book were H. R. Bellinson, G. R Gause, L. W. Shaw, L. E. Simon, and A. Stein.

Ordnance, the Army Quartermaster Corps, the Army Ordnance Department, and the Rocket Division (Division 3) of the National Defense Research Committee, was asked by the Navy to prepare a manual on sampling inspection, including tables, procedures, and principles. The request was for a comprehensive treatment of various types of sampling inspection, based on both attributes and variables, but it was agreed that the manual on attributes should be prepared first. The attributes manual was completed, reproduced, and distributed just ninety days after the work started; preparation of sampling tables and procedures based on variables was well under way by Aug. 10, 1945 but was abandoned then.

The Navy manual was written for use by a Government agency for acceptance sampling only. In preparing it, however, the Statistical Research Group assembled a wide range of material from its own experience in sampling inspection and from the work of many other groups, including those referred to above. The present book presents, in a form suitable for the peacetime needs of industry, sampling procedures for use in either acceptance inspection or process control. The material of the Navy manual has been expanded in four directions: the addition of new sampling plans; the inclusion of a good deal of supplementary information about the basic sampling plans; a considerable amount of new material on process control; and an account of the principles underlying the preparation of the tables and procedures.

AUTHORSHIP

So many members of the Statistical Research Group contributed directly or indirectly to the preparation of this volume that its authorship can only be ascribed to the group as a whole. Preparation of the volume was possible only because of the large and varied amount of practical work done in the group on acceptance inspection and related problems; yet some of the individuals who participated most extensively in such work and whose ideas and experience are reflected in the book did none of the actual writing or editing. Thus, the book is in a real sense a group product.

The manuscript was prepared by H. A. Freeman, Milton Friedman, Frederick Mosteller, Leonard J. Savage, David H. Schwartz, and W. Allen Wallis. The work was planned, directed, and reviewed by the editors listed on the title page; Mr. Friedman bore the main burden of planning the Navy manual, of integrating the contributions of others, and of finally editing the entire manual, and Mr. Freeman has borne the corresponding responsibilities for this volume. Part IV is almost entirely the work of Mr. Friedman, Chaps. 6 and 13 are primarily the work of Messrs. Mosteller and Schwartz, and the final revisions of Part III, to which all the authors contributed, have been carried out in large part by Messrs. Friedman and Schwartz. The index was prepared by Messrs. Freeman and Mosteller. For all parts of the book, however, the editors have shared jointly the final authority and responsibility.

Invaluable professional assistance with the tables and procedures was rendered by Harriet Levine, Myra Levine, and Edward Paulson. In addition to assistance too diverse to mention specifically, Harriet Levine was responsible for much of the mathematical analysis in Sec. 3 of Chap. 17 and prepared the first draft of that section; Myra Levine supervised the derivation of the acceptance and rejection numbers for the sequential-sampling plans, prepared the first draft of Sec. 2 of Chap. 17, and carried through most of the revisions to the tables of the Navy manual for Part V of this book; and Edward Paulson organized and supervised the computations on which Tables 2, 3, and 4 are based.

Of those within the Statistical Research Group whose ideas and experience influenced the work, Kenneth J. Arnold, Churchill Eisenhart, and M. A. Girshick deserve explicit mention.

COLLABORATION

The Navy manual was prepared in close collaboration with sampling-inspection personnel of the Navy; in fact, two preliminary editions were prepared and discussed in detail with Navy representatives. Lieutenant Commander John H. Curtiss of the Bureau of Ships made numerous valuable suggestions, many of which are reflected in the book; in particular, the Appendix to Part II is an adaptation of material prepared by him, and the Supplement to Table 3 was suggested by him and his colleague, Lieutenant Commander Joseph F. Daly. Lieutenant Harris A. Squire of the Bureau of Ordnance also contributed substantially at many points. Lieutenant Commander James R. Slocum of the Office of Procurement and Material, who had immediate responsibility for the Navy manual, gave useful advice on the administrative aspects of inspection, as did his associates, Lieutenant Commander W. Frank Persons and Lieutenant Earl Brooks. Commander Bruce S. Old, Captain Robert D. Conrad, and Rear Admiral J. A. Furer of the Office of Research and Inventions were instrumental in initiating the work, and Lieutenant Mary E. Wallace in facilitating it. Commodore Augustus J. Wellings of the Office of Procurement and Material was in general charge of preparing the Navy manual.

ACKNOWLEDGMENTS

Thanks are due Warren Weaver, Chief of the Applied Mathematics Panel, for his leadership and assistance in all aspects of the Statistical Research Group's work, including in particular the preparation of the Navy manual. The present volume would probably have been impossible, and certainly would have been most difficult of accomplishment, without the indefatigable, efficient, and unstinting assistance of Mina Rees of the Applied Mathematics Panel. S. S. Wilks of the panel likewise facilitated the work greatly. George B. Pegram, chairman of the Columbia University Committee on War Research, gave encouragement

and material assistance which was important in launching and carrying through the project. Harold Hotelling, as Official Investigator for the Statistical Research Group contract, also encouraged and inspired the work.

The entire staff of the Statistical Research Group willingly worked long hours, sometimes under adverse conditions, in order to complete the Navy manual in so short a time. Albert H. Bowker supervised the computing staff, and Eula Blair the clerical and secretarial staff.

The Navy, in addition to collaborating in the preparation of the Navy manual, provided a great deal of practical information about the administrative aspects of inspection. At a five-day conference in Hershey, Pa., in June, 1945, inspection authorities from all naval inspection districts throughout the country and from technical bureaus concerned with procurement discussed the Navy manual in detail, contributing several valuable suggestions. Discussion with the Army Quartermaster Corps Inspection Service, both in connection with its own work and in connection with the Navy manual, proved of great value, as did similar discussions with the staffs of the Army Ordnance Department, the Army Signal Corps, the Navy Bureau of Ordnance, the Navy Bureau of Ships, and the Bell Telephone Laboratories. Many contacts with quality-control work in industry, especially in connection with sequential sampling, proved useful in preparing this book.

The double-sampling plans in AQL class 6.4 to 8.5 percent defective for sample-size letters B, C, D, E, F, J, and K, and their OC curves, were computed by the Quartermaster Corps Inspection Service.

The figures, except those of Table 4, were drawn by H. Irving Forman. The figures of Table 4 were drawn by the Navy, except those for AQL class 6.4 to 8.5 percent defective, which Mr. Forman drew. We are indebted to the Publications Division in the Office of the Secretary of the Navy for permitting us to use these figures, which were drawn under our supervision and from our data, and we are especially grateful to Robert H. Perry, Jr. of the Treasury Procurement Division for securing this permission.

The publisher has absorbed the extra costs incurred by the use of four colors in the charts. Royalties, at usual rates, are being paid to the United States Treasury; no royalties are being paid to Columbia University or to any private institution or individual. The publisher has agreed that ten years after the date on which this volume is issued the copyright thereon shall be relinquished and the work shall become part of the public domain.

W. ALLEN WALLIS
Director of Research
Statistical Research Group

CHICAGO, ILL.,
December, 1946.

ACCURACY OF CHARTS

The accuracy of the operating-characteristic curves of Table 4 should be sufficient for practical purposes. It is, however, limited by the following factors:

1. *Mathematical approximations were used instead of precise formulas in some of the computations underlying the curves.* These approximations are explained in Chap. 17, especially Secs. 1.2 to 1.4, pages 184 to 187.

2. *Only about 12 to 15 points were computed for each curve, the course of the curve between these points being determined by the draftsman.* The draftsman's plotting of the computed points was checked against the computation sheets by a statistician who also reviewed the smooth curves passed through these points. For single sampling, the number of computed points was typically about 14, ranging from 12 to 17; for double sampling, it was typically about 12, ranging from 7 to 22; and for sequential sampling there were 15 for each curve.

3. *Limitations on the accuracy of four-color printing introduce errors in the percentage of submitted lots accepted, as read from the vertical scales of the charts.* These errors are usually within the range ± 3; for example, if the computed percentage is 90, the chart will nearly always read between 87 and 93. More specifically, when the percentage of submitted lots accepted is in the neighborhood of 90, the error (deviation of the chart reading from the computed value) has a standard deviation of about 1.1; in the neighborhood of 50 the standard deviation is about 1.5; and in the neighborhood of 10 the standard deviation is about 1.0. These statements are based on 360 measurements made on a sample selected from the finished sheets for the first printing of this book just prior to binding. The curve readings were equally distributed among the three levels of percentage of submitted lots accepted (10, 50, and 90) and among the three types of sampling (single, double, and sequential), were all made on charts having vertical scales about 3 in. long, and were compared with the computed values that had been given the draftsman. Only 5 of the 360 measurements showed errors as large as 3.0; the maximum error was 4.2.

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Part I

INTRODUCTION

CHAPTER 1

NATURE, PURPOSES, AND SCOPE OF THIS BOOK

1. NATURE AND USES OF SAMPLING INSPECTION

Evaluation of the quality of product by inspecting some but not all of the product is called "sampling inspection." Sampling inspection is widely applicable as a method for determining the quality and acceptability of product. It is almost always possible to use sampling inspection for this purpose, and if inspection is destructive or costly sampling inspection may be the only feasible method.

The four principal uses of sampling inspection in industry are

1. To determine the quality and acceptability of incoming raw material, incoming semifinished product, and incoming finished product.
2. To determine the quality and acceptability of outgoing product.
3. To determine the acceptability, for further processing within the plant, of semifinished product made in the plant.
4. To improve and control the quality of product.

2. PURPOSES OF THE BOOK

One purpose of this book is to explain the general principles of sampling inspection as a method for determining the quality and acceptability of material and as a method for controlling the quality of material. A second purpose is to describe in detail a standard sampling-inspection procedure that may be readily applied to a wide variety of products. A third purpose is to provide an extensive catalogue of sampling plans, usable both by those who adopt the standard procedure and by those who construct tailor-made procedures.

The book is intended for technical, production, contracting, quality-control, and inspection authorities who are concerned with the planning, installation, operation, and review of inspection plans and with the improvement and control of the quality of product. An effort has been made to give them an understanding of the principles and practical aspects of sampling inspection and to delineate their responsibilities in sampling inspection. The book is also intended for use in training programs in sampling inspection and quality control.

It is not particularly intended for inspectors of items (line inspectors). They are not usually concerned with sampling inspection as a whole, but only with the *operation* of specific sampling-inspection plans; so for line

inspectors considerably briefer and simpler instructions are not only sufficient but preferable.

3. SCOPE OF THE BOOK

3.1. Possible Measures of Quality

3.1.1. Quality of a Lot.—Sampling plans are used in inspection to accept or reject product submitted in batches or groups called "lots"; efficient sampling plans will generally accept lots of high quality and reject lots of low quality. Whenever sampling plans are used to reach decisions on lots of product, it is clearly necessary to decide what shall be meant by high quality and low quality, or, more generally, to decide how the quality of a lot shall be described.

There are many ways of describing the quality of a lot of product with respect to the characteristic or characteristics being inspected; the following are perhaps the most common:

a. The average of the characteristic among the items in a lot, the characteristic being measured along a scale (arithmetic mean).

b. The variability of the characteristic among the items in a lot, the characteristic being measured along a scale (standard deviation).

c. The ratio of the variability to the average of the characteristic of the items in a lot (coefficient of variation).

d. The average number of defects per item in a lot (defects per item).

e. The percentage of defective items in a lot (percentage defective).

Each of these methods gives a gradation of quality. The line between lot quality that is considered high and lot quality that is considered low depends largely on the use to be made of the product.

3.1.2. Quality of an Item.—The problem of describing the quality of an item (unit) of product is somewhat independent of the problem of describing the quality of a lot. The most common methods of describing the quality of an item are

1. By variables, that is, by measurement of some characteristic of an item along a continuous scale. *Examples:* Hardness in Rockwell units, strength in pounds, porosity in cubic feet per minute per square foot.

2. By counting along a discrete scale. *Example:* Number of loose connections per electric motor; number of flaws per linear yard of textiles; number of errors per page of bookkeeping.

3. By attributes, that is, by classification of the quality of an item into one of two classes.* *Examples:* Correctly or incorrectly assembled, strong or weak, within or outside tolerances.

* The term "attributes" is sometimes used to refer to any qualitative classification containing any number of classes. In this book, the term is used to refer to a classification containing two classes.

3.2. Measures of Quality Used in This Book

This book is concerned only with the combination *e*-3, that is, the quality of a lot is expressed by the percentage of defective items it contains and the quality of an individual item is expressed as defective or nondefective. The combination *e*-3 is very practical for most products for which sampling inspection can be used. Other methods of describing lot quality or item quality are, however, used in sampling inspection.*

3.3. Sampling Plans in the Book

3.3.1. Selection of a Plan.—Sampling-inspection plans may be tailor-made; that is, it is possible to construct special plans for particular products, such plans differing among themselves in many respects. For this purpose a thorough knowledge of Part II (Principles of Sampling Inspection for Attributes), plus some knowledge of Part IV (Construction of Sampling Tables and Standard Procedure) and of the tables in Part V is required. Many detailed questions of operation are not answered in Part II; those who prefer tailor-made plans will have to answer these questions out of their own experience in inspection and quality control.

Sampling-inspection plans may be ready-made; that is, it is possible to construct one or two standard procedures for selecting a sampling plan; these procedures may be applied to many products by persons who have only a minimum knowledge of principles and moderate experience in inspection and quality control. Such a detailed standard procedure, together with a suggested division of the responsibilities for various phases of the procedure, is described in Part III (A Standard Sampling-inspection Procedure). The standard procedure described in Part III is sufficiently flexible to meet a wide variety of quality requirements and administrative and technical conditions, and it has proved relatively simple and economical to use.

* For (*a*-1) see, for example, Harry G. Romig, *Allowable Average in Sampling Inspection*, doctoral thesis, Columbia University, New York, 1939; for (*b*-1) see B. L. Welch, "Specifications of Rules for Rejecting too Variable a Product, with Particular Reference to an Electric Lamp," *Supplement to the Journal of the Royal Statistical Society*, Vol. 3 (1936), pp. 29–48. Examples of (*a*-1), (*b*-1), (*c*-1), and (*e*-3) are given by H. A. Freeman, *Indsutrial Statistics*, Chap. V, John Wiley & Sons, Inc., New York, 1942. A sequential-sampling plan covering (*d*-2) is given by the Statistical Research Group, Columbia University, *Sequential Analysis of Statistical Data: Applications*, Sec. 7, Columbia University Press, New York, 1945. The very useful combination (*e*-1) is described in detail by the Statistical Research Group, Columbia University, *Techniques of Statistical Analysis*, Chap. 1, McGraw-Hill Book Company, Inc., New York, 1947. The combination (*e*-3) covered in this book is also discussed in Harold F. Dodge and Harry G. Romig, *Sampling Inspection Tables*, John Wiley & Sons, Inc., New York, 1944, and in Eugene L. Grant, *Statistical Quality Control*, Chaps. XIII–XV, McGraw-Hill Book Company, Inc., New York, 1946.

3.3.2. Operation of a Plan.—The *selection* of sampling-inspection plans depends on whether tailor-made or ready-made plans are to be used. But the *operation* of sampling-inspection plans described in this book is the same no matter how they are selected. The sampling-inspection plans described in this book provide for accepting or rejecting material in groups called "inspection lots" on the basis of the quality of representative items selected from each inspection lot. The operation of such a sampling-inspection plan involves

a. Formation of inspection lots.

b. Selection of a sample or of several samples from each inspection lot.

c. Inspection of the items in the sample or samples to ascertain their quality.

d. Acceptance or rejection of each inspection lot on the basis of the quality of the items in the sample or samples from that inspection lot.

These four steps are discussed at length in Parts II and III.

Part II

PRINCIPLES OF SAMPLING INSPECTION FOR ATTRIBUTES

CHAPTER 2

SAMPLING INSPECTION AS AN ACCEPTANCE PROGRAM

1. OBJECTIVES OF AN ACCEPTANCE PROGRAM

A number of items of a product are contracted for by a buyer; these items are made by a supplier subject to specifications and are submitted to the buyer for acceptance. It is almost certain that some of the items submitted are defective, though how many and which items are defective is not known. The buyer is faced with the problem of deciding which items to accept and which to reject.

A perfect acceptance program would enable the buyer to accept all nondefective items and to reject all defective items; the program would encourage and assist the supplier to improve the quality of his product; the program would be easy to administer and economical in cost. This ideal will seldom, if ever, be achieved in practice. It serves to illustrate, however, the objectives in terms of which any acceptance program should be judged.

2. USES OF AN ACCEPTANCE PROGRAM; MEANING OF TERMS

We have used the terms "buyer," "supplier," "accept," and "reject"; these terms are in part holdovers from the widespread use of sampling inspection by the Army and Navy during the war—the government as buyer, industry as supplier, and the objective of sampling inspection being acceptance or rejection of submitted product. But the uses of sampling inspection are many and the four terms named above have broad meanings in this book. The "supplier" may refer to a plant furnishing raw material, semifinished, or finished product to another plant (the "buyer," which in this book is replaced by a more general term, "receiver"); or "supplier" may refer to one division of a plant furnishing semifinished product to another division that will use this product at the next stage of manufacture.*

The terms "accept" and "reject" also have broad meanings. The result of sampling inspection is to make one of two decisions on each submitted lot of product; these decisions are commonly acceptance and

* Such use of sampling inspection along the production line may be very fruitful. It is almost certain that there will be some defective items on which further operations either cannot be performed or would be wasted if performed. The producer is faced with the problem of judging the work of the department submitting the product and deciding whether the product should be passed on for further work.

rejection, but broader interpretation (such as replacing rejection by acceptance at reduction in price) is in order. The terms acceptance and rejection may also be used as judgments of the process of manufacture.

3. TYPES OF ACCEPTANCE PROGRAMS

3.1. Introduction

A method of procuring high-quality product is to purchase only from suppliers who have in the past consistently supplied high-quality product or, within the plant, to use only top-grade personnel, machines, and materials; but clearly this does not constitute a complete acceptance program in itself. Further steps are necessary to maintain the quality of product produced and accepted. While these may take many forms, three are of special interest because of their widespread use: process inspection, screening, and lot-by-lot sampling inspection.

3.2. Process Inspection

Process inspection, that is, studying the manufacturing process and spot-checking the material at various stages of production from the introduction of raw material to the completion of the product, can be one of the most reliable acceptance programs. Process inspection is almost always a desirable component of internal production control. It is often not feasible, however, as, for example, with incoming material. Further it does not provide any systematic evaluation of the quality of items actually produced. It can, perhaps, be of greatest value as a means of following up the results obtained from the use of sampling-inspection plans at several stages of the production process rather than as a substitute for such plans.

3.3. Screening

It is often necessary to inspect each submitted item of product, rejecting all defective items. Such 100 percent inspection accompanied by rejection of all defective items is called "screening." Screening is the only program that can possibly guarantee the rejection of all defective items and only defective items. Furthermore, such perfection can be approached only if suitable automatic machines are available to do the inspecting or the inspector is not subjected to a load so excessive as to impair his accuracy.* Suitable automatic machines are rarely available, so screening is ordinarily expensive in personnel and time; moreover,

* A statistical study made at Frankford Arsenal, Philadelphia, indicates very clearly that the percentage of items classified erroneously increases sharply with the volume of inspection.

screening will rarely, in practice, eliminate all defective items. Obviously if the quality test for each item is destructive, screening is out of the question.

Screening, by itself, is not likely to contribute much to the improvement of the quality of product submitted in the future; this is particularly true when the supplier and receiver are independent plants and the receiver is carrying out acceptance inspection. Under screening, the supplier has only defective items returned to him; all nondefective items are accepted. He therefore has little incentive to improve the quality of the product or to cull out defective items himself. In effect, the receiver is doing part of the supplier's work for him, since it is the responsibility of the supplier to deliver satisfactory product.

3.4. Lot-by-lot Sampling Inspection

3.4.1. Description of Lot-by-lot Sampling Inspection.—In many cases, it is not necessary to screen the product, and an adequate control of quality may be attained by subjecting the product to lot-by-lot sampling inspection. In screening, all submitted items of product are inspected, and each item is accepted or rejected; in lot-by-lot sampling inspection, only some of the submitted items of product are inspected, and groups of submitted items called inspection lots are accepted or rejected as a whole. In lot-by-lot sampling inspection, the product is divided into appropriate inspection lots (which may or may not be exactly the number of items in a container, or the number of items produced in a day, or the number submitted by the supplier at a given time); one sample (that is, a collection of items drawn at one time) or several samples are drawn from the inspection lot, and the inspection lot is accepted or rejected according to the number of defective items found in the sample or samples.

3.4.2. Risks in Lot-by-lot Sampling Inspection.—No sampling plan can assure the complete separation of defectives from nondefectives among the submitted items, for not all of the items are inspected. Accepted inspection lots may contain some defective items; rejected inspection lots may contain some nondefective items. Moreover, sometimes a low-quality inspection lot (that is, an inspection lot containing many defective items) will be accepted and sometimes a high-quality inspection lot will be rejected, for while good sampling plans prevent such errors from being made often, it must be remembered that samples obey the laws of chance and, therefore, will occasionally be of a quality that is far out of line with the quality of the inspection lots from which they were drawn. When these risks are balanced against the great saving in cost, sampling inspection will generally be preferred to screening. If the product can be tested only destructively, sampling inspection is necessary and will still provide protection against accepting more than a small number of low-quality

inspection lots and against rejecting more than a small number of high-quality inspection lots.

3.4.3. Effect of Lot-by-lot Sampling Inspection on Quality of Product. The acceptance or rejection of an entire inspection lot on the basis of the quality of a sample drawn from that inspection lot provides a strong incentive for the supplier to improve the quality of his product. If large inspection lots are even occasionally rejected, the supplier may find much of his profit (budget, if the supplier is a division of the plant) eaten up by screening, reworking, or in scrap. The way to reduce the number of inspection lots rejected and thereby reduce these costs is to submit product of higher quality.

Both sampling and screening provide information that, if properly acted upon, helps to improve the quality of submitted product. The recording and processing of this information are neither difficult nor unduly time-consuming, and it is the experience of those who have used sampling inspection plans that such recording and processing pay dividends. Slight information can be valuable, as is illustrated by the following example. Inspection records showed that the number of incorrectly labeled assemblies was increasing. A short investigation revealed that newly hired operators had been poorly instructed in labeling requirements; this situation was remedied before serious trouble could occur.

CHAPTER 3

PROPERTIES OF SAMPLING-INSPECTION PLANS

1. EXAMPLES OF SAMPLING-INSPECTION PLANS

1.1. Introduction

There are many kinds of sampling-inspection plans. The plan that is best for one product and supplier may not be best for another. The chief problem in sampling inspection is to choose the plan that is best suited to the product and supplier in question. The suitability of a plan can be judged only in terms of its properties: the assurance given by the plan that inspection lots of high quality will be accepted and inspection lots of low quality rejected, the amount of inspection required by the plan, the extent to which the plan encourages the supplier to improve the quality of product submitted, the ease with which the plan can be administered and operated: Accordingly, this chapter gives examples of different sampling plans and describes their properties.

The many kinds of sampling-inspection plans may be grouped into three types, which are here called "single," "double," and "sequential" plans. Each type is illustrated below; the three types of sampling plans and their relative merits are discussed in detail in Chap. 4. Examples I and II are single-sampling plans, Example III is a sampling program containing many single-sampling plans, Example IV is a double-sampling plan, and Example V is a sequential-sampling plan. These examples will be used to illustrate many points throughout the rest of this book. For simplicity, the examples refer to inspection for a single defect. However, the principles they illustrate and the standard sampling-inspection procedure given in Part III of this book are general and apply to inspection for any number of defects. In these examples, all samples are assumed to be drawn at random from inspection lots that are not too small relative to the sample.

1.2. Single Sampling

Example I: Inspection lots of 3,400 items are formed, and from each inspection lot the inspector draws a sample of 225 items. Each of the 225 items is subjected to a quality test. If 14 or fewer items fail the test, the inspection lot is accepted; if 15 or more items fail the test, the inspection lot is rejected.

Example II: Inspection lots of 100 items are formed, and from each inspection lot the inspector draws a sample of 2 items. Both items are subjected to a quality test; if both pass the test, the inspection lot is accepted; if either or both items fail, the inspection lot is rejected.

Examples I and II are called "single-sampling plans" because only a single sample is drawn from each inspection lot. The acceptance or rejection of the inspection lot depends on the number of defective items in the single sample. Such a single-sampling plan is completely described by its inspection-lot size, sample size, acceptance number, and rejection number. In Example I the inspection-lot size is 3,400, the sample size is 225, the acceptance number is 14, and the rejection number is 15. In single-sampling plans the acceptance number is always one less than the rejection number; thus a definite decision on the inspection lot is always reached by the time all items in the sample have been inspected. In Example II the inspection-lot size is 100, the sample size is 2, the acceptance number is 0, and the rejection number is 1.

From the discussion that follows it will be apparent that the plan in Example I is an appropriate plan for some purposes. On the other hand, it will be shown that, while the plan in Example II will accept most inspection lots containing a low percentage of defectives, it cannot be depended on to reject inspection lots containing a fairly high percentage of defectives. Under the plan in Example II, as is shown below, inspection lots in which as many as 29 of the 100 items are defective are more likely to be accepted than rejected.

Example III: Inspection lots of various sizes are formed and from each inspection lot the inspector draws a sample consisting of 10 percent of the items in the inspection lot. Each item in the sample is subjected to a quality test. If 2 percent or fewer of the items in the sample fail the test, the inspection lot is accepted; otherwise, the inspection lot is rejected.

In Example III, the sample size and the acceptance and rejection numbers cannot be determined until the inspection-lot size is known. Example III illustrates a kind of procedure that has been widely used for selecting a single-sampling plan for each inspection-lot size.

The simple way in which Example III covers all possible inspection-lot sizes is deceptive. If an inspection lot contains 10 items, the sample consists of only 1 item. Inspection lots of 10 items containing 5 defective items (50 percent defective inspection lots) have just as much chance of being accepted as of being rejected, for the probability is ½ that a single item drawn from each such inspection lot is defective. On the

other hand, an inspection lot of 10,000 items requires a sample of 1,000 items, and the inspection lot is accepted if 20 or fewer defective items are found in the sample. Inspection lots of this size containing only 2.1 percent defective items are rejected more often than accepted, because a sample of 1,000 items from such an inspection lot is more likely to contain 21 or more defective items than 20 or fewer defective items. The great fault of such a fixed-percentage plan is that the chance of accepting an inspection lot of any given quality varies much too widely with inspection-lot size. It is often appropriate to draw somewhat larger samples from larger inspection lots, but a plan like that in Example III makes the sample size vary too much with inspection-lot size. For small inspection lots, this leads to too little inspection for adequate protection, while for larger inspection lots the expense of inspection is unjustifiably high. As soon as one understands that the protection derived from inspection depends very largely on the total *number* of items inspected from an inspection lot rather than the *percentage* of the inspection lot inspected, interest in any fixed-percentage plan dies.

1.3. Double Sampling

Example IV: Inspection lots of 3,400 items are formed and the inspector draws a first sample of 150 items and inspects each of them. If the first sample contains 9 or fewer defective items, the inspection lot is accepted; if it contains 24 or more defective items, the inspection lot is rejected; if it contains more than 9 but fewer than 24 defective items, no decision is reached, and the inspector draws a second sample of 300 items. If the total of 450 items inspected in both samples contains 23 or fewer defective items, the inspection lot is accepted; if it contains 24 or more, the inspection lot is rejected. This double-sampling plan can be summarized in tabular form as follows:

Inspection-lot size: 3,400

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Double	First	150	150	9	24
	Second	300	450	23	24

As this example illustrates, the acceptance and rejection numbers for the first sample in double sampling lead to three possibilities rather than two as in single sampling. The inspection lot may be rejected, it may be accepted, or the decision may be deferred until evidence from a second

sample is gathered. If the inspection lot is neither accepted nor rejected on the basis of the evidence of the first sample, it is sure to be accepted or rejected after the second sample has been inspected, since in double sampling the second acceptance number is always one less than the second rejection number.

1.4. Sequential Sampling

Example V: The following is a sequential-sampling plan:

Inspection-lot size: 3,400

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Sequential	First	50	50	1	6
	Second	50	100	3	9
	Third	50	150	7	13
	Fourth	50	200	10	16
	Fifth	50	250	13	19
	Sixth	50	300	16	22
	Seventh	50	350	19	25
	Eighth	50	400	24	25

If this plan is used, the inspector selects from an inspection lot of 3,400 items a first sample of 50 items and inspects each of them. If the first sample contains 1 or 0 defective items, the inspection lot is accepted then and there. If it contains 6 or more defective items, the inspection lot is rejected then and there. If it contains 2, 3, 4, or 5 defective items, no decision is reached, and the inspector draws a second sample of 50 items and inspects each of them. If the total number of defective items in the 100 items examined is 3 or fewer, the inspection lot is now accepted; if the total number of defective items is 9 or more, the inspection lot is now rejected; if the total number of defective items is more than 3 but less than 9, he takes a third sample of 50 items; and so on. This process is continued until a decision is reached, which will be after the eighth sample at the latest. The sequential-sampling plans discussed in this book all have a final sample at which the acceptance number is one less than the rejection number; so a decision is reached, at the latest, after the final sample has been inspected.

It will be shown later that the sampling plans in Examples I, IV, and V all give any particular inspection lot about the same chance of being

accepted. In spite of their greater complexity, the plans in Examples IV and V are often preferable to the plan in Example I because they ordinarily require less inspection.

2. THE OPERATING-CHARACTERISTIC (OC) CURVE OF A SAMPLING PLAN

2.1. The Relation between the Percentage of Inspection Lots Accepted and Their Quality

Each of the sampling plans in the above examples will accept some inspection lots and reject others. The chance that an inspection lot will be accepted depends on its quality. If the inspection lot contains no defective items, it is certain to be accepted by each of the plans. If it contains only defective items, it is certain to be rejected by each of the plans. If however, as is almost always the case, it contains some defective items and some nondefective items, then usually it is neither certain to be accepted nor certain to be rejected.* If a large number of identical inspection lots, that is, inspection lots containing the same number of items and the same number of defective items, are submitted, some inspection lots will be accepted and some rejected. The percentage of inspection lots accepted depends on how many defective items each inspection lot contains and on the plan used. The smaller the number of defective items in the inspection lots, the larger the percentage of inspection lots each plan will accept. Fortunately it is unnecessary to perform an extensive experiment to determine how frequently each plan will accept inspection lots of any given quality; instead, the behavior of any clear-cut plan can be accurately predicted from mathematical formulas. Once the inspection-lot size, the sample size (or sample sizes for double and sequential sampling) and acceptance and rejection numbers are known, it is possible to compute the percentage of inspection lots of any given quality that will be accepted.

2.2. Illustrative Computation of the Percentage of Inspection Lots Accepted

For the sampling plan in Example II above, the computation of these percentages is relatively easy. To illustrate: if each of a large number of submitted inspection lots has 29 defectives among the 100 items, what percentage of the inspection lots would be accepted under the plan? According to the plan an inspection lot will be accepted if neither of two items drawn is defective. Since each inspection lot contains 71 nondefective items and 29 defective items, the first item drawn will be nonde-

* Strictly speaking, if there are sufficiently few defective items (nondefective items) in the inspection lot some sampling plans will be certain to accept them (reject them). For example, if an inspection lot contains exactly 16 defective items and the lowest acceptance number of the plan being used is 23, the inspection lot will certainly be accepted.

fective for $\frac{71}{100}$ of the inspection lots. Each of *these* inspection lots still has a chance of being accepted; and from each of these (each now contains 99 items, 29 of which are defective), a second item is drawn. This second item will be nondefective for $\frac{70}{99}$ of these inspection lots. Therefore, of all inspection lots originally submitted, $\frac{71}{100} \times \frac{70}{99} = 0.502$, or 50.2 percent, will be accepted; this incidentally is the basis for the remark made earlier that inspection lots of 100 items containing 29 defective items are more likely to be accepted than rejected under the plan in Example II.

2.3. Definition of the Operating-characteristic (OC) Curve of a Sampling Plan

Such computations can be made for any percentage defective in the submitted inspection lots of 100 items, and Fig. 3.1 summarizes the results

OC Curve for Sampling Plan in Example II

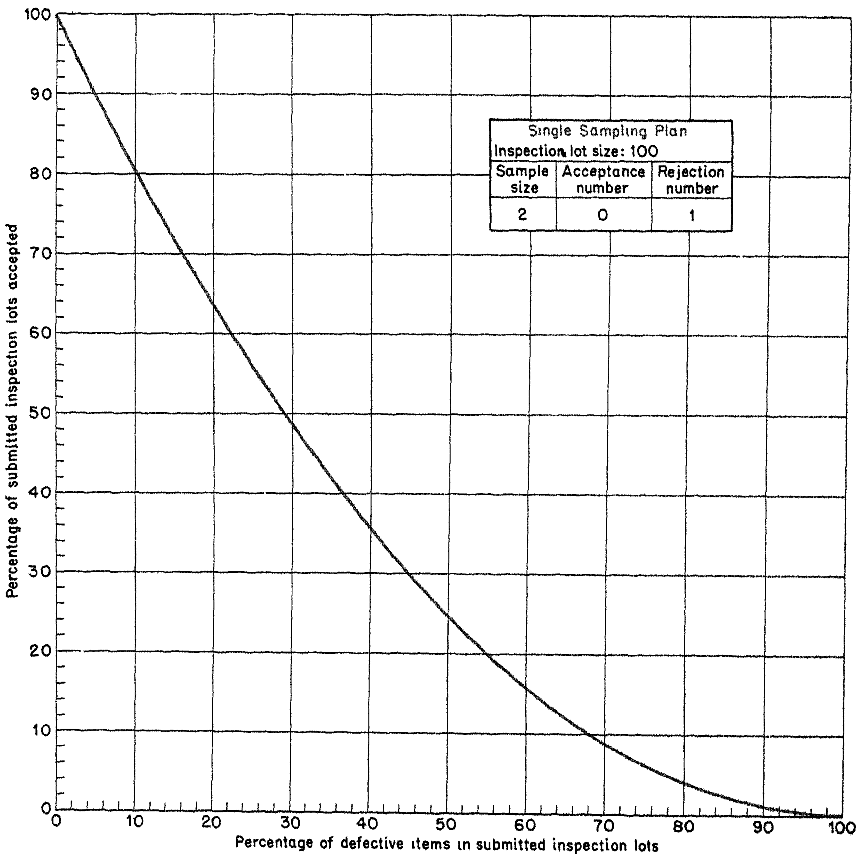


FIG. 3.1.

of these computations.* The curve in Fig. 3.1 is called the "operating-characteristic curve" (abbreviated to OC curve). Each point on it shows, for submitted inspection lots containing a specified percentage of defective items, the percentage of such inspection lots that will be accepted by the plan in Example II.

An OC curve like that in Fig. 3.1 can be computed for any sampling plan, though generally the computations are more difficult. Examination of the OC curve of a plan reveals the chance that a submitted inspection lot of any given quality will be accepted.

2.4. Effect of Inspection-lot Size on the Operating-characteristic (OC) Curve

It is no coincidence that if the same sample size (2), acceptance number (0), and rejection number (1) are used for inspection lots of 1,000 containing 29 percent defective items, that is, 290 defectives, the percentage of such inspection lots accepted turns out to be 50.4, scarcely different from the previous result. This is a very important point. When inspection lots are large compared with the sample, variation in inspection-lot size has little effect on the percentage of inspection lots accepted.†

2.5. Operating-characteristic (OC) Curves of Sampling Plans in Examples I, III, IV, and V

Figure 3.2 shows the close agreement between the OC curves of the plans in Examples I, IV, and V, reaffirming the remark made earlier that these three plans give an inspection lot of any specified quality about the same chance of being accepted.

Figure 3.3 shows how the OC curves for the plans in Example III (the 10 percent plans) vary with inspection-lot size; the inspection-lot sizes are 100, 1,000, and 10,000 items. The OC curve is considerably steeper for inspection lots of 1,000 items than for inspection lots of 100 items and steeper still for inspection lots of 10,000 items. The steepness of the OC curve of the sampling plan for inspection lots of 10,000 items shows that this sampling plan discriminates sharply between inspection lots that contain fewer than 2 percent defective items and inspection lots that contain more than 2 percent defective items; that is, it is almost certain to accept inspection lots that contain only slightly fewer than 2

* This curve should properly be represented as a series of points, but it is convenient to use a smooth curve.

† For this reason, the OC curve of a sampling plan is often given without specifying the inspection-lot size. Such OC curves are computed on the assumption that the inspection lot is large compared with the sample and are therefore only approximate when this assumption is not valid.

OC Curves for Single Sampling Plan in Example I,
Double Sampling Plan in Example IV, and
Sequential Sampling Plan in Example V

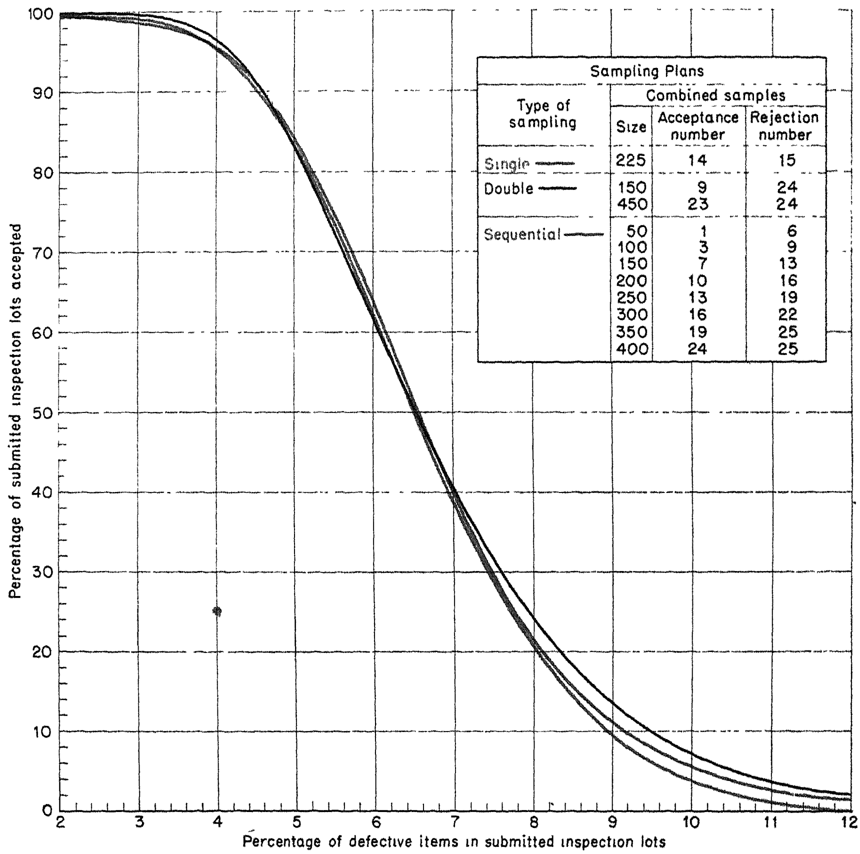


FIG. 3.2.

percent defective items and almost certain to reject inspection lots that contain only slightly more than 2 percent defective items. Such sharp discrimination is obtained for inspection lots of 10,000 items because a large sample, 1,000 items, is used. Such a large sample gives sharp discrimination whatever the size of the inspection lot or whatever the percentage of the inspection lot contained in the sample.

2.6. Ideal Operating-characteristic (OC) Curve

A sampling-inspection program might be considered ideal if for a given inspection-lot size it were able to discriminate perfectly between inspec-

tion lots containing, say, 2 percent or fewer defectives and inspection lots containing more than 2 percent defectives. Such a plan would have the

OC Curves for 10% Sampling Plans in Example III for Inspection Lots of 100, 1,000, and 10,000 Items

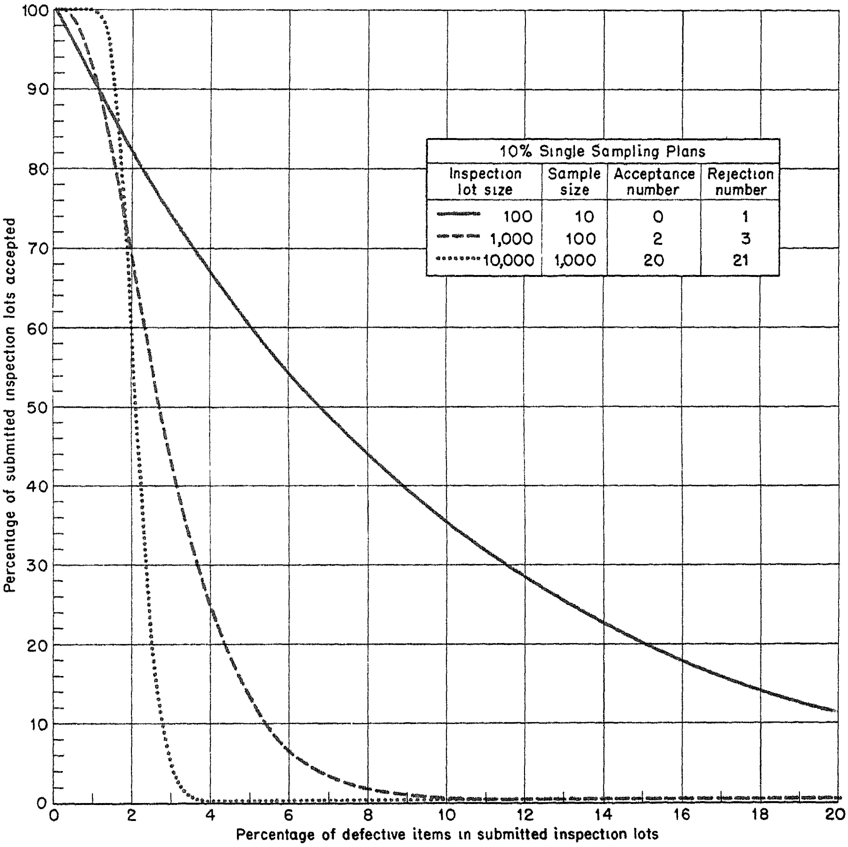


FIG. 3.3.

OC curve of Fig. 3.4.* There is only one way to obtain this OC curve, and that is to do 100 percent inspection. However, this OC curve is

* That such a plan is ideal implies that the user is able to select a precise level of quality (2 percent defective) that shall separate acceptable and nonacceptable lots and that he has good reason to want to accept *all* inspection lots containing 2 percent or fewer defectives and *no* inspection lots containing more than 2 percent defectives. The user will not often know at what level of quality he wants such sharp discrimination, and consequently he may be indifferent if offered a choice between a steep and a perpendicular OC curve. Hence, it is impractical to attach much importance to this ideal OC curve.

closely approached when the sample size is very large (not necessarily near the inspection-lot size), for example, when a sample of 1,000 items is drawn from an inspection lot of 10,000 items. Such a close approach will often involve an unjustifiable amount of inspection.

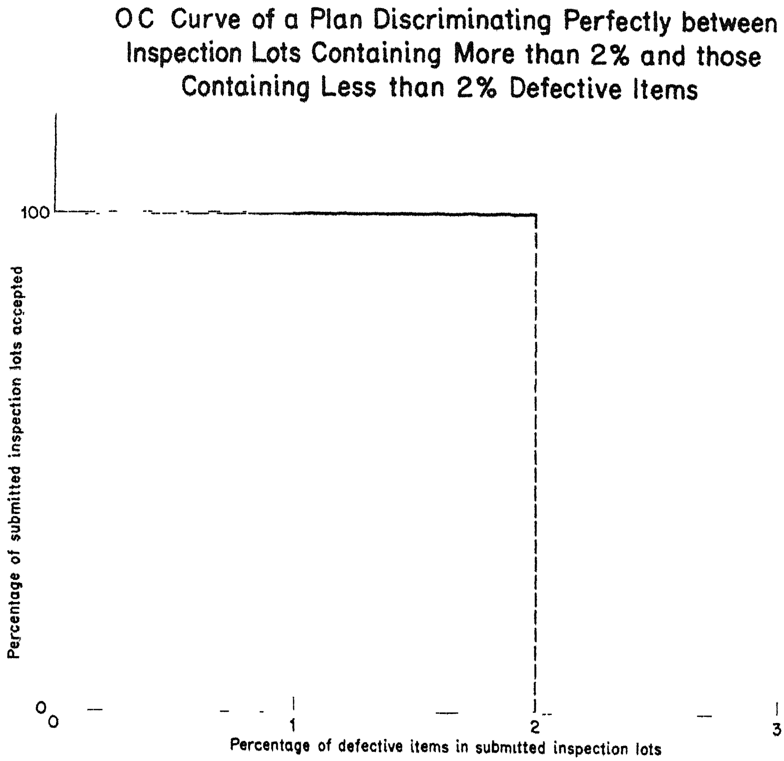


FIG. 3.4.

2.7. Classifications of Sampling Plans and Their Operating-characteristic (OC) Curves by Acceptable-quality Level (AQL) and Lot Tolerance Percent Defective (LTPD)

OC curves have been computed for many sampling plans. Convenient collections of sampling plans and their OC curves are available in this book and elsewhere.* These collections frequently classify sampling plans by one or two key points on their OC curves. For example, one way in which sampling plans are classified in this book is by the acceptable-quality level (AQL). The AQL of a sampling plan is used in this book to mean a quality of product (expressed in percent defective)

* See, for example, Army Service Forces, Ordnance Department, Industrial Service, *Ordnance Inspection Handbook on Standard Inspection Procedures, Quality Control*, Ord-M608-8, Issue 2, 3 February 1944.

such that the sampling plan will result in the acceptance of 95 percent of submitted inspection lots of that quality.* Another point on the OC curve sometimes used to classify sampling plans is the lot tolerance percent defective (LTPD); the LTPD of a sampling plan is used in this book to mean a quality of product (expressed in percent defective) such that the sampling plan will result in the rejection of 90 percent of submitted inspection lots of that quality.

2.8. Effect of Inspection Errors on the Operating-characteristic (OC) Curve

In even the best run inspection departments, some defective items are classified nondefective, and some nondefective items are classified defective. Since the OC curves of sampling plans are computed under the assumption that there are no errors in classification, it is worth while to consider the effects of errors in classification.

Suppose that the probability of a nondefective being misclassified as defective is p_1, and that of a defective being misclassified as nondefective is p_2. For example, $p_1 = 0.005$ and $p_2 = 0.020$ means that out of 1,000 nondefectives the inspector calls 5 items defective, on the average, while out of 1,000 defectives he calls 20 nondefective, on the average. Now if the true proportion defective is $p = 0.03$, in the long run the inspector will classify as defective the proportion

$$p_1(1 - p) + p(1 - p_2)$$

which is the proportion of the lot that is nondefective but incorrectly classified as defective, plus the proportion of the lot that is defective and correctly classified as defective.

In the example under consideration he classifies as defective the proportion $0.005 \times 0.97 + 0.03 \times 0.98 = 0.03425$. In other words, the effect of the errors in classification is the same as changing the percent defective in the lot from 3.0 to 3.4. Consequently this material, when examined by this inspector, has less chance of being accepted than the OC curve indicates for submitted lots having 3.0 percent defectives; in fact, its chance of acceptance is that shown by the OC curve for submitted lots having 3.4 percent defectives. In general, the effect of errors in classification is that an inspection lot with $100p$ percent defective will be treated by the sampling plan as if it had $100[p_1(1 - p) + p(1 - p_2)]$ percent defective.

When p is small and p_2 is not too large (say less than 0.10) this effective percent defective can be approximated by $100(p_1 + p)$. This means

* In some other collections of tables, including the one in the publication cited in the previous footnote, the term "acceptable-quality level" has a slightly different meaning.

that the error of classifying nondefective items as defective may constitute nearly the whole change from the true percent defective to the effective percent defective. In the example given above this approximation works very well, indicating an effective percent defective of 3.5 percent instead of the 3.425 percent given by the more careful calculation. If the inspection department has a policy that practically never misclassifies a nondefective item, the effective percent defective will become $100p(1 - p_2)$. Undoubtedly p_2 will increase as a result of this policy, so the chance of rejecting submitted inspection lots of poor quality will decrease. On the other hand, the chance of accepting submitted inspection lots of good quality will increase. The moral is: keep *both* p_1 and p_2 as small as possible, but especially p_1.

3. DISPOSITION OF REJECTED INSPECTION LOTS AND EFFECT OF DISPOSITION ON QUALITY

3.1. Quality of Product Accepted by the Sampling Plan

A sampling plan alone cannot guarantee that the quality of product finally accepted will be high. The quality of product finally accepted depends on the quality of product submitted and on the method used to dispose of inspection lots that have been rejected on the basis of a sampling plan, as well as on the sampling plan itself.

Examination of the OC curves of sampling plans reveals that even if the submitted inspection lots are of low quality a small percentage of them will be accepted. If only low-quality inspection lots are submitted by the supplier, there will be nothing for the sampling plan to accept except low-quality inspection lots. If some of the submitted inspection lots are of higher quality than others, the sampling plan will accept a larger percentage of the higher quality inspection lots than of the lower quality inspection lots. Thus when inspection lots vary in quality, the average quality of the inspection lots accepted will be higher than the average quality of the inspection lots submitted. This superiority in the average quality of inspection lots accepted over the average quality of inspection lots submitted is the only direct quality-improving effect gained from the acceptance or rejection of inspection lots on the basis of a sampling plan.*

3.2. Disposition of Inspection Lots Rejected by the Sampling Plan

The disposal of inspection lots rejected by the sampling plan offers new possibilities for improving the quality of product finally accepted; the rejected inspection lots may be disposed of in such a manner as to induce

* In addition, realization by the supplier that good lots will generally be accepted while bad lots will generally be rejected (in short, that the sampling plan is efficient) usually has a profound effect on the quality of product that he will submit in the future.

the supplier to improve the quality of his submitted product. For example, he may be required to correct each rejected inspection lot so that it is converted into a perfect inspection lot; this will set an upper limit on the *average* percentage of defective items in the product finally accepted. Other methods of disposing of rejected inspection lots include acceptance "as is" at reduced price and outright refusal, which may lead to sale in other markets or to scrapping; obviously the former is not relevant if the inspection program deals with semifinished product within the plant. In any case, if the rejection of submitted inspection lots is to provide the supplier with an incentive to improve quality, such rejection should always involve a penalty to the supplier.

3.3. Effect of Disposition of Rejected Inspection Lots on the Quality of Product Accepted

The action taken on rejected inspection lots affects the quality of product finally accepted by the receiver. Suppose, to take an extreme example, that the supplier persistently submits inspection lots containing the same percentage of defective items and that inspection lots rejected on the basis of the sampling plan are never resubmitted to the receiver; that is, they are otherwise disposed of. Under a sampling plan a certain percentage of the submitted inspection lots will be accepted. The inspection lots accepted will all have the same percentage of defective items as the inspection lots submitted, and, consequently, the inspection lots accepted will be of the same quality as the inspection lots submitted.* Which inspection lots are accepted and which are rejected will of course be determined by chance. This example shows that a sampling plan cannot by itself guarantee that the accepted inspection lots will have an average percentage defective less than any fixed quantity.

In practice, submitted inspection lots vary in quality from inspection lot to inspection lot, and the quality of the inspection lots accepted on the basis of the sampling plan will be better, on the average, than that of the submitted inspection lots. This is illustrated by the following example. Suppose that 1,000 inspection lots containing 2 percent defective items,

* If, as assumed above, identical inspection lots are submitted, each containing the same number of items and the same number of defective items, the inspection lots accepted will be those in which relatively few of the defective items in the inspection lots happen to be included in the samples, and the inspection lots rejected will be those in which relatively many of the defective items happen to be included in the samples. If, further, the defective items in the sample are always discarded or replaced by nondefective items, the inspection lots rejected will be of *better* quality than those accepted, since more defective items are removed from the rejected inspection lots than from the accepted inspection lots, while all inspection lots initially contained the same number of defective items. This paradox depends on the highly unrealistic assumption that all inspection lots submitted contain exactly the same percentage of defective items.

1,000 inspection lots containing 4 percent defective items, and 1,000 inspection lots containing 8 percent defective items are submitted for acceptance, that all inspection lots are the same size, and that the sampling plan in Example I is used. The OC curve of Fig. 3.2 indicates that practically all the inspection lots containing 2 percent defective items will be accepted, about 960 of the inspection lots containing 4 percent defective items will be accepted, and only about 210 of the inspection lots containing 8 percent defective items will be accepted. The average quality of the submitted inspection lots is

$$\frac{(1{,}000 \times 2 + 1{,}000 \times 4 + 1{,}000 \times 8)}{3{,}000} = 4.67 \text{ percent defective}$$

while the average quality of the inspection lots accepted on the basis of the sampling plan is

$$\frac{(1{,}000 \times 2 + 960 \times 4 + 210 \times 8)}{2{,}170} = 3.47 \text{ percent defective.}$$

To simplify this example, the quality of the inspection lots has been confined to three values (2, 4, and 8 percent defective). In practice, the quality of submitted inspection lots will cover many more values, but the main result will be the same; that is, the quality of accepted inspection lots will, on the average, be better than the quality of submitted inspection lots. Notice that this improvement is an improvement in the average; the quality of certain accepted inspection lots may still be very low.

If a rejected inspection lot is to be resubmitted, its quality should be improved by the supplier before resubmission. If it were permissible for the supplier to submit an inspection lot time after time without improvement, a sampling-inspection plan would present little or no barrier to the acceptance of low-quality product. Suppose an inspection lot is of such low quality that it has only one chance in two of being accepted each time it is submitted. If the supplier were permitted to submit it as many as three times it would have seven chances out of eight of being accepted. Obviously it would be accepted sooner or later if submitted often enough, and this is true of product of almost any quality.

3.4. Average Outgoing Quality (AOQ)

One way of setting an upper limit on the percentage of defective items in the product finally accepted by the receiver is to require that all rejected inspection lots be improved so that every defective item in the rejected inspection lot is made nondefective, either by replacement, correction, repair, or reworking. Consider how much improvement of rejected inspection lots, in conjunction with the sampling plan in Example I, will affect a stream of inspection lots of the same size, each of which contains 6 percent defective items. The OC curve of this plan (Fig. 3.2) indicates that about 620 inspection lots will be accepted out of every 1,000

submitted; this accepted product will, of course, be as defective as the incoming product, namely, 6 percent. It is now required that the 380 inspection lots that are rejected out of every 1,000 submitted be screened

Average Outgoing Quality (AOQ) after Screening of Rejected Inspection Lots: Single Sampling Plan in Example I

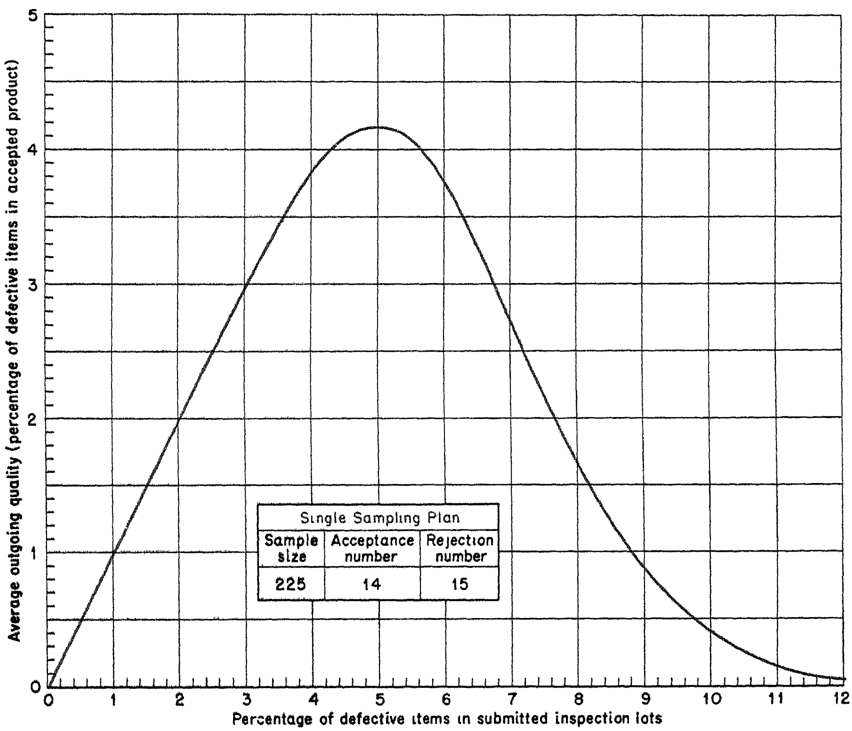


FIG. 3.5.

of all defective items and resubmitted as perfect inspection lots. The average percentage of defective items in the product accepted after this screening process will be approximately

$$\frac{(620 \times 6 + 380 \times 0)}{1000} = 3.72 \text{ percent}$$

This 3.72 percent defective is called the average outgoing quality (AOQ).*

* This standard nomenclature is unfortunately a little confusing in that a low numerical value of AOQ means high quality.

The value 3.72 percent is approximate because it does not take account of the defective items found in the samples inspected from the 620 accepted inspection lots. In practice, these defective items would generally be replaced, corrected, repaired, or reworked, still further reducing the average percentage of defective items in product finally accepted.

For every quality of incoming product, the sampling plan, together with the screening of rejected inspection lots, will in general yield an AOQ that is lower than the percent defective in the incoming product. As has just been illustrated, it is easy to compute from the OC curve the AOQ for any incoming quality. If this is done, an AOQ curve can be drawn showing the AOQ for all qualities of incoming product. Such an AOQ curve is shown in Fig. 3.5 for the sampling plan in Example I.

3.5. Average Outgoing-quality Limit (AOQL)

The AOQ curve always has a maximum value, which is called the "average outgoing-quality limit" (AOQL). This means that when a sampling plan is used together with the requirement that inspection lots rejected on the basis of the sampling plan shall be made perfect and then accepted, it is certain that the product finally accepted will, *on the average*, be no worse than some fixed percent defective, namely, the AOQL. For example, the AOQL in Fig. 3.5 is 4.2 percent.

The average quality of product accepted from processes to which a sampling plan and screening are applied may be expected to run much less than the AOQL. In principle, this might occur in either of two ways: the supplier might submit high-quality product or very low-quality product. In practice, it generally occurs in the first way; the supplier finds it worth while to establish and maintain a process such that almost all the inspection lots he submits will contain a sufficiently low percentage of defective items to pass the sampling inspection.

When screening of rejected inspection lots is required, the AOQL may provide a convenient basis for selection of a sampling plan. For this reason, one of the ways in which the sampling plans in Part V are classified is by AOQL.

4. AMOUNT OF INSPECTION

4.1. Factors on Which the Amount of Inspection Depends

The amount of inspection required by a sampling plan depends on the particular sampling plan, on the curtailing or completion of the inspection of each sample, and on the quality of submitted product.

4.2. Single Sampling

4.2.1. Inspection of Complete Sample.—In single sampling, it is often advisable to inspect every item in the sample. If this is done, the number of items inspected per inspection lot is simply the number of items in the sample.

4.2.2. Curtailed Inspection.—If inspection had no other purpose than to determine which inspection lots to accept and which to reject, it would

be feasible to stop inspection as soon as the rejection number is reached or as soon as it is known that the acceptance number will not be exceeded, since further inspection would not affect the acceptance or rejection of the inspection lot. For example, an inspector examining a sample of 225 under the plan of Example I may find 15 defectives—enough to reject

Average Amount of Inspection for Single Sampling Plan in Example (I), if the Entire Sample Is Inspected, and if Inspection Is Curtailed as soon as a Decision Can Be Reached

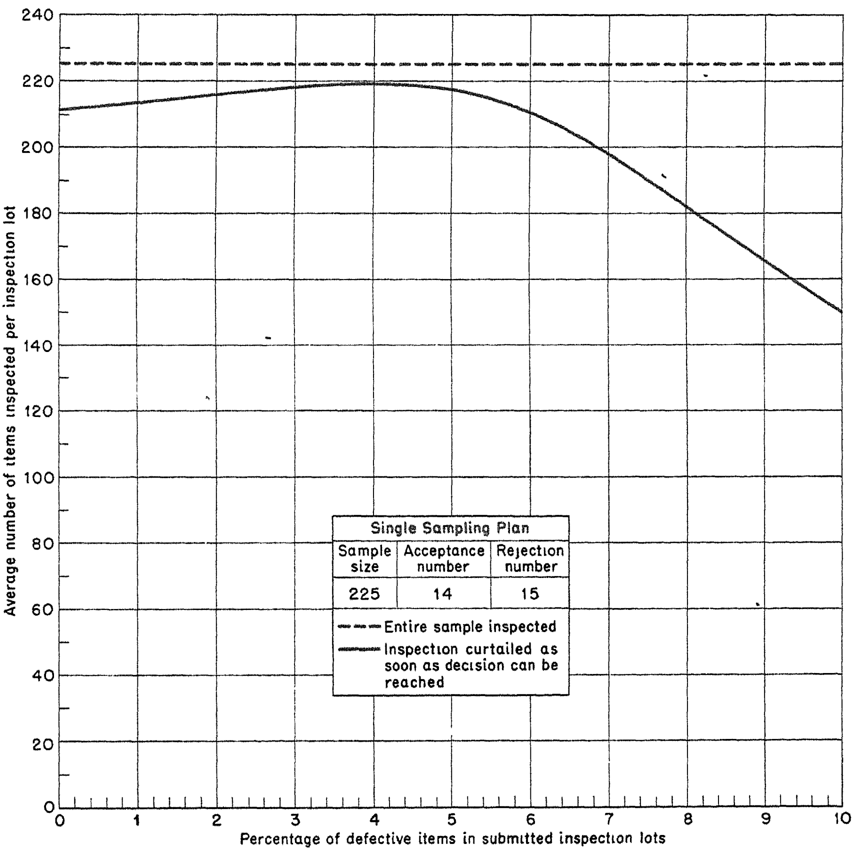


FIG. 3.6.

the inspection lot—among the first 35 items inspected. Or he may inspect 215 items and find only 4 defectives, in which case he cannot reach the rejection number 15 even if all the remaining 10 items of the sample are defective.

It may not be advisable, however, to curtail inspection in this way. The main reason is that it may be desirable to obtain information about

the quality of product, in addition to deciding whether to accept or reject each inspection lot. Knowledge of the number of defective items in the *entire* sample facilitates estimation of the quality of the inspection lot. There are formulas by which the quality of the inspection lot can be estimated under curtailed inspection, but these formulas require not only that the items be selected at random from the inspection lot but that they be inspected in a random order. Random selection is necessary for valid acceptance or rejection; but random order of inspection within samples is necessary only if the process average is to be estimated and sampling is curtailed. Ordinarily, random order of inspection within samples will not be burdensome, since it is only necessary to inspect the items in the order drawn. Sometimes, however, retention of this order (or randomizing if the order is lost) may cost more than is justified by the saving achieved from curtailing inspection.

4.2.3. Effect of Curtailed Inspection on Amount of Inspection.—When curtailed inspection is used, the average number of items inspected per inspection lot depends on the quality of the submitted inspection lots, primarily because the rejection number tends to be reached sooner for very poor inspection lots than for inspection lots of intermediate quality. This is illustrated in Fig. 3.6 for the plan in Example I. Figure 3.6 plots the average number of items inspected per inspection lot against the quality of the inspection lots submitted. The saving over uncurtailed inspection is small for inspection lots of good or intermediate quality but considerable for inspection lots of poor quality. For example, even for inspection lots containing 6 percent defective items, which, according to the OC curve in Fig. 3.2, are rejected almost 40 percent of the time, the average amount of inspection is reduced only from 225 to about 210, while for inspection lots containing 10 percent defective items, which would be rejected slightly more than 95 percent of the time, the average amount of inspection is reduced to 150, a saving of $\frac{1}{3}$.

4.3. Double and Sequential Sampling

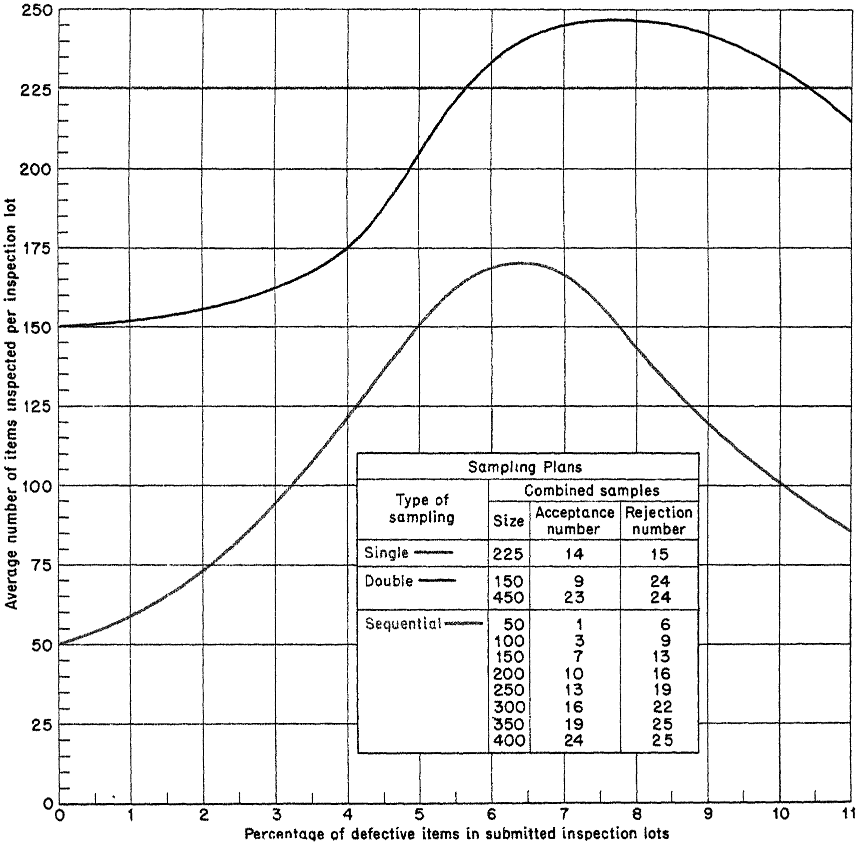
An effective way to reduce the average amount of inspection is to use a suitably chosen double- or, still better, sequential-sampling plan. In double and sequential sampling, it may be advisable to inspect the first sample in full in order to facilitate estimation of the quality of the inspection lot. It is not necessary to inspect the later samples in full for this purpose; it may therefore be advisable to curtail inspection of the later samples as soon as a decision is reached.

The saving from the use of double or sequential sampling instead of single sampling is illustrated in Fig. 3.7, in which the double-sampling plan in Example IV and the sequential-sampling plan in Example V are compared with the single-sampling plan in Example I.

Though these three plans have practically the same OC curves (see Fig. 3.2), the average amount of inspection for inspection lots of higher quality than the AQL (very close to 4 percent defective) is substantially lower for double sampling than for single sampling and lower yet for

Average Amount of Inspection for Single Sampling Plan in Example I, Double Sampling Plan in Example IV, and Sequential Sampling Plan in Example V

For Single Sampling, Entire Sample Inspected; for Double and Sequential Sampling, Entire First Sample Inspected, Inspection of Later Samples Curtailed as soon as Decision Can Be Reached



Sampling Plans

Type of sampling	Combined samples: Size	Acceptance number	Rejection number
Single ——	225	14	15
Double ——	150	9	24
	450	23	24
Sequential ——	50	1	6
	100	3	9
	150	7	13
	200	10	16
	250	13	19
	300	16	22
	350	19	25
	400	24	25

FIG. 3.7.

sequential sampling. For these high-quality inspection lots, the double-sampling plan in Fig. 3.7 requires about ¼ to ⅓ less inspection on the average than the single-sampling plan, and the sequential-sampling plan about ⅓ less than the double-sampling plan. The sequential-sampling

plan in Fig. 3.7 thus requires only about $\frac{1}{2}$ as much inspection as the single-sampling plan for high-quality inspection lots. For some intermediate-quality inspection lots, the double-sampling plan in Fig. 3.7 requires slightly more inspection than the single. The sequential-sampling plan in Fig. 3.7 always requires less inspection than the single, the minimum saving being about $\frac{1}{4}$. For poor-quality inspection lots, the saving over single sampling is greater for both the double and the sequential plans in Fig. 3.7 than for intermediate quality.

CHAPTER 4

SINGLE-, DOUBLE-, AND SEQUENTIAL-SAMPLING PLANS

1. SINGLE-SAMPLING PLANS

A typical single-sampling plan has already been introduced as Example I of Sec. 1.2 of Chap. 3. Under this plan, inspection lots of 3,400 items are formed, samples of 225 items are drawn and inspected, and the acceptance or rejection of the inspection lot is decided by determining whether the number of defective items in the sample is equal to or less than the acceptance number 14, or equal to or more than the rejection number 15. In general, a single-sampling plan is described by the size of the inspection lot, the size of the sample to be drawn from each inspection lot, and the acceptance and rejection numbers. If the size of the inspection lot is large compared with the size of the sample, the size of the inspection lot has a negligible effect on the percentage of inspection lots accepted. A given sample size and given acceptance and rejection numbers will therefore give the same OC curve for a considerable range of inspection-lot sizes.

In single-sampling plans, it may be desirable, in order to obtain adequate information on the quality of product, to inspect every item of the sample, even if the acceptance or rejection of the inspection lot is determined before the last item is inspected.

2. DOUBLE- AND SEQUENTIAL-SAMPLING PLANS

Examples IV and V of Sec. 1.3 and 1.4 of Chap. 3 illustrate double- and sequential-sampling plans. Just as a single-sampling plan may be described by its sample size and its acceptance and rejection numbers, a double- or sequential-sampling plan may be described by its sample sizes and the acceptance and rejection numbers associated with each sample. This information may be conveniently tabulated as shown in Secs. 1.3 and 1.4 for Examples IV and V.

In double- and sequential-sampling plans it may be advisable to inspect every item in the first sample in order to obtain adequate information on the quality of product but to curtail later samples whenever a rejection number is reached or an acceptance number cannot be exceeded.

3. COMPARISON OF SINGLE-, DOUBLE-, AND SEQUENTIAL-SAMPLING PLANS

No one of the three types of sampling plans—single, double, or sequential—can be categorically recommended for all products and all suppliers

and receivers. Each has its merits and each its defects, and which is best depends on the particular situation at hand. The considerations that need to be taken into account in choosing among the three types of sampling plans are solely those affecting the administration and operation of the plans and not those affecting the protection provided by the plans. The protection provided by the plans against the rejection of high-quality inspection lots and the acceptance of low-quality inspection lots (this protection is shown by the OC curves of the plans) need not be taken into account since single-, double-, and sequential-sampling plans *need* not differ significantly in their OC curves. That is, if a particular plan of one of the three types provides a particular OC curve, it is always possible to find plans of the other two types that provide almost the same curve.

Three such plans, that is, a single, double, and sequential plan having almost the same OC curve, may differ in (*a*) their sampling-inspection costs; (*b*) the amount of information they provide about the quality of the product; (*c*) the administrative costs involved in using them.

3.1. Sampling-inspection Costs

The costs of sampling inspection are of two kinds: those incurred in selecting items or in selecting samples, and those incurred in inspecting the selected items or in inspecting the selected samples. These costs sometimes depend simply on the average number of items selected and inspected per inspection lot; sometimes, however, they also depend on the number of samples, on the maximum number of items or samples that may ever be needed from an inspection lot, or on the extent to which the amount of inspection varies from inspection lot to inspection lot. The three types of sampling plans differ in all these respects.

3.1.1. Average Amount of Inspection.—The average number of items inspected per inspection lot is typically less for double sampling than for single sampling and less for sequential sampling than for double sampling. The saving from using double or sequential sampling cannot be described simply, since it depends on the quality of product submitted and on the particular plans compared. Figure 3.7 shows the saving for one set of three plans; Fig. 9.2 shows the saving for several additional sets of plans. For high-quality product, double sampling frequently requires something like $\frac{1}{4}$ to $\frac{1}{3}$ less inspection than single sampling, and sequential sampling requires something like $\frac{1}{3}$ to $\frac{1}{2}$ less inspection than single sampling.

3.1.2. Number of Samples Inspected.—In single sampling, it is necessary to inspect only one sample (generally fairly large); in double sampling, it may be necessary to inspect two samples; while, in sequential sampling, it may be necessary to inspect a series of samples (generally small) in order to determine whether to accept or reject the inspection lot.

3.1.3. Maximum Amount of Inspection.—The maximum number of items that may ever have to be inspected from any inspection lot is least for single sampling and greater for double and sequential. In the plans in this book the maximum sample size is always greater for double than for sequential sampling.

3.1.4. Variability in Amount of Inspection.—In single sampling (uncurtailed) the amount of inspection does not vary; that is, if a particular single-sampling plan is used, the same number of items must always be inspected in order to reach a decision on an inspection lot; in double and sequential sampling, the amount of inspection does vary from inspection lot to inspection lot.*

3.1.5. Summary of Differences among Plans Affecting Sampling-inspection Costs.—Differences, affecting sampling-inspection costs, among plans of the three types having almost the same OC curves are summarized below.

COMPARISON, WITH RESPECT TO SAMPLING-INSPECTION COSTS, OF SINGLE-, DOUBLE- AND SEQUENTIAL-SAMPLING PLANS HAVING ALMOST THE SAME OPERATING-CHARACTERISTIC (OC) CURVES

	Type of sampling		
	Single	Double	Sequential
Average number of items inspected per inspection lot	Most	Less	Least
Number of samples per inspection lot	One	Two	Several
Maximum possible number of items inspected from an inspection lot	Least	More	
Variability in amount of inspection among inspection lots	None	Some	

3.1.6. Circumstances Affecting Significance of Inspection-cost Differences among the Plans.—The particular circumstances under which a plan operates determine which differences among the plans have a significant effect on sampling-inspection costs. It is impossible to consider all possible circumstances; the paragraphs that follow, however, deal with a few of the more important.

When it is practicable to hold the inspection lot stationary during sampling and inspection operations, samples can be drawn as they are needed. In this case the drawing of samples will generally present no obstacle to the profitable use of double or sequential plans.

Practical considerations sometimes require, however, that all items

* For further discussion, see Statistical Research Group, *Techniques of Statistical Analysis*, Chap. 6, "Variability of Amount of Inspection for Double, Multiple, and Sequential Sampling," McGraw-Hill Book Company, Inc., New York, 1947.

that may need to be inspected be set aside in advance of inspection. This necessity arises when, for example, it is expedient to move the inspection lot immediately after the inspector has selected some items from it, whereupon the inspection lot is inaccessible to the inspector for further sampling. Under such circumstances, double or sequential sampling requires that the maximum number of items that might need to be inspected before reaching a decision on the inspection lot be drawn in advance. This number is generally considerably larger than the number of items required by a single-sampling plan with approximately the same OC curve. In many cases the necessity of selecting a larger number of items in advance is not a serious obstacle to the use of double or sequential plans, but single sampling is preferable when the cost of selecting the items and making them available to the inspector is not small compared with the cost of inspecting the items once selected and made available.

If items are inspected one or a few at a time, the cost of inspection is smaller for double and sequential sampling than for single sampling, because the average number of items inspected per inspection lot is generally smaller for these types of sampling. The saving is generally greater for sequential sampling than for double sampling. Sequential sampling may be especially profitable when the inspection of each item is costly; an outstanding example of costly inspection is the use of expensive equipment for the destructive testing of valuable items.

Sometimes the cost of inspecting items depends not only on the number of items inspected but also on the number of samples inspected. This may be so if inspection is carried out on a large group of items at once; for example, life tests on electric light bulbs by a battery of test machines. In double or sequential sampling it is not known whether to inspect another group of items until the result of the inspection of the current group is available; in cases like these, single sampling is usually desirable.

These circumstances are not, of course, equally common. The circumstance that is perhaps most common is that in which the cost of inspection depends primarily on the average number of items inspected per inspection lot, and other factors affecting sampling-inspection costs are of secondary importance. Under this circumstance, the lower average amount of inspection under sequential sampling gives it a decided advantage over the other two types of sampling.

3.2. Information about Quality

A valuable by-product of a sampling plan is the quantitative information it gives about the quality of each submitted inspection lot and therefore about the process producing these inspection lots. This information is ordinarily derived solely from the first sample drawn from each inspection lot; hence the three types of sampling differ in the amount of information they provide, since they differ in the size of their first samples.

Single sampling provides the most precise information since the single sample is larger than the first sample of a double- or sequential-sampling plan having almost the same OC curve. Double sampling provides more precise information than sequential sampling, since the first sample is considerably larger in double than in sequential sampling. These differences may often be unimportant, since information on quality can be accumulated from the first samples of several inspection lots. When it is important to have precise information about the quality of each inspection lot separately and at the same time to economize on the average amount of inspection per inspection lot, double sampling may be a useful compromise.*

3.3. Administrative Costs

Double and sequential sampling have certain administrative disadvantages. These plans are somewhat more difficult to teach to line inspectors than single sampling, and they require longer forms, which make it more laborious to enter and check the results of inspection. A more serious disadvantage is that double and sequential sampling, because of their inherently variable inspection load, require sufficiently flexible assignment of inspectors to permit the meeting of occasional peak loads.

A minor adminstrative advantage of double and sequential plans is that they are often preferred by suppliers who feel that these plans are more equitable because "they give the inspection lot a second chance." This attitude is of course erroneous, because the chance of accepting an inspection lot depends only on its quality and the OC curve of the plan being used and not on the number of samples that are associated with the sampling plan.

3.4. Summary

The following is a summary of the factors that influence the choice of the type of sampling plan:

a. Sampling-inspection Costs.—Sequential or double sampling is indicated if costs depend principally on the average number of items inspected per inspection lot; single sampling, if costs depend principally on the maximum number of items or the maximum number of samples that may ever need to be inspected from a single inspection lot.

b. Information about Quality.—Single sampling provides slightly more information than double and much more than sequential about the quality of *each* inspection lot.

* See also Chap. 11, p. 117, footnote.

c. Administrative Costs.—Single sampling is administratively simpler than double or sequential.

For most products and for most suppliers and receivers, sampling inspection costs will be most important, since information on quality can be accumulated over several inspection lots and the administrative differences among the types of sampling are rather minor.

CHAPTER 5

INSTALLATION AND OPERATION OF SAMPLING PLANS

1. THE ITEM OF PRODUCT

1.1 Determination of What Is an Item of Product

The first step in installing a sampling plan for a particular product is to determine what is to be an item or unit of product for purposes of inspection. It is frequently easy to determine what an item is; examples are a hinge, a life preserver, a vacuum tube. Even these apparently simple cases, however, suggest problems that may arise in defining an item. A receiver may obtain vacuum tubes to use as spare parts or to assemble in radio sets. If they are to be used as spare parts, a single vacuum tube is probably the relevant item for purposes of inspection. If they are to be assembled in radio sets, the group of tubes going into one set may be the relevant item. There are many products for which it is even more difficult to decide what shall constitute an item. Examples are paint, coal, paper, rope, wire, shoes, and textiles.

1.2. Classification of Defects in an Item of Product

Once it is determined what is to be an item, the next step is to list the defects an item may have, to classify them, and to prescribe the method of inspecting an item for these defects. There are several kinds of defects. Some defects, such as wrong blowing time of a fuse, could lead to serious results, and it is therefore particularly important that product accepted be practically free from them. Other defects, such as scratches on the surface of a fuse, may be of much less importance. In addition to listing all possible determinable defects in an item, it is therefore desirable to classify them according to importance. Three classes are frequently used: major defects, minor defects, and irregularities. Defects are sometimes classed as major if they will cause a failure of the item to function as intended and as minor if they impair the efficiency or shorten the life of the item. Defects that represent departures from good workmanship but do not affect the performance, efficiency, or life of an item are sometimes classed as irregularities.*

* For simplicity in exposition, only major and minor defects are considered throughout the remainder of Part II.

1.3. Inspection of an Item of Product

An item is called defective if it contains one or more defects of *any* class. The class of defect the item contains determines what kind of defective it is called. If it contains one or more major defects and no defects of other classes, it is called a major defective. If it contains one or more minor defects and no defects of other classes, it is called a minor defective. If it contains both major and minor defects, it is both a major defective and a minor defective. Because it is important that product accepted be freer from major defectives than from minor defectives, there may be two sampling plans for the same product, a severe one for major defectives and a less severe one for minor defectives.

For some products it may be necessary to take account of the cumulative effect of several defects of either the same or different classes. For such products, provision may be made for classifying an item as a major defective if it contains a specified number or combination of minor defects and irregularities.

It is essential that an inspector know exactly what is a defective item of product and how to test an item of product for each defect. It is essential that he inspect each item for *all* quality characteristics and that he classify observed defects properly. It is harder to teach inspectors to inspect an item properly than to teach them the few new ideas associated with the operation of a sampling plan. All inspection plans rest squarely on proper inspection of items by the inspector.

2. FORMING THE INSPECTION LOT

2.1. Definition of Inspection Lot

The common operating feature of all lot-by-lot sampling-inspection plans is that they accept or reject inspection lots of product on the basis of the quality of samples drawn from the inspection lots. The term "inspection lot" is used in this book to designate the group of items accepted or rejected as a whole on the basis of a sampling plan. It should not be confused with other terms including the word "lot," for example, "shipping lot" or "order lot"; such terms have special meanings and are not related to the principles of or procedures for sampling inspection as described in this book. An inspection lot may consist of only part of a lot as otherwise defined, may contain some items from one such lot and some from another, or may contain several such lots.

2.2. Effect of Method of Forming Inspection Lots on Quality of Product Accepted

The quality of accepted product depends to some extent on how inspection lots are formed. The average quality of accepted product

will be higher if inspection lots are formed without mixing product of different quality. To illustrate this principle by an extreme example, suppose that there are two sources of product producing the same number of items, that one produces only defective items, the other only nondefective items, and that product from both sources flows into the same bin and is thoroughly mixed. Inspection lots formed from this mixed product then contain approximately 50 percent defective items. These inspection lots are accepted or rejected on the basis of the sampling plan being used, the percentage of inspection lots accepted being as shown on the OC curve of that plan. Whatever inspection lots are accepted, however, are 50 percent defective. Suppose, now, that product from the two sources is not mixed and inspection lots are formed from each source of product. Half of the inspection lots so formed contain only nondefective items and half only defective items. All inspection lots with no defective items are accepted on the basis of the sampling plan, and all inspection lots with only defective items are rejected. Thus, when inspection lots are formed from unmixed product, the product accepted contains no defective items and the product rejected, no nondefective items, whereas when inspection lots are formed from mixed product, the product accepted contains 50 percent defective items and the product rejected, 50 percent nondefective items.

As this example illustrates, both the supplier and receiver benefit if product is segregated according to the conditions under which it is produced, in so far as differences in these conditions result in product of different quality.

2.3. Factors to Be Taken into Account in Forming Inspection Lots

No hard and fast rules on the formation of inspection lots can be laid down, since the best way to form inspection lots of a product depends on the particular factors that are most likely to lead to variation in the quality of that product. Certain general rules on the formation of inspection lots have, however, been found helpful.

Do not combine product produced from different batches of raw material or from component parts obtained from different sources.

Do not combine product from different production lines or produced by different methods.

Do not combine product of different shifts.

Do not combine product produced from different sets of molds, patterns, or dies.

Do form inspection lots of product from natural portions of the production process, such as the product from one production line for one shift, the product produced from one batch of raw material, and so forth.

In forming inspection lots, it is ordinarily not possible to take into account all factors that might lead to variation in quality. In the first place, it is frequently technically impossible to do so; for example, the production process may be such that, after the item is made, there is no way of identifying the batch of raw material used in making a particular item. In the second place, taking all factors into account may make the inspection lots so small that it is necessary to inspect a prohibitively large percentage of the product. The general principle that should be followed is to form as large inspection lots as possible while at the same time taking account of the most important factors that might lead to variation in quality.

2.4. The Size of the Inspection Lot

Let us consider why large inspection lots are desirable. The OC curve of a sampling plan depends primarily on the *number* of items inspected per inspection lot; the larger this number, the better the protection that the sampling plan gives against the rejection of high-quality inspection lots and the acceptance of low-quality inspection lots. But the total cost of inspection depends primarily on the *percentage* of the submitted items that are inspected. The larger this percentage, the higher the total cost of inspection. Since we want both a large *number* of items in the sample (for good protection) and a small *percentage* of items (for low cost) it follows that large inspection lots are desirable.

The number of items inspected from a large inspection lot can be larger than the number of items inspected from a small inspection lot and yet constitute a smaller percentage of the large inspection lot than the number of items inspected from the small inspection lot constitutes of the small inspection lot. If this is so for the sampling plans used for large and small inspection lots, the protection will be better for large than for small inspection lots, while at the same time the percentage of items inspected will be less. For example, a sample of 300 items from an inspection lot of 10,000 gives better protection than a sample of 115 items from an inspection lot of 1,000; and it requires inspection of only 3 percent instead of 11.5 percent of the items submitted.

The gains from forming large inspection lots may be more than offset if the formation of large inspection lots requires the inclusion in each inspection lot of product differing widely in quality. For example, the formation of inspection lots of 10,000 instead of 1,000 items might require that each inspection lot be formed from the product of 10 machines that differ widely in the percentage of defective items they produce. If this were so, the loss from not being able to accept or reject the product of each machine separately might more than offset the gain from larger inspection lots.

If, as is desirable, such factors as work shift or production line are used in forming inspection lots, the number of items in an inspection lot will not be exactly the same from inspection lot to inspection lot. This will not in general affect adversely the operation of a sampling plan.

2.5. Moving Inspection Lots

Whenever possible, it is desirable that the entire inspection lot be formed before sampling inspection is begun. This is not always possible; the inspector may have to select his single* sample as the items produced reach him—as, for example, when the items are subassemblies coming from a moving line. If inspection is done on a moving inspection lot by single sampling, it will sometimes happen that the rejection number, and therefore a decision, is reached before the entire inspection lot has been completely produced; the rejection of the entire inspection lot would then involve the rejection of items not yet completely produced. This appears unreasonable, particularly if the factor or factors responsible for the defectives found by the inspector are located and eliminated before production is completed on the remaining items in the inspection lot. One alternative to rejection of the entire inspection lot, including the unfinished items, is to reject only the part of the inspection lot that was available to the inspector for sampling. This alternative also is undesirable. First, it in effect changes the protection given by the sampling plan, since the uncompleted part of such a rejected inspection lot has more than one chance of being accepted. Second, rejection of part of an inspection lot gives the supplier less incentive to improve the quality of his product than does rejection of the entire inspection lot.

It is not easy to decide which course of action to follow. One possible compromise is to have the inspector request the supplier to take corrective action on the factors responsible for the defect or defects that caused rejection. When the inspector is satisfied that corrective action has been taken, a new inspection lot should be started at the point in the line at which the defect or defects occurred. Material that has been produced prior to the corrective action should be included in the previous inspection lot and rejected.

Perhaps the best procedure is to use sampling plans that are especially designed to handle continuous production.†

* Double and sequential sampling are harder to use on moving inspection lots, because it is difficult both to obtain and to inspect second and later samples from moving inspection lots.

† See, for example, H. F. Dodge, "A Sampling Plan for Continuous Production," *Annals of Mathematical Statistics*, Vol. 14 (1943), pp. 264–279; also, A. Wald and J. Wolfowitz, "Sampling Inspection Plans for Continuous Production Which Insure a Prescribed Limit on the Outgoing Quality," *Annals of Mathematical Statistics*, Vol. 16 (1945), pp. 30–49.

3. SELECTING A SAMPLING-INSPECTION PLAN

3.1. Factors Influencing the Choice of a Sampling Plan

The next operation in the installation of a sampling plan is the choice of the sampling plan itself. Ideally, the plan chosen should achieve the best possible balance between the costs of administering and operating the plan and the gains resulting from the acceptance of high-quality inspection lots and the rejection of low-quality inspection lots. The information necessary for a perfect balance between the costs and returns of sampling inspection is rarely available, but experience with sampling inspection for products similar to the product in question is helpful.

In choosing a sampling plan, both tangible and intangible factors must be considered. The former will be considered later; among the latter are the supplier's record and reputation, the quality of product that he may be expected to submit, and the good-will and other value that should be attached to the acceptance of high-quality inspection lots and the rejection of low-quality inspection lots.

3.2. Classification of Sampling Plans

Sampling plans may be classified into three types: single-, double-, and sequential-sampling plans. These three types may be further classified in several ways; for example, by acceptable-quality level (AQL), by average outgoing-quality limit (AOQL), by lot tolerance percent defective (LTPD), by steepness of OC curves, and by amount of inspection.

As an example of one way in which plans may be classified, consider the plans in Part V of this book that are summarized in Tables 2 and 3 and given in detail in Table 4. These plans are classified in two ways: by AQL and amount of inspection (Tables 2 and 4) and by AOQL and amount of inspection (Table 4). Table 3 shows the amount of inspection for various combinations of AQL and AOQL.

It is not practicable to have a separate plan for each possible AQL or for each possible AOQL value. Accordingly, AQL and AOQL values are grouped into fairly narrow class intervals, and plans are provided for each such class.

A typical page of Table 4 contains three sampling plans—single, double, and sequential—having approximately the same OC curves. The upper corner of the page gives the AQL class and the AOQL class for the plans on the page and a sample-size letter. The *sample-size letter*, which may be any letter from A to O, stands for a particular sample size or sequence of sample sizes. For example, the letter J means a sample of 150 items for single sampling, a first sample of 100 items and a second

sample of 200 items for double sampling, and seven samples of 40 items each for sequential sampling. Pages are included in Table 4 for a large number of combinations of AQL classes and sample-size letters or AOQL classes and sample-size letters. Plans are not available for every combination because it is not possible to attain very low AQL or AOQL values with small samples, and it is unnecessary to use very large samples for high AQL or AOQL values.

3.3. Selection of Quality Requirement

The choice of a sampling plan for a particular product requires a decision on the quality requirement. For example, if a set of plans classified by AQL is being used, it is necessary to choose an appropriate AQL.* The choice of an AQL requires the balancing of quality desired against quality attainable. If the AQL is too exacting to be attainable with current production methods, excessive rejections will result. On the other hand, if the AQL is not exacting enough, inferior product may be accepted. The quality of product that the supplier may reasonably be expected to submit with current production methods is therefore a primary consideration in setting an AQL.†

If the currently attainable quality is better than the quality needed, the AQL should be higher (that is, less exacting) than the currently attainable quality in order to reduce rejections. If the currently attainable quality is poorer than the quality needed, the AQL may need to be lower (that is, more exacting) than the currently attainable quality. If it is so set, however, measures will have to be taken to improve the production process so as to reduce the number of defectives produced; otherwise excessive rejections will occur.

The importance of avoiding rejections depends on the urgency of need (or intensity of the demand) for a product. It is therefore appropriate that, other things being equal, the AQL be higher for urgently needed product than for product of less immediate urgency.

3.4. Selection of Type of Sampling

The selection of a specific sampling plan also requires a decision on the type of sampling—single, double, or sequential—to be used. This decision should be made in accordance with the principles given in Sec. 3 of Chap. 4.

3.5. Determination of Amount of Inspection

For sampling plans of each type, single, double, and sequential, it is possible to have any amount of inspection. The larger the amount of

* Recall that the AQL is the quality of product such that the sampling plan accepts 95 percent of submitted inspection lots of that quality.

† Similar statements could be made if the choice were based on LTPD or AOQL.

inspection, the better the discrimination provided by the sampling plan between high-quality inspection lots and low-quality inspection lots; that is, the steeper the OC curve of the sampling plan.

Other things being equal, it is appropriate that the amount of inspection be smaller when the cost of inspection is high. If the same product is to be inspected in various inspection-lot sizes, the amount of inspection should be somewhat larger for large inspection lots than for small ones in order to achieve a proper balance between cost and protection. To keep protection the same for all inspection-lot sizes, that is, to have the same OC curve, would require the inspection of approximately the same number of items from each inspection lot regardless of the size of the inspection lot. This would mean that the percentage of items inspected, and therefore the cost of inspection *per item submitted*, would be much larger for small inspection lots than for large ones. On the other hand, to keep cost of inspection per item submitted the same, the average number of items inspected would have to be proportional to the size of the inspection lot. This would mean much less protection for small inspection lots than for large inspection lots, since the number of items inspected from each inspection lot would be much smaller for small inspection lots. It is generally appropriate, in fixing the amount of inspection, to compromise between these two extremes of constant protection and constant cost. This is done in Table 1 of Part V; the number of items inspected per inspection lot increases with the size of the inspection lot, but the percentage of items inspected decreases.

3.6. Sampling Plans for Different Kinds of Defectives

When items are to be inspected for more than one kind of defective, say major and minor, separate sampling plans are chosen for each kind of defective. The sampling plan for majors will ordinarily have a lower AQL than the sampling plan for minors. For convenience both in line inspection and in the administration of inspection, it is desirable to select plans that have the same sample size or sequence of sample sizes for different classes of defectives; these plans will, of course, usually differ in their acceptance numbers and in their rejection numbers.

4. NORMAL, REDUCED, AND TIGHTENED SAMPLING PLANS

4.1. Normal Inspection

Inspection under the sampling plan initially introduced for a particular product and supplier is often called "normal inspection." It is continued in use as long as the quality of product submitted is in the neighborhood of the AQL. (Even if the plan is chosen on the basis of AOQL or LTPD, it will have an AQL, which can be read from its OC curve,

since the AQL is a quality of product such that 95 percent of submitted inspection lots of that quality will be accepted.) If the quality of product submitted is consistently better than the AQL, reduced inspection may be used; if consistently worse than the AQL, tightened inspection may be used.

4.2. Reduced Inspection

If the quality of product submitted is consistently better than the AQL, the high quality of submitted product itself provides assurance that the inspection lots being accepted are also of high quality. Under such circumstances, it may at times be desirable to tolerate a higher risk of accepting inspection lots of low quality, if submitted, since there is, evidence that such inspection lots are rarely submitted. If this is desirable, the amount of inspection can be reduced. Of course, "reduced inspection" should be continued only so long as the supplier continues to submit high-quality product. If the quality of the product submitted deteriorates, normal inspection should be resumed.

4.3. Tightened Inspection

If the quality of product submitted is consistently worse than the AQL, a large proportion of submitted inspection lots will be rejected. At the same time, those accepted are likely to be of low quality, since, if only low-quality product is submitted, only low-quality product can be accepted (unless, of course, provision is made for correcting rejected inspection lots). Under such circumstances, normal inspection may not give sufficient protection against the acceptance of inferior inspection lots. An inspection plan may be needed that will reject so high a percentage of the inferior inspection lots being submitted as to force the supplier to improve the quality of his product. Inspection under such a plan is called "tightened inspection."

5. THE SAMPLING PROCEDURE

5.1. Work of the Inspector

Once a sampling plan has been selected, the inspector of items comes into the picture. His job is, essentially, to carry out the provisions of the sampling plan—to draw items from the inspection lot, to inspect each of these items for all defects, and to determine whether the inspection lot is acceptable or not by comparing the number of defectives he finds with the acceptance and rejection numbers of the sampling plan he is using. The second and third of his duties have already been discussed; we shall now consider the problems associated with drawing items from an inspection lot.

5.2. Biased and Unbiased Sampling

The inspector should attempt to draw items from an inspection lot so that these items constitute a sample that fairly represents the quality of that inspection lot. This is easier said than done, but with some guidance the inspector should soon be able to avoid such biased methods as taking items from the same positions in containers, stacks, or piles every day, taking all items from the top of a box, taking no items from the top of the box, taking items from the output of certain machines and not from others, taking items that appear to be defective. If the inspector succeeds in avoiding such biased sampling procedures, what he *does* do is likely to yield a sample (or samples) that fairly represents the quality of the inspection lot.

5.3. The Meaning of Random Sampling

For every sampling-inspection plan described in this book, the OC curve is calculated on the assumption that random sampling is used. Technically, a random sample is a sample drawn in such a manner that the chance that any given item in the inspection lot will be included in the sample is independent of the quality of that item and independent of the quality of other items selected for the sample. Randomness is assured if each item in the inspection lot has the same chance of being the first item in the sample and if, after the first item in the sample is drawn, each of the remaining items in the inspection lot has the same chance of being the second item in the sample, and so on.

The only way to guarantee that the method of selecting items is random is by the use of some device that eliminates personal discretion from the selection of items. For example, the particular items to be included in the sample might be determined as follows: number consecutively the items in the inspection lot, number cards correspondingly, thoroughly shuffle these cards, select blindly from the thoroughly shuffled deck as many cards as there are items to be included in the sample, and take as the sample the items in the inspection lot whose numbers are on the cards so drawn. Another possible method is to rotate a wheel of chance having as many numbers as there are items in the inspection lot and take as the sample the items corresponding to the numbers at which the wheel stops. The use of such devices leaves entirely to chance the determination of the particular items to be included in the sample and therefore gives each item in the inspection lot the same chance of being included.

Actually numbering the items in an inspection lot may be impractical, both because of the work involved in attaching the numbers and because of difficulties in the inspector's finding the item having a given number.

Practical schemes can, however, usually be devised for any particular inspection operation. Presentation of the lot for inspection often involves fixed positions into each of which one item is placed, as when there are fixed racks, trays, or shelves, or the items are arranged in some geometric pattern. Suppose, for example, that the inspection lots are of size 420 and are presented in 12 piles of 35 each. If a single sample of 55 is to be selected, a series of 55 code designations such as 8–29 (designating the twenty-ninth item in the eighth pile) can be prepared in such a way that the first part of each code designation is equally likely to be any number from 1 to 12 and the second part any number from 1 to 35. The series of 55 code numbers can be prepared in advance from random mechanisms by clerks not concerned with the inspection of items and given to the inspector in a sealed, opaque envelope not to be opened until the lot is completely in position for inspection. Although the clerks would draw the numbers for each sample in a random order, they would list them for the inspector in the order most convenient for drawing—all those in pile 1 together and in serial order, and so forth. The details of any such randomizing procedure—indeed of any procedure for drawing a sample—must be adapted to the particular physical surroundings, as well as to the characteristics of the item and of the inspection operation, involved in each particular sampling plan.

5.4. Rules for the Selection of Samples

5.4.1. Three Rules.—Since the use of such devices is ordinarily not practicable in inspection, several rules have been devised to increase the likelihood that a sample will fairly represent the inspection lot from which it is drawn. The first of these is

Rule I. Draw proportional samples.—According to this rule, inspection lots should, wherever possible, be divided into sublots on the basis of factors that are likely to lead to variation in the quality of the product. Examples of such sublots are items produced by the same machine, items made from the same batch of raw material, items made by the same shift. In general, the factors to be considered when dividing the inspection lot into sublots are those that were considered in forming inspection lots but, because of practical difficulties or because of the necessity of large inspection lots, could not be taken into account in forming the inspection lots.

From each sublot into which the inspection lot is divided a subsample should be selected. The size of the subsample from each sublot should be proportional to the size of that sublot. The total number of items drawn from all the subsamples should, of course, be equal to the size of the sample.

The following are examples of the application of rule I.

Suppose an inspection lot consisting of 2,000 items comes to the inspector in 20 trays of 100 items each and the sampling plan calls for a sample of 100 items. Each tray can be considered a sublot of the inspection lot. There are then 20 sublots, so, according to rule I, a subsample of 5 items should be drawn from each of the 20 trays.

Suppose that inspection is done as items move along a conveyer belt, that an inspection lot consists of 2,000 successive items, and that the sampling plan calls for a single sample of 100 items from each inspection lot. A group of 20 successive items might be considered a sublot of the inspection lot; if so, one item should be drawn from each such group. Care should be taken not to draw *exactly* every twentieth item, as this would provide opportunity for someone to place good (bad) items at these points and thus bias the sample.

In the preceding example, single sampling is used. Double or sequential sampling cannot easily be used with moving inspection lots, as noted in Sec. 2.5 of this chapter.

Two additional rules are

Rule II. Draw sample items from all parts of each sublot of the inspection lot.

Rule III. Draw sample items blind.—It is essential that the items selected from each sublot of an inspection lot be selected in a random manner. While, as indicated above, only the use of devices that eliminate personal discretion from the selection of items can guarantee random selection, a reasonable assurance of randomness is obtained by following rules II and III.

The purpose of rule II is to assure every item in each sublot of the inspection lot an equal chance of being included in the sample. Violation of this rule makes it possible for personnel intentionally or unintentionally to bias the sample. To illustrate: an inspection lot consists of 500 heavy brass rods; a certain supplier submits each inspection lot in a single box. The rods in the bottom of the box may have less chance of being drawn in the sample than others. Supplier personnel might realize this and place all defective rods in the bottom of the box; or, because of some physical cause, defective rods might tend to lie lower in the box than nondefective rods. The remedy: inspection lots should be presented in such a way that the inspector is able to draw sample items from all parts of the inspection lot.

As another illustration of the application of rule II, consider the first example discussed under rule I. Five items are drawn from each tray. The location from which these 5 items are drawn should be varied from tray to tray and from inspection lot to inspection lot so that items are not always taken from one location, such as the center or one particular

corner of the tray, and so that the positions from which the 5 items are drawn are neither always close together nor always widely scattered.

The purpose of rule III, like that of rule II, is to assure every item in the inspection lot an equal chance of being included in the sample. To illustrate: an inspector begins to draw sample items from a particular inspection lot but immediately "spots" a number of obviously defective items. It is humanly impossible for him to ignore these defective items; he is bound to make a conscious effort either to draw or to avoid drawing them for his sample. Either decision injects personal discretion into the selection of the sample and means that not every item in the inspection lot has the same chance of being included in the sample. What should he do? One desirable possibility is that he decide in advance which items he is going to draw. For example, if the material comes in piles, he might decide in advance to take the second item in the first pile, the sixth item in the second pile, and so forth, varying these numbers, of course, from inspection lot to inspection lot, preferably according to a card-shuffling or similar device. If an advance decision is not possible, he should remove, before beginning to draw items for his sample, any obviously defective items that he happens to notice.* The inspector should not, of course, make a thorough inspection of the inspection lot before beginning to draw items for his sample nor, indeed, make any attempt to find defectives but should simply remove obviously defective items that he happens to notice.

5.4.2. Relation of the Rules to the Principle of Randomness.—From a strictly logical point of view rules I and II contradict the principle of randomness given in Sec. 5.3. In practice they result in a closer approach to randomness than can be achieved by other methods except those suggested in Sec. 5.3. The methods of Sec. 5.3 are sometimes felt to be impractical although they have not, as far as is known, had an adequate trial in the industrial field. The methods of Sec. 5.3 would probably be simpler to teach and administer than the concepts and procedures involved in rules I and II.

Use of rules I and II leaves the average number of defective items in samples unchanged but reduces the variation of the number of defectives from sample to sample. The result is a steepening of the OC curve so that good inspection lots are accepted more often, while bad ones are accepted less often than if the sampling were random. In other words there is a tendency toward the ideal OC curve described in Chap. 3, Sec. 2.6. In practice this tendency is negligible, as is illustrated by the following examples.

* These should be returned to personnel responsible for producing them, with appropriate comments.

Suppose a single-sampling plan with sample size 300, acceptance number 27, rejection number 28 is used; and suppose that the items of an inspection lot are equally divided among 30 trays, of which 5 contain 20 percent defectives each and 25 contain 5 percent defectives, a total of 7.5 percent defectives in the inspection lot. It turns out that sampling according to rules I and II will result in accepting an average of one more inspection lot out of 200 such inspection lots submitted than would be accepted by random-sampling procedures. Similarly if 10 trays have 20 percent defectives and 20 trays have 5 percent defectives, one fewer inspection lot per 200 will be accepted when rules I and II are followed than if random sampling were used.

These examples suggest, and more extensive investigations show, that, in practical situations likely to arise in industrial sampling, the use of rules I and II will seldom have much effect on the OC curve. Of course if the differences among the sublots are extreme there will be a real difference between the OC curve under rules I and II and that under random sampling. For example, if some trays have all defective and the rest all nondefective items, inspection lots of identical quality will be either always accepted or always rejected under the rules.

The point has been made that use of rules I and II leaves the average number of defectives in the sample unchanged, that is, the procedure is unbiased. That biased procedures may make serious changes in the OC curve is suggested by the fact that a small change in the percent defective at a position where the OC curve is steep makes a considerable change in the probability of an inspection lot's being accepted. For example, if 20 trays are 5 percent defective and 10 are 20 percent defective, but the sampling is done randomly from 27 trays, always omitting 3 trays of poor quality, the average percent defective in samples will change only from 10 percent to 9 percent. But the percentage of such inspection lots accepted leaps from 32 to 57, under the plan mentioned in the paragraph at the top of this page.

5.5. Practical Aspects of Sampling

The principles of good sampling are rather simple but they are not always easy to apply. Each product and each supplier presents special problems. It is, for example, one thing to sample rivets and quite another to sample cotton fibers. An hour's instruction of each inspector at his place of work is a good way to achieve an unbiased yet practicable method of sampling the material that he is to inspect.

5.6. Inspection of Resubmitted Material

When a rejected inspection lot is screened or reworked, say for major defectives, it is important for the inspector to assure himself, as far as

may be practicable, that the resubmitted inspection lot is free of major defectives and that the number of minor defectives has not been increased. If supervision of the supplier's screening or reworking is practicable, it may be appropriate to inspect the resubmitted inspection lot only for the class of defects for which it was originally rejected or sometimes only for a particular defect.

CHAPTER 6

USE OF SAMPLING INSPECTION FOR QUALITY CONTROL

1. TWO PURPOSES OF INSPECTION

Inspection of material may be performed to determine (a) whether material already produced should be accepted or rejected or (b) whether corrective action should be taken on the production process.

Sampling inspection for the first of these purposes is often called "acceptance sampling" and, for the second, "control sampling." Sometimes, sampling inspection is used both to determine the quality of material already produced and to judge the production process.

2. AIM OF SAMPLING INSPECTION

While acceptance and control sampling differ in some respects, their mutual aim is to compare the results of inspection with an objective criterion and thereby decide whether corrective action need be taken. The nature of the corrective action depends on whether sampling is for acceptance or control. If for acceptance, corrective action may consist of screening the rejected inspection lot and scrapping, reworking, or replacing the defective items. If for control, corrective action may consist of investigating the production process and determining and removing the cause of the trouble.

3. OBJECTIVE CRITERIA IN ACCEPTANCE AND CONTROL SAMPLING

Since the action to be taken on the product or on the process is not primarily a statistical problem, this chapter is confined to a discussion of objective acceptance and control criteria. One such criterion, already discussed, is to specify the amount of protection and choose a sampling plan whose OC curve assures this protection. This criterion characterizes most acceptance-sampling procedures, including the standard sampling-inspection procedure of Part III.

Most control-sampling procedures employ a somewhat different criterion. This chapter (*a*) describes the criterion of one of the most popular control-sampling procedures, (*b*) indicates how OC curves may be used to obtain a criterion for control sampling, and (*c*) shows how the sampling plans in Part V may be used for control as well as acceptance sampling.

4. CONTROL FEATURES OF ACCEPTANCE SAMPLING

Ordinary acceptance sampling has control features. Acceptance sampling contributes substantially to superior quality by discriminating between high- and low-quality inspection lots, thereby giving suppliers an incentive to achieve and maintain a high level of quality. In addition, acceptance sampling provides information that, if properly interpreted and acted upon, can help to improve quality.

The quality information provided by acceptance-sampling inspection for attributes is of several kinds: the number of defective items found in the sample or samples inspected from each inspection lot, the average percentage of defective items in samples from a series of inspection lots, that is, the process average, and tallies of the particular kinds of defects found in the course of inspection. Prompt review of the number of defective items found in the sample or samples inspected from each inspection lot can lead to the detection of inspection lots that are of exceptionally low quality or exceptionally high quality, compared with the general quality level of inspection lots the process is currently producing. Immediate investigation of the conditions under which these abnormal inspection lots were produced may lead to the detection of steps in the process that can be improved or of faults that can be removed or of the existence of factors that should be permanently incorporated into the production process. Continuing surveillance of sampling-inspection data can lead to the prompt detection of an increase in the process average, that is, a trend away from high-quality product, and to the elimination of the causes of such a trend before a serious deterioration in quality takes place. Analysis of tallies of the particular kinds of defects found in the course of inspection can suggest the kinds of defects it will be most profitable to attempt to eliminate.

The quality information derived from acceptance inspection may also be used to adjust the sampling program to meet changes in the quality of product being submitted by the supplier. A sufficient increase in the process average may call for tightened inspection, while a sufficient decrease in the process average may call for reduced inspection.

Coordinated quality information from several plants or processes can assist in the construction of additional sampling programs, in the establishment and adjustment of specifications, and in the writing of equitable contracts.

Despite these control features of acceptance sampling, the importance of the slogan, "Quality cannot be inspected into a product, quality must be *built* in," should be thoroughly recognized. The best way to assure the acceptance of nothing but good product is to be certain that nothing but good product is manufactured. More practically, if it is possible to improve the process, then for the same amount of production a larger

volume of acceptable product will be produced, and the average quality of product accepted will be higher.

5. THE CONTROL-CHART METHOD

5.1. Introduction

The control chart is a method of determining when to take corrective action on the production process; it is more direct than a review of the results of acceptance sampling.

The control-chart method for attributes* operates as follows: Arrangements are made for the formation of inspection lots, selection of samples, and inspection of items; a record (such as that shown in Table 6.1) is kept of the number of defectives found in the first† sample from each inspection lot. After samples from, say, 25 inspection lots have been inspected, the sample data are analyzed to obtain information about the quality level and homogeneity of the production process.

The data of the first 25 samples in Table 6.1 will later be used to illustrate the analysis. These data (also shown in Fig. 6.1) suggest, among other things, that the samples from inspection lots 5 and 12 had abnormally high numbers of defectives. The first step in the analysis of these data is to establish an objective criterion by which such suggestions may be tested, that is, to decide whether the number of defectives observed in a particular sample could likely have arisen from the present production process by chance or whether such a number of defectives is so unlikely to have arisen by chance that it is more reasonable to suspect some cause other than chance.

Before it can be decided objectively whether the number of defectives found in a sample is abnormal, the current level of quality must be determined, for it is with respect to the current level of quality that a decision on the abnormality of the sample is made.

5.2. Quality Control Based on the Number of Defectives in a Sample

One way of measuring the current quality level of a process is by determining the average number of defectives, $\bar{c}$, in samples of size n. The number of defectives in a sample of size n is then compared with $\bar{c}$.

* Other control chart methods are available for variables inspection. See, for example, American Standards Association, *Guide for Quality Control and Control Chart Method of Analyzing Data*, New York, 1941, and *Control Chart Method of Controlling Quality during Production*, New York, 1942; also Holbrook Working and Edwin G. Olds, *Manual for an Introduction to Statistical Methods of Quality Control in Industry*, Office of Production Research and Development, War Production Board, Washington, 1944, and Eugene L. Grant, *Statistical Quality Control*, Part Two and Chap. XVI, McGraw-Hill Book Company, Inc., New York, 1946.

† This control-chart method generally involves single sampling because double and sequential sampling introduce complications.

The following are formulas for the upper control limit (UCL) and the lower control limit (LCL) on the number of defectives likely to be found in samples of size n if all variation in the number of defectives is attributable to chance. When all variation is attributable to chance, the observed number of defectives will almost always lie between these limits. These formulas can be used when the average percentage of defectives per sample is small (say less than 10 percent).

$$\text{Upper control limit (UCL)} = \bar{c} + 3\sqrt{\bar{c}}$$
$$\text{Lower control limit (LCL)} = \bar{c} - 3\sqrt{\bar{c}}$$

If LCL is negative or zero, no LCL exists.

TABLE 6.1
NUMBER OF DEFECTIVES FOUND IN FIRST SAMPLES OF 150 ITEMS FROM EACH OF 40 INSPECTION LOTS

Inspection lot number	Number of defectives in first sample	Inspection lot number	Number of defectives in first sample
1	1	26	1
2	0	27	0
3	1	28	0
4	0	29	0
5	7	30	2
6	0	31	1
7	1	32	0
8	1	33	1
9	0	34	1
10	0	35	0
11	0	36	1
12	6	37	0
13	1	38	3
14	1	39	0
15	0	40	1
16	0	41	
17	0	42	
18	1	43	
19	1	44	
20	1	45	
21	0	46	
22	0	47	
23	2	48	
24	1	49	
25	0	50	
Total defectives in first 25 samples	25		

If the average percentage of defectives per sample is not small, the following formulas are preferable:

$$\text{Upper control limit (UCL)} = \bar{c} + 3\sqrt{\bar{c}\left(1 - \frac{\bar{c}}{n}\right)}$$

$$\text{Lower control limit (LCL)} = \bar{c} - 3\sqrt{\bar{c}\left(1 - \frac{\bar{c}}{n}\right)}$$

where n is the sample size. A process is assumed to be out of control if at any time the number of defectives in a sample of size n falls outside these control limits.

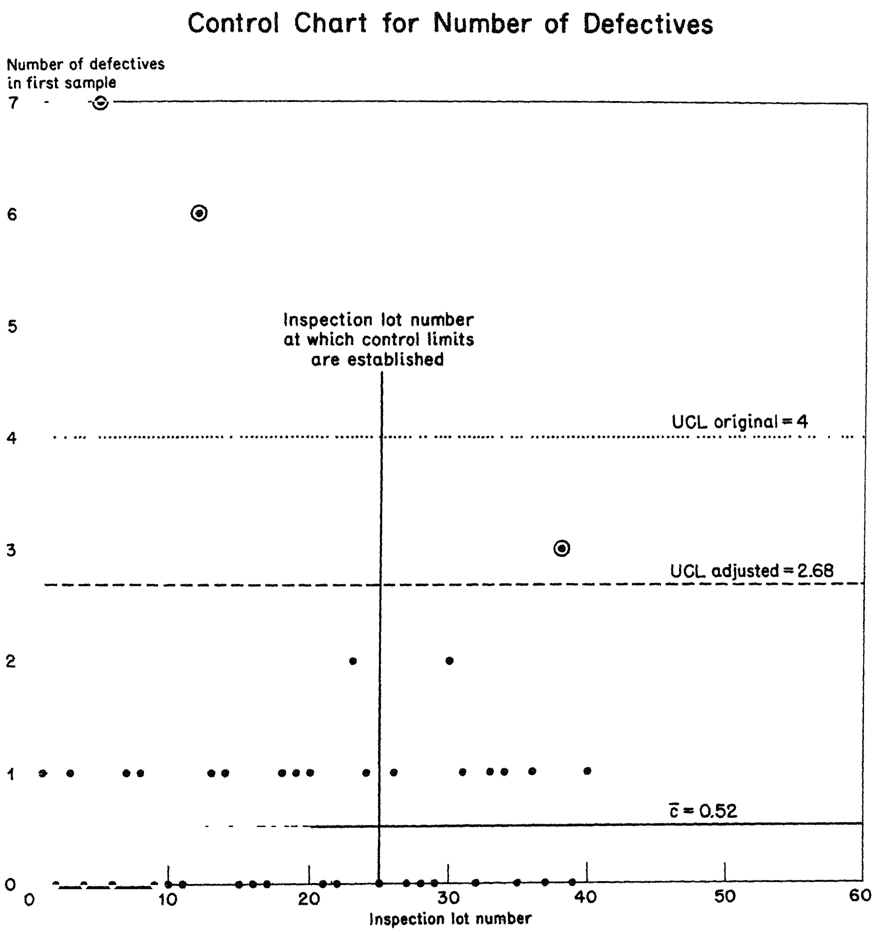


FIG. 6.1.

5.3. Quality Control Based on the Proportion of Defectives in a Sample

A second way of measuring the current quality level of a process is by determining the average proportion $\bar{p}$ of defectives in samples. The proportion of defectives in samples of size n is then compared with $\bar{p}$.

The number of defectives in a sample is of course easier to determine and therefore easier to plot than the proportion of defectives in a sample. However, different sample sizes are generally used for different products or processes, and if the quality levels of different processes or products are to be compared it is preferable to use proportion defective.

The following are formulas for the UCL and the LCL on the proportion of defectives likely to be found in samples of size n if all variation in the proportion of defectives is attributable to chance.

$$\mathrm{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\mathrm{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

A process is assumed to be out of control if at any time the proportion of defectives in a sample falls outside these control limits.

The average *percentage* of defectives per sample is called the "process average" (PA); that is,

$$\mathrm{PA} = 100\bar{p}$$

and since

$$\bar{p} = \frac{\bar{c}}{n}$$

we have

$$\mathrm{PA} = \frac{100\bar{c}}{n}$$

5.4. Example of the Use of a Control Chart

In the first 25 samples in Table 6.1 there are 25 defectives, so $\bar{c} = 25/25 = 1$, the process average is $100\bar{c}/n = 0.67$ percent, the upper control limit (UCL) on the number of defectives is $1 + 3\sqrt{1} = 4$, and there is no lower control limit (LCL) on the number of defectives since $1 - 3\sqrt{1}$ is negative; in this example the process is considered to be out of control only when too many defectives are observed. The samples from inspection lots 5 and 12 are outside the control limits; that is, they had more defectives than are likely to have arisen by chance alone. An assignable cause for the large number of defectives should be sought and if found should, if possible, be removed. A bad batch of raw material, an operator's error, a machine failure are a few of the possible assignable

causes. In any case, information has been obtained which may be useful to both the supplier and the receiver:

a. It has been determined that the tentative process average is approximately 0.67 percent defective.

b. The tentative UCL on the number of defectives is 4 and there is no LCL.

c. The process is not in control.

If a process is not in control it is not possible to predict with confidence that future inspection lots will be of the same average quality as past inspection lots.

If assignable causes for the extraordinary samples from inspection lots 5 and 12 are found and *eliminated,* a new $\bar{c}$, a new process average, and new control limits are computed, omitting these samples. A misunderstanding on the part of the inspector of items caused the high number of defectives in inspection lot 5, and a worn bearing was the troublemaker in inspection lot 12. The inspector is now satisfactorily informed, and the bearing was replaced at once, so we may regard the assignable causes as found and eliminated. Omitting the results of these two abnormal samples we recompute. The new $\bar{c} = 12/23 = 0.52$, the new process average is 0.35 percent defective, the new

$$\text{UCL} = .52 + 3\sqrt{.52} = 2.68,$$

and no LCL exists; these are plotted on Fig. 6.1 and extended for future use. Unless new assignable causes occur, it is reasonably certain that the supplier's process will remain in control at a quality level of approximately 0.35 percent defective; that is, nearly all samples from future inspection lots will fall within the new control limits.*

If an assignable cause had been found and eliminated for sample 5 *or* sample 12 only, tentative control limits and a new process average would be computed omitting that sample, while if assignable causes were found for neither sample, the original control limits and the original process average must stand. It should be recognized that in either case an unhealthy state of affairs exists, since there is evidence that the supplier cannot consistently produce material within these limits.

When a point falls above the UCL a search for the cause is always worth while. When an LCL exists and a point falls below the LCL, it is just as worth while to look for a cause. In the latter case, something may be learned that will benefit the production process. In exceptional cases, a factor may be discovered that will lead to a permanent improvement in quality.

* If the new process average, 0.35 percent defective, is not satisfactory, *major* changes in the process will probably be needed to reduce it.

6. TALLIES OF DEFECTS AND CONTROL CHARTS

A tally of defects of various types is often valuable. Control charts can be set up for separate defects rather than for defective items, provided the defects of various types are recorded separately. There are usually a large number of possible defects. In practice, however, defects may be classified into a few groups. Some of these groups may have assignable causes that control charts on those groups can reveal and good engineering can identify and economically remove.

7. RELATION OF CONTROL TO ACCEPTANCE

The acceptance or rejection of an inspection lot does not depend on whether a point falls inside or outside the control limits. If a supplier is producing very high-quality product, it may easily happen that a point for an accepted inspection lot falls above the UCL. Such a point calls for immediate investigation, because it may be a symptom of impending trouble, but the inspection lot remains accepted. On the other hand, a supplier may be consistently producing low-quality product, and some inspection lots may be rejected though the points for all inspection lots are within control limits. In this case, the control chart is usually of little assistance in locating the cause of the defectives; though control charts at earlier stages of the process may be useful.

8. OPERATING-CHARACTERISTIC (OC) CURVES OF QUALITY-CONTROL PLANS

8.1. When the Lower Control Limit (LCL) Does Not Exist

Sampling plans have OC curves, as explained in the earlier chapters of Part II. The control-chart plans already described in this chapter also have OC curves. Consider two examples in both of which the control chart has no LCL and the sample size n is 50; in one example $\bar{c}$ is 0.5 while in the other $\bar{c} = 1.0$. For $\bar{c} = 0.5$, the UCL is between 2 and 3, and for $\bar{c} = 1.0$, the UCL is just under 4; that is, in the first example the rejection number is 3 while in the second example the rejection number is 4. The control limits are equivalent, in effect, to the following single-sampling plans:

SAMPLING PLAN FOR $\bar{c} = 0.5$

Sample size	Acceptance number	Rejection number
50	2	3

SAMPLING PLAN FOR $\bar{c} = 1.0$

Sample size	Acceptance number	Rejection number
50	3	4

The OC curves of these sampling plans are found in Part V and are shown in Fig. 6.2. Notice that sampling plans used for control have

acceptable-quality levels (AQL's) just as have sampling plans used for acceptance; the sampling plan for $\bar{c} = 0.5$ has an AQL of 1.6 percent defective, and the sampling plan for $\bar{c} = 1.0$ has an AQL = 2.8 percent.

When no LCL exists, therefore, control charts are very similar to acceptance-sampling plans.

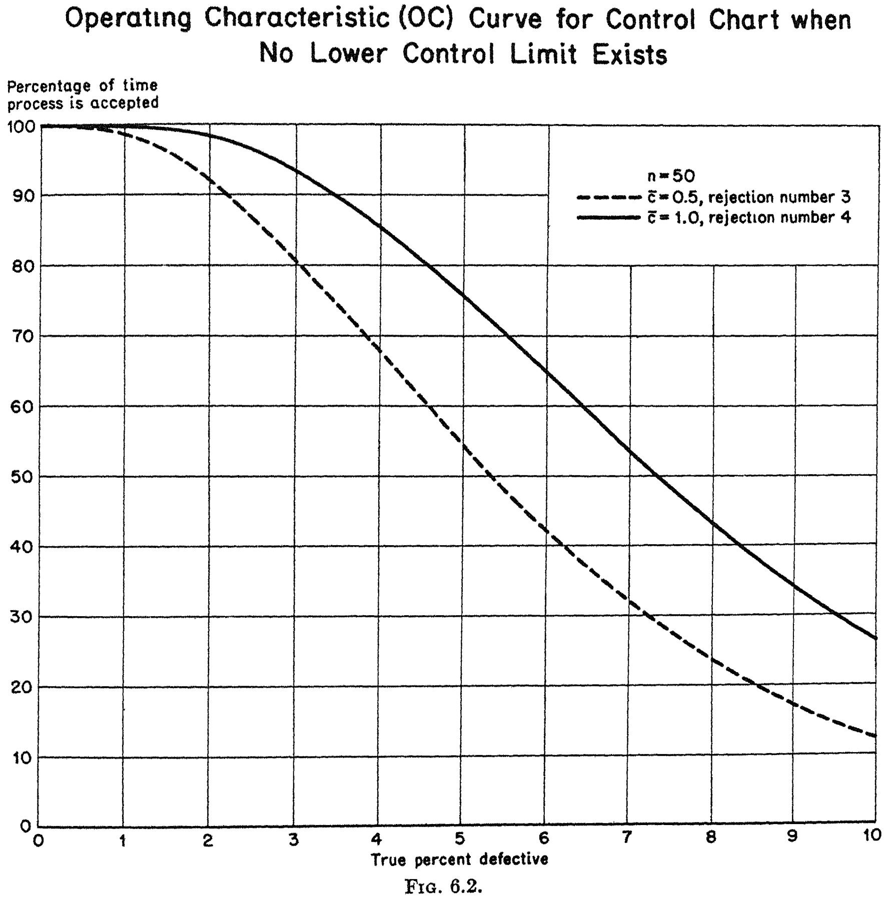


FIG. 6.2.

It should be noted that although the number 3 in the expression $\bar{c} \pm 3\sqrt{\bar{c}}$ provides an easy rule for determining control limits, it may sometimes be economical to look for trouble in the process more often or less often than provided by this arbitrary constant. Those who proposed the ± 3 rule were well aware of this, but some users of control-chart methods seem to believe that no other number should ever be considered.

8.2. When the Lower Control Limit (LCL) Exists

Consider the control chart for which an LCL exists. There are now two situations that may call for investigation, (a) too many defective items in the sample, (b) too few defective items in the sample. Two acceptance curves—one for the UCL and one for the LCL—must be combined to construct the OC curve of this control-chart plan.

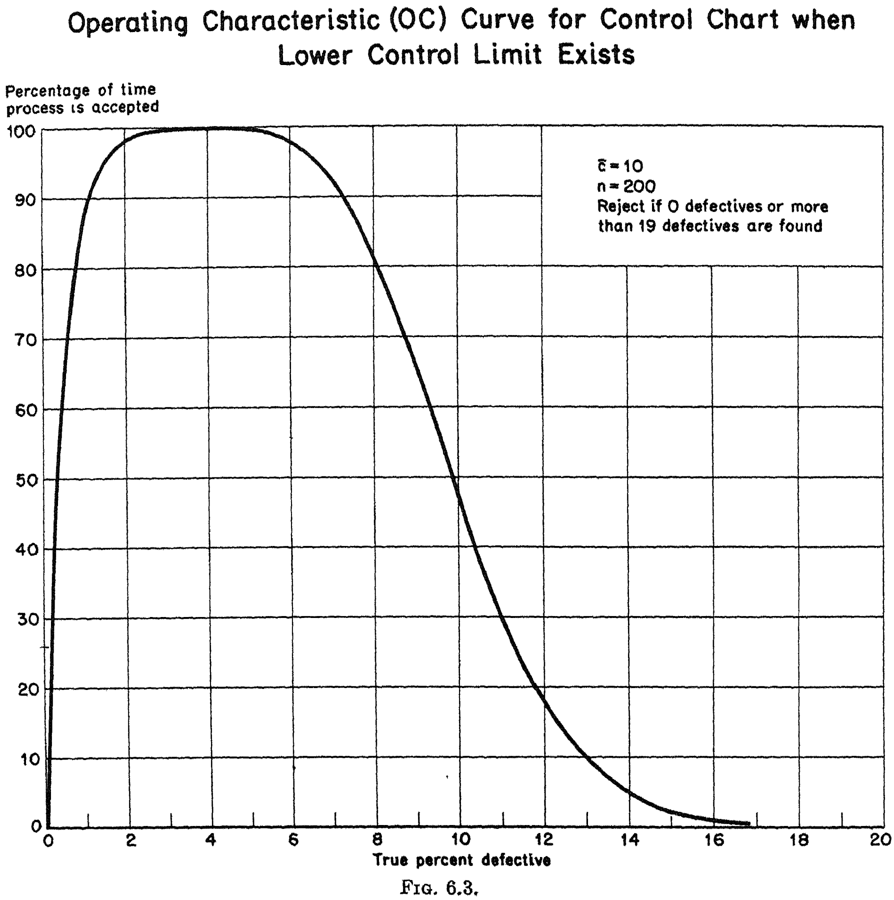


FIG. 6.3.

For example, for $n = 200$, $\bar{c} = 10$, the control limits are

$$\bar{c} \pm 3\sqrt{\bar{c}\left(1 - \frac{\bar{c}}{n}\right)} = 10 \pm 3\sqrt{9.5} = 0.76,\ 19.24.$$

Thus the process is not in control if no defectives, or if more than 19 defectives are found in the sample of 200 items. Figure 6.3 shows the OC curve of this control plan. The ordinate of the OC curve takes the

value 0.95 twice, first at 1.5 percent defective and again at 6.5 percent defective, so there are two AQL's. This means that if the true percent defective is between 1.5 and 6.5, the process is rejected as out of control less than 5 percent of the time, while, if the true percent defective is less than 1.5 or more than 6.5, the process is rejected as out of control more than 5 percent of the time.

Computation of the OC curve is slightly more complicated when a control plan has both a UCL and an LCL. If the inspection lot is large compared to the sample and if the proportion defective is relatively small, it is possible to approximate the required probabilities by using Molina's tables of the Poisson distribution.* Suppose that the sample size is n and that the true proportion of defectives is p; let $np = a$. Then the probability of getting exactly c defectives in the sample and the probability of getting c or more defectives in the sample can be obtained directly from Molina's tables. The OC curve of the control-chart plan given above ($n = 200$, $\bar{c} = 10$) is obtained as shown in Table 6.2.

TABLE 6.2
CALCULATION OF THE OPERATING-CHARACTERISTIC (OC) CURVE FOR A CONTROL-SAMPLING PLAN WHEN A LOWER-CONTROL LIMIT (LCL) EXISTS
$n = 200$, $\bar{c} = 10$

True p	$a = np$	(1) Probability of 0 defectives	(2) Probability of more than 19 defectives	(3) Sum of (1) and (2)	(4) Probability of accepting process [1 − (3)]
0	0	1.00000	0.00000	1.00000	0.00000
0.005	1	0.36788	0.00000	0.36788	0.63212
0.010	2	0.13534	0.00000	0.13534	0.86466
0.015	3	0.04979	0.00000	0.04979	0.95021
0.020	4	0.01832	0.00000	0.01832	0.98168
0.030	6	0.00248	0.00000	0.00248	0.99752
0.040	8	0.00034	0.00025	0.00059	0.99941
0.050	10	0.00004	0.00345	0.00349	0.99651
0.060	12	0.00001	0.02128	0.02129	0.97871
0.080	16	0.00000	0.18775	0.18775	0.81225
0.100	20	0.00000	0.52974	0.52974	0.47026

The entries in Table 6.2 are carried to five places, not because the accuracy of the approximation justifies such a procedure, but to illustrate better the order of magnitude of the contributions of columns (1) and (2).

Columns (1) and (2) have been read directly from Molina's table. Their sum (3) is the probability of rejecting the process because either too few or too many defectives occur in the sample. Column (4) is the

* See References p. 381.

probability of accepting the process; that is, one minus the probability of rejecting the process.

A graph of column (4) is shown in Fig. 6.3.

9. SAMPLING PLANS FOR CONTROL SAMPLING

9.1. Method of Selection

Although control limits set at $\bar{c} \pm 3\sqrt{\bar{c}}$ do not assure a fixed risk of taking unnecessary corrective action, they have nevertheless been used with excellent results as criteria for judging a wide variety of production processes. Some processes may, however, require better control over the sampling risks than is possible when the criterion is obtained by simply adding to $\bar{c}$ this arbitrary multiple of $\sqrt{\bar{c}}$. In such cases, the OC curve must be considered before the criterion can be adopted as a basis for action.

Since the sampling plans in Part V are given with their OC curves, these plans may be conveniently used for control purposes, for a choice may be made among them by comparing their OC curves. The arrangement of sampling plans in Part V is such that the task of selecting a sampling plan for control purposes is quite simple. The sampling plans in Part V are catalogued by AQL, the AQL being the percent defective which the plan will reject about 5 times out of 100. If the user is willing to take corrective action on the process 5 times out of 100 when the true percent defective of the process is actually not different from the process average, a plan whose AQL is approximately equal to the process average can be selected. It is then relatively easy to find the particular sampling plan whose OC curve furnishes the desired amount of discrimination.

In this procedure, whenever the process average changes, a new sampling plan, based on a new AQL, is selected. This parallels the ordinary control-chart procedure of recomputing the process average and control limits at periodic intervals.

This method of selecting a sampling plan cannot be used unless one is willing to take corrective action 5 times out of 100 when the true percent defective of the process is equal to the AQL. If one is not willing to take unnecessary corrective action that frequently, a sampling plan for control purposes can be obtained by examination of the various OC curves in Part V.

A simple procedure would be to decide first the highest percent defective at which one is willing to look for trouble as seldom as 5 percent of the time. Regard this as the AQL. If the sample size has already been determined, merely look up the acceptance and rejection numbers in Part V. If the sample size has not been determined it is a good plan to try to decide at what level of quality it would be worth

while to look for trouble a large proportion of the time, say 90 percent (LTPD). Having decided this, examine the OC curves of the chosen AQL class until a plan appropriate to these decisions is discovered. Unless an AQL outside the limits of the charts in Part V has been chosen or the difference between LTPD and AQL is unreasonably small, a satisfactory plan should be available.

To illustrate the method, suppose that we want an AQL of 2 percent and an LTPD of 9 percent. A short examination of the OC curves of Part V shows that the plan (sample-size letter H, AQL class 1.2 to 2.2 percent defective) with sample size 75, acceptance number 3, rejection number 4, fills the bill admirably.

9.2. Advantages and Disadvantages

The principal advantage of using the sampling plans in Part V for control sampling is that their OC curves are given. Another advantage is that double- and sequential-* as well as single-sampling plans are available; this makes possible savings in inspection costs. Finally, computation of upper and lower control limits is unnecessary; a criterion can be obtained simply by choosing an appropriate sampling plan from among those in Part V.

These advantages are offset by certain disadvantages. If the process average and sample size are large enough, a *lower* control limit exists, and the sampling plans in Part V cannot be used. But since the process average and sample size are often so small that no lower control limit exists, this disadvantage may often be irrelevant. A more serious disadvantage appertains to double and sequential sampling. An important feature of the control-chart method is graphic presentation of inspection results; this facilitates detection of trends in quality. If one of the single-sampling plans in Part V is used for control sampling, the acceptance number of the sampling plan can be inserted on the graph as the UCL. But if one of the double- or sequential-sampling plans is used, graphic presentation is not effective. For one thing, the acceptance numbers in double and sequential sampling cannot be plotted as the UCL in any simple manner.

It is necessary to weigh the sampling plans in Part V against quality-control schemes with control limits fixed simply by $\bar{c} \pm 3\sqrt{\bar{c}}$. If graphic analysis of inspection results is important, single-sampling plant are desirable. If an LCL is needed, the sampling plans in Part V cannos be used at all. If only a UCL is needed, the sampling plans in Part V are useful. If graphic presentation is not too important, double- and

* There is no reason, other than difficulty of computation, why double- and sequential-sampling plans could not be furnished in terms of $\bar{c}$, as is done for single sampling under the usual control-chart procedure.

sequential-sampling plans may be used for control sampling in essentially the same manner they are used for acceptance sampling.*

* For a comprehensive and up-to-date treatment of quality control, see Eugene L. Grant, *Statistical Quality Control*, McGraw-Hill Book Company, Inc., New York, 1946.

APPENDIX TO PART II

RELATION OF SAMPLING INSPECTION TO DESIGN SPECIFICATIONS

1. DESIGN AND ACCEPTANCE REQUIREMENTS

In considering the general problem of specifications, it is necessary to distinguish carefully between two entirely different kinds of specification requirement. The first kind of requirement defines what is really wanted, the second kind defines the method of deciding whether or not that which was wanted has been really obtained. In other words, one kind of requirement is concerned with *description*, the other with *enforcement.* The two types of requirement will be called *design requirements* and *acceptance requirements* respectively. They may be described in more detail as follows.

1.1. Design Requirements

Design requirements define the dimensions, physical and chemical properties, and performance characteristics that the product offered under the specification is supposed to have. While such requirements may sometimes refer to the average quality of the product, more often they refer to tolerances and minimum-quality levels that apply to each unit or item of the product. For example, a typical requirement in a specification for half-hard brass rods might state that the tensile strength of the product must be 58,000 lb. per sq. in. This generally means that if every rod in a lot were pulled in tension, each test result should exceed this minimum.

1.2. Acceptance Requirements

Acceptance requirements define the amount and kind of evidence considered necessary to establish that the product offered under the specification does or does not comply to a satisfactory extent with the design requirements. There are two parts to acceptance requirements.

a. Methods of Test.—These comprise the test procedure to be used on individual items of product. Methods of testing are primarily in the province of engineering; nevertheless, there is always the problem of deciding whether a given test is sufficiently correlated with the corresponding design requirement, and thus the field of probability and statistics is involved in the study of test methods.

b. Sampling Requirements.—These comprise the lot-by-lot sampling procedure whereby each inspection lot of product is accepted or rejected. These procedures are based primarily on the laws of probability and statistics, as shown in Parts II and IV.

2. WRITING ACCEPTANCE SPECIFICATIONS

The problem of formulating design requirements is very different from that of formulating acceptance requirements. In brief, the design portion of a specification must be written by one who knows exactly what purpose the product is to serve and what tolerances and characteristics can be attained and maintained without undue expense. On the other hand, the acceptance portion of the specification must be written with the economics of inspection, in its broadest aspect, as the basic consideration. The fundamental problem in the acceptance portion of the specification is to determine a sampling-inspection procedure that balances the cost of inspection against losses from accepting subspecification and rejecting specification product. Some aspects of this problem that must be taken into account include availability and distribution of inspection and laboratory personnel, need of or demand for product regardless of quality level, percentage of defective product considered acceptable, likelihood that defective product will be submitted for inspection, maximum risks of rejecting good product and accepting bad product, degree of correlation of methods of test with design requirements. The writer of the acceptance portion of a specification does not need to know the exact engineering details of the manufacture and intended application of the material. His essential prerequisites are

a. General engineering knowledge and "horse sense."

b. Some general knowledge of the process of manufacture of the product in question.

c. Close liaison with and knowledge of the capabilities of line-inspection personnel and close liaison with the writer of the design specification.

d. Familiarity with modern statistical methods, in particular, the mathematical theory of acceptance sampling.

Frequently, acceptance requirements are written by engineers who are primarily concerned with design requirements; the result is that the acceptance requirements are in general far cruder than the design portions of the specifications. The sampling methods used may be unsound from the point of view of either the receiver or the supplier or of both, the protection afforded by the sampling plan may be slight, inspection loads may be erratic, the specified inspection procedures may in large measure be unsound and unworkable and not tied in properly with the design requirements. The consequence is inability to determine whether or not the submitted product is satisfactory.

On the other hand, good sampling inspection not only solves these problems but contributes to improvement in design specifications. The reason is simple: when an engineer stops to think carefully how he is going to find out whether he has bought what he wanted to buy, he soon realizes the necessity for a clear definition of what he really wanted to buy.

3. ORGANIZATION OF DESIGN AND INSPECTION ACTIVITIES IN THE PLANT

It is often desirable for a plant to organize its design and inspection activities along the following lines:

a. All design requirements for specifications, contracts, and orders should be the responsibility of the respective appropriate design or technical units. These units should prescribe the engineering details of the test methods.

b. An inspection unit should have cognizance over all acceptance requirements in specifications; this unit should prepare and review appropriate, efficient sampling-inspection plans both within the plant and in contracts and orders (excluding methods of test). All specifications (and all contracts and orders that require inspection) should be automatically routed through this unit for insertion of sampling-inspection requirements.

c. Whenever possible, the inspection unit should make technical suggestions to the design unit as to possible improvements in the design requirements of specifications and should assist writers of design requirements to obtain realistic quantitative design requirements by means of statistical analysis of inspection data.

Part III

A STANDARD SAMPLING-INSPECTION PROCEDURE

CHAPTER 7

GENERAL DESCRIPTION OF THE STANDARD PROCEDURE

1. PURPOSE OF PART III

The set of tables given in Part V of this book can be used simply as a catalogue of sampling plans from which, as the need arises, a particular plan can be selected in accordance with the general principles outlined in Part II. Alternatively, these tables can be used as the basis for an integrated inspection program involving the coordinated selection and operation of several or many sampling plans. The inspection program may apply to incoming product from several suppliers or from the same supplier at different times, to semifinished products produced by different departments of the same firm and inspected prior to further processing, to finished products inspected prior to final release, or to products inspected in order to check the functioning of the processes producing them. Inspection may be conducted by a relatively small concern inspecting only small quantities of a few products; by a large, nation-wide concern inspecting many products purchased in large quantities from many suppliers or produced by many different departments of the firm; or by a governmental organization purchasing large quantities of many products from many suppliers.

Whatever the kind of product inspected or the nature of the organization conducting inspection, an integrated inspection program requires a standard procedure for selecting sampling plans from the tables of Part V, for operating these plans, and for using the results of sampling inspection to change the sampling plans. Part III describes such a standard procedure. This standard procedure cannot, of course, apply in minute detail to every organization conducting inspection. Many of the details of the procedure will have to be adjusted to the particular circumstances of the particular organization—the chain of responsibility, the division of authority, the physical facilities available for inspection, special features of the products inspected, and so on. The broad outlines of the standard procedure described in this part, and even many of the details, are, however, applicable to most inspection programs. About all that is necessary for the procedure to be applicable is that there be some separation of responsibility for two sets of decisions: the establishment of standards applicable to several sampling plans and the adjustment of each sampling

plan to these standards in view of the special circumstances under which the sampling plan is to be used. Section 5 of this chapter indicates more concretely the kind of separation of responsibility to which the standard procedure is adapted.

This part is intended primarily for those concerned with the "how" rather than the "why" of sampling. An effort has been made to include in this part all information needed to use sampling plans for inspection of attributes, so that this part may be used, if desired, without reference to Part II. Most users of this part will, however, find careful study of Part II well worth while; it will give them a fuller appreciation of what sampling inspection can and does accomplish and will put them in a far better position to adapt the standard procedure to their own needs.

2. APPLICABILITY OF THE STANDARD PROCEDURE

The standard procedure is applicable to inspection on an *attributes* basis, that is, where each item inspected is merely classified defective or nondefective. It is not applicable to inspection on a *variables* basis, that is, where the quality of each item of product is expressed as a value on a continuous scale. The procedure is designed primarily to determine the acceptability of inspection lots of product (acceptance sampling) although it may also be used to determine whether corrective action on the process of manufacture is needed to improve and maintain quality of product (control sampling). The standard procedure will probably be found most useful for relatively large-scale sampling-inspection and quality-control programs.

Under this procedure, the quality of product is measured by the percentage of items that are defective. If the percentage of defective items is low, the product may be referred to as good product; if high, as bad product.

3. MEANING OF "ACCEPTANCE" AND OF "SUPPLIER"

The words "acceptance" and "supplier" are used here in a broad sense. "Acceptance" may have any one of three meanings:

a. Acceptance of incoming product.

b. Acceptance of finished product as suitable for shipment.

c. Acceptance of partly finished product at an intermediate point in the process of manufacture as suitable for further processing.

Similarly, the word "supplier" is used to refer to the source of the product inspected, whether that source is another concern from which the product inspected is purchased or a department of the concern conducting the inspection.

4. STEPS IN APPLYING THE STANDARD PROCEDURE

The major steps involved in applying the procedure are

a. Establishment of standards.
b. Installation of the procedure.
c. Operation of the procedure.
d. Review of past results.

These steps may be further subdivided as shown in Table 7.1.

TABLE 7.1
STEPS IN APPLYING THE STANDARD SAMPLING-INSPECTION PROCEDURE

Step	Where discussed	
	Part II	Part III
	PAGE	PAGE
Establishment of standards:		
Decide what shall be an item of product	39	81
Prepare a list of defects and, if needed, a classification of defects	39	82–83
Fix an acceptable quality level (AQL) for each class of defects	45	83–85
Fix an inspection level for the product	45-46	85–86
Installation of procedure:		
Arrange for the formation of inspection lots	40-43	87–90
Decide what type of sampling shall be used: single, double, or sequential	33–38	91–97
Extract sampling plan(s) to be used by the inspector	44–45	97–99
Operation of procedure:		
Draw sample items from each inspection lot	47–53	101–102
Inspect each sample item	40	103
Determine whether to accept or reject the inspection lot (if sampling for acceptance) or whether to take corrective action on the process (if sampling for control)	13–16 55–67	103–109
Review of past results:		
Compute process average(s)	60–61	115–120
Determine whether to tighten or reduce inspection on future inspection lots	46-47	120–125

Chapters 8 to 12 discuss these steps for acceptance sampling. Chapter 13 outlines the modifications necessary to make the standard procedure applicable to control sampling.

5. RESPONSIBILITY FOR DIFFERENT PHASES OF THE STANDARD PROCEDURE

The division of responsibility for these steps can best be indicated by a concrete example. Assume that a firm is purchasing the same product from a number of suppliers and that the acceptability of the submitted product is to be determined by sampling inspection. It is assumed that the firm wishes to apply the same standards to the different suppliers.

Some executive in the firm will have responsibility for establishing standards for the product—let us call him the "inspection executive." The inspection executive will have responsibility for deciding what shall be considered an item of product, for preparing a list of defects, and for fixing an acceptable-quality level (AQL) for each class of defects and an inspection level for the product. These decisions will then apply to all suppliers of the product. Both economic and technical considerations are involved in these decisions. Accordingly, the inspection executive will establish the defects classification and acceptable-quality level in collaboration with the design, production, purchasing, and selling executives and others concerned with the quality desired in the product and the cost of maintaining that quality.

The installation of the procedure for a particular supplier subject to the standards established by the inspection executive will be the function of the person responsible for supervising inspection—let us call him the "inspection supervisor." The inspection supervisor will arrange for the formation of inspection lots, decide on the type of sampling to be used, and extract sampling plans. These decisions require detailed knowledge of the product of each supplier and of the conditions under which inspection is to be conducted. The inspection executive cannot be expected to possess this knowledge.

The actual operation of the procedure on each submitted inspection lot will be the responsibility of the inspector of items—let us call him the "line inspector." He will draw items from each inspection lot, inspect each sample item, and determine from the sampling plan he is using whether the results of inspection require the acceptance or the rejection of the submitted inspection lot. He will do his work in accordance with instructions furnished by the inspection supervisor and under his direct supervision.

The review of past results is in part the function of the inspection supervisor—in so far as it is used to determine whether a change should be made in the sampling plan for an individual supplier; in part, the function of the inspection executive—in so far as it is used to determine whether all suppliers are meeting the standards initially laid down, whether these standards should be revised, or whether changes in purchasing arrangements are desirable. In general, the inspection supervisor will compute (or supervise the computation of) the process averages and will use the process averages to determine whether to tighten or reduce inspection. The computed process averages will then be turned over to the inspection executive who may, in turn, send them to, or summarize them for, other departments such as the purchasing or production departments.

This broad description of responsibilities is applicable to a very wide

variety of inspection programs. At one extreme, it may apply to a small firm purchasing a single product from two or three suppliers in relatively small amounts and inspecting the product after it is received at the home plant. At the other extreme, it may apply to a national concern or a branch of the Federal government (for example, the Navy or Army), purchasing hundreds or thousands of products, each from many suppliers, and inspecting many products at the supplier's premises before shipment. In a small organization some of the functions described above may be combined. The inspection executive and the inspection supervisor, or the inspection supervisor and the line inspector, may be the same person. In large organizations, there may be even further specialization and division of functions. The essential feature that is likely to characterize almost all organizations conducting integrated inspection programs is the division of responsibility for the establishment of standards, on the one hand, and for the administration of inspection, on the other.

With minor changes, this description is also applicable to inspection programs for product other than incoming material—the inspection of partly finished product to determine suitability for further processing, the inspection of finished product to determine suitability for shipment, the inspection of product to determine whether a process is satisfactory.

CHAPTER 8

ESTABLISHMENT OF STANDARDS

1. DECIDING WHAT SHALL BE AN ITEM OF PRODUCT

An item of product is defined as a single member of the inspection lot, usually though not necessarily a single article. The first step in establishing standards is to determine exactly what the word "item" is to mean for the particular product to be inspected.

Products that are manufactured in discrete pieces generally offer little difficulty; for example, in the inspection of coats, an item is obviously one coat; in the inspection of electric fuses, an item is obviously one fuse.

There are some products, however, for which an item is not so readily defined—products produced in a continuous stream, such as textiles, and those produced in large batches, such as chemicals. These products present no natural division into discrete pieces and the meaning of the word "item" must therefore be decided more or less arbitrarily. In general, the appropriate definition of an item for such products depends on the use to be made of the product. For example, if a bolt of cloth is to be cut into pieces of approximately a yard for manufacture into garments, the yard is the appropriate item, since the percentage of defective yards is an estimate of the percentage of garments made from this bolt that will contain defective cloth. Such an estimate could be derived from the percentage of defective feet, say, only if the relation among defects in contiguous feet were known.

The use to be made of the product may affect the definition of an item even when the product consists of discrete pieces. For example, in the inspection of shoes, a pair, rather than a single shoe, is generally the appropriate item. However, if the two members of a pair are combined at random from the items of a given size (as is usually the case with socks but not with shoes), the individual member of a pair may be taken as the item, since the percentage of defective pairs can then be estimated from the percentage of defective lefts and the percentage of defective rights.* The use of the individual member as the item may then have the advantage of increasing the size of the inspection lot.

* If p_L is the proportion of defective lefts, p_R, of defective rights, and p, the estimated proportion of defective pairs, then, under the assumed conditions,

$$p = 1 - (1 - p_R)(1 - p_L)$$

2. PREPARING A LIST OF DEFECTS AND, IF NEEDED, A CLASSIFICATION OF DEFECTS

Having defined the item of product, the inspection executive next lists the defects an item may have; if necessary, he classifies the defects as major defects, minor defects, or irregularities; and prescribes the method of inspecting an item for these defects. In this step he ordinarily needs technical assistance, particularly to determine the effect of any deviation from specifications on the performance of an item or on further operations. The list of defects should be comprehensive, since otherwise line inspectors may fail to notice even fairly obvious imperfections in the product; and it should be detailed and explicit, to attain uniformity and consistency among line inspectors.

A defect is any deviation from the requirements of a specification, design, contract, or order. A defect is usually classified as major if it will cause a failure of the item to function as intended, and as minor if it will impair the efficiency, shorten the lifetime, or otherwise reduce the value of an item. A defect that represents a departure from good workmanship but does not affect the performance or life of an item is usually classed as an irregularity. For simplicity in exposition, we shall consider only major and minor defects throughout the remainder of Part III.

A specimen list and classification of defects is shown in Table 8.1.

TABLE 8.1
SPECIMEN LIST AND CLASSIFICATION OF DEFECTS FOR COOKING SPOON
Model No. Z-2437X

Quality characteristics	Major defect	Minor defect
1. Formation of bowl and curvature of handle (Check with special gauge)	Outside major limits on dial of gauge	Outside minor limits on dial of gauge but not outside major limits
2. Workmanship	Burrs or ragged edges on bowl or handle	Embossing too deep, causing marks on reverse side
3. Position of hole in handle (Check with plug gauge)	Out of position enough to overlap embossing	Out of position over $\frac{1}{32}$ in. but not overlapping embossing
4. Finish Should be equal to a No. 6 commercial finish for corrosion-resisting steel (Compare with approved specimens)	Cracked, nicked, or pitted	Scratched, wrinkled, or discolored Not within limits of No. 6 finish
5. Identification marking	Missing	Distorted or illegible

An item is called defective if it contains one or more defects, a major defective if it contains one or more major defects, and a minor defective if it contains one or more minor defects. An item may be a major defective only, a minor defective only, or both a major defective and a minor defective. If an item has neither a major nor a minor defect it is called nondefective.

3. FIXING AN ACCEPTABLE-QUALITY LEVEL (AQL) FOR EACH CLASS OF DEFECTS

After listing and classifying possible defects in the individual items, the inspection executive next fixes quality standards for the aggregate of product, or the inspection lot. This is done by deciding what acceptable-quality level (AQL) to use for each class of defects.

The acceptable-quality level (AQL) of a sampling plan is a quality such that the sampling plan will result in the acceptance of 95 percent of submitted inspection lots of that quality. A choice may be made among the fourteen AQL classes listed below.

AQL CLASSES (% defective)
0.024 and under 0.035
0.035 and under 0.06
0.06 and under 0.12
0.12 and under 0.17
0.17 and under 0.22
0.22 and under 0.32
0.32 and under 0.65
0.65 and under 1.2
1.2 and under 2.2
2.2 and under 3.2
3.2 and under 4.4
4.4 and under 5.3
5.3 and under 6.4
6.4 and under 8.5

If there are two classes of defects, major and minor, a separate AQL must be established for each class. The AQL, major, is generally more exacting (smaller) than the AQL, minor; for example:

AQL, major—0.15 percent defective
AQL, minor—2.2 percent defective

In fixing an AQL, a balance must be struck between quality desired and quality attainable. If the AQL is more exacting than can be attained under existing production conditions, an excessive number of inspection lots will be rejected. On the other hand, if the AQL is not exacting enough, an excessive amount of inferior product may be accepted.

Perhaps the best way to go about fixing an AQL is first to estimate the best quality of product (that is, the smallest percent defective) that it is reasonable to expect suppliers of this product (that is, firms from whom the product is being purchased, production lines the output of which is being inspected, and so forth) to submit under existing conditions of manufacture and at the current price of the product. Such an estimate should be based on the past performance of suppliers of this product and of similar products.

If the AQL were set equal to this estimate of currently attainable quality, too many rejections might occur since (*a*) all suppliers might not achieve this quality, (*b*) even suppliers who did achieve it for a time might not at other times, and (*c*) even if all suppliers achieved it most of the time, 5 percent of submitted inspection lots would be rejected (recall the definition of AQL) and this may be a larger percentage of rejections than is justified. In order to reduce rejections, it may sometimes, therefore, be desirable to fix the AQL at a higher percentage of defectives than the currently attainable quality. On the other hand, the currently attainable quality may sometimes be so poor that the value of the aggregate of product would be greatly reduced by the large number of defectives it contains. In such cases, it may be desirable to fix the AQL at a lower percentage of defectives than the currently attainable quality; if this is done, however, rejections will be large (definitely above 5 percent) until the suppliers succeed in changing their production or inspection processes so as to reduce the percentage of defectives in submitted inspection lots.

The following factors influence the amount by which it is desirable to have the AQL depart from the estimate of currently attainable quality:

a. Reduction in value of product occasioned by defectives. This is of fundamental importance. Sometimes the loss occasioned by a defective is so large that if there are more than a small percentage of defectives the product will be worth less than it costs. In such cases, it may be desirable to fix the AQL at or below this break-even percentage of defectives even if this should involve the rejection of a large proportion of submitted inspection lots.

b. Class of defects. This follows from (*a*). Major defects ordinarily reduce the value of product more than minor defects. Consequently, the AQL should ordinarily be lower (that is, more exacting) for major defectives than for minor defectives.

c. Effect of defective product on later processing and assembling. If defective product results in marked waste of material and time during later processing or assembling, the AQL should be more exacting (lower). The number of items that are assembled may also play a part. For example, consider an inspection lot of radio tubes of which 5 percent are

defective; if these tubes are assembled at random for use in three-tube radio sets, approximately 14 percent of the sets will contain at least one defective tube, whereas if they are assembled for five-tube sets, approximately 23 percent of the sets will contain at least one defective tube. Consequently, if it is important to keep the percentage of defective sets as low for five-tube as for three-tube sets, the AQL will have to be more exacting (lower) when tubes are intended for use in five-tube sets than when they are intended for use in three-tube sets.

d. Suppliers' average quality and urgency of demand for product. If the quality that suppliers can furnish is poor and cannot readily be improved and if output is needed badly, the AQL may have to be higher than otherwise desired; if it is not higher, excessive rejections may occur. If the suppliers' average quality can be expected to improve over a period of time, gradual lowering of the AQL may be desirable.

e. Kind of defects included in the defects list. In order to permit consistent inspection and to keep close control over the quality of product submitted, it will sometimes be desirable to include in the defects list defects whose effect on functioning is questionable or to define defects more stringently than is strictly necessary for the use to which the product is to be put. When this is done, inspection subjects the item to a severer test than the item will receive when it is used and the AQL should accordingly be more liberal than if each item were subjected to a less severe test.

4. FIXING AN INSPECTION LEVEL

In order to determine which of the sampling plans in this book to use on a particular product, it is necessary to fix an inspection level for the product as well as an AQL. The term "inspection level" is used here to designate the relative amount of inspection given the product.

The standard procedure permits a choice among five different inspection levels, identified by Roman numerals from I to V. Inspection level III is considered the normal amount of inspection. For a given inspection lot size and type of sampling, the relative amount of inspection required by each of the other levels is given in Table 8.2.

TABLE 8.2
RELATIVE AMOUNT OF INSPECTION REQUIRED BY EACH INSPECTION LEVEL

Inspection level	Relative amount of inspection
I	½ as much as level III
II	¾ as much as level III
III	The normal amount
IV	1½ times as much as level III
V	Twice as much as level III

The risk of rejecting an inspection lot whose quality is exactly equal to the AQL is the same (5 out of 100) for all inspection levels, but the risk of accepting an inspection lot whose quality is worse than the AQL varies from level to level. A high inspection level means relatively more inspection and consequently less risk of accepting inspection lots whose quality is worse than the AQL. The higher the inspection level, the greater the protection against acceptance of low-quality inspection lots.

The following factors should, therefore, be considered in choosing the inspection level for a particular product:

a. The risk that can be run of accepting inspection lots whose percent defective is worse than the AQL. If a small risk is desired, the inspection level must be high.

b. Availability of inspectors and inspection facilities. If these are limited, the amount of inspection must be small and the inspection level consequently low.

If there are two classes of defects for a particular product, it will often seem desirable to require a lower inspection level (that is, less inspection and therefore less protection) for minor defects than for major defects. This procedure has, however, the serious practical disadvantage that the inspector has to perform a different amount of inspection for each class of defects. To avoid this complication, it is generally better to establish one inspection level for a given product and use it for both major defects and minor defects.

CHAPTER 9

INSTALLATION OF THE PROCEDURE

1. ARRANGING FOR THE FORMATION OF INSPECTION LOTS

1.1. Introduction

An inspection lot is a collection of items that are submitted for inspection and that are to be accepted or rejected *as a group*. Arranging for the formation of inspection lots is the first step in the installation of the standard sampling-inspection procedure.

Arrangements for the formation of inspection lots should include provision for the following:

a. Type of inspection lot.
b. Inspection-lot size.
c. Contents of inspection lot.
d. Subdivision of inspection lot.
e. Physical arrangement of product in inspection lot.
f. Identification and segregation of inspection lot.

1.2. Type of Inspection Lot

Inspection lots may be either stationary or moving. The differences between the two are summarized below:

	Manner of presentation	Manner of drawing samples
Stationary inspection lot	All items in inspection lot are presented for inspection simultaneously; inspection lot is stacked up for inspection	All sample items may be drawn at one time
Moving inspection lot	Items in inspection lot are presented for inspection a few at a time; inspection lot flows past the point of inspection	Sample items must be drawn a few at a time as the inspection lot flows past the point of inspection

Stationary inspection lots are distinctly superior to moving inspection lots for inspection purposes and should be used whenever feasible. The advantages of a stationary inspection lot are (*a*) it is easier to control its disposition, (*b*) it is easier to preserve its identity, (*c*) it is easier

to take successive samples from the entire inspection lot, which greatly facilitates the use of sequential sampling. Indeed, this economical type of sampling is not likely to be feasible for moving inspection lots.

Stationary inspection lots, however, may sometimes have disadvantages sufficiently serious to require resort to moving inspection lots. Stationary inspection lots represent the accumulation of sizable inventories of product and, when stacked in such a way that every item in the inspection lot is accessible to the inspector, may take up considerable space. When the item is valuable or space is limited, it may be necessary to resort to moving inspection lots. Moving inspection lots offer few space problems and require minimum accumulation of inventory.

These considerations ordinarily lead to the use of stationary inspection lots for receiving inspection and end-product inspection and the occasional use of moving inspection lots for intermediate inspection of partly finished product or component parts.

1.3. Inspection-lot Size

The term "inspection-lot size" means the number of items in the inspection lot. Arrangements for the formation of inspection lots should include a statement of the usual number of items to be included in each inspection lot and the permissible variation in this number from inspection lot to inspection lot, for example:

Usual inspection-lot size	1,400 items
Minimum inspection-lot size	1,200 items
Maximum inspection-lot size	1,600 items

The general rule is to make the usual inspection-lot size as large as possible and the permissible variation as small as possible.

Making the usual inspection-lot size as large as possible promotes economy of inspection because the sampling plans in Part V require inspection of a smaller percentage (though a larger number) of sample items from large inspection lots than from small ones. Keeping the variation in inspection-lot size down to a minimum permits close control over the inspector's workload, for stable inspection-lot sizes reduce the fluctuations in the percentage of product inspected. Large inspection-lot size also results in greater protection against the acceptance of product whose percent defective is worse than the AQL because the *absolute* number of sample items per inspection lot required by the sampling plans in Part V is larger for a large inspection lot than for a small one, even though the *relative* number of sample items per inspection lot is smaller. This is illustrated by Table 9.1.

TABLE 9.1
ADVANTAGES OF DOUBLING THE INSPECTION-LOT SIZE
(When AQL is 4%, the inspection level is III, and single sampling is used)

	Inspection-lot size	
	600	1,200
Number of items inspected per inspection lot	75	115
Percentage of items inspected per inspection lot	12.5%	9.6%
Chance of accepting an inspection lot containing 10% defective items	37 out of 100	20 out of 100

1.4. Contents of Inspection Lot

An effort should be made to segregate items for inspection in such a way that each inspection lot is as homogeneous as possible. A homogeneous inspection lot is one in which no one part has a substantially larger percentage of defectives than any other part, fluctuations in the percentage defective from part to part of the inspection lot being only such as would occur by chance.

While no method of assembling inspection lots can guarantee their homogeneity, the likelihood of obtaining homogeneous inspection lots is greatest when each inspection lot is confined to product manufactured under essentially the same conditions.

By "product manufactured under essentially the same conditions" is meant

a. Product turned out from the same batch of raw material.

b. Product manufactured by the same production or assembly line with the same molds, dies, patterns, and so forth.

c. Product turned out during a natural unit of time, such as an hour, a day, a shift, and so forth.

Unfortunately, it is rarely feasible to satisfy all these conditions when arranging for the formation of inspection lots because a high degree of homogeneity can usually be obtained only at the cost of a great reduction in inspection-lot size. This would have the undesirable consequence of increasing the cost of inspection, since the percentage of items inspected from a small inspection lot is greater than from a large one.

It is, therefore, necessary to strike a compromise between homogeneity and economy of inspection. Inspection lots must be large enough to permit economical inspection but small enough to be fairly homogeneous. The exact point at which this balance should be struck depends on the conditions under which inspection is conducted. If quality is poor or extremely variable it is desirable to have small homogeneous

inspection lots; if quality is good it is desirable to have large inspection lots even at the expense of homogeneity.

1.5. Subdivision of Inspection Lot

When economic considerations necessitate the formation of large nonhomogeneous inspection lots, an effort should be made to separate the product in each inspection lot into homogeneous sublots, each sublot consisting of product of common origin in the process of manufacture. These sublots represent what would have been individual homogeneous inspection lots if it had not been necessary to combine them into one large inspection lot in order to reduce the cost of inspection.

Even though an inspection lot is subdivided into a number of homogeneous sublots, acceptance or rejection is still to be of the inspection lot as a whole and not of the individual sublots. The purpose of subdivision is to make it possible to give each sublot proportional representation in the sample or samples. This is explained in Sec. 1.3 of Chap. 10.

1.6. Physical Arrangement of Inspection Lot

The items in a stationary inspection lot should be stacked so that all are equally accessible to the inspector. Furthermore, it should be possible for the inspector to draw any item in the inspection lot with reasonable ease. In moving inspection lots, the arrangement of product can be practically ignored, since sample items are drawn as the inspection lot is produced and before it is stacked up en masse.

1.7. Identification and Segregation of Inspection Lots

Arrangements for the formation of inspection lots should include provision for identification of each inspection lot, by means of a serial number, for at least as long as it takes to decide whether the inspection lot is to be accepted or rejected. This is relatively easy with stationary inspection lots since such inspection lots may be identified simply by segregating them during inspection.

Identification of a moving inspection lot is somewhat more difficult. One method is to tag the first and last items of each inspection lot with the serial number of the inspection lot and allow these tags to remain until it has been determined whether the inspection lot is to be accepted or rejected. When this is done, it is necessary, of course, to be sure that product from other inspection lots does not find its way between the two tagged items.

2. DECIDING THE TYPE OF SAMPLING TO USE

The sampling plans in Part V are of three types: single, double, and sequential. The second step in the installation of the standard procedure is to decide which type of sampling to use.

In single sampling, a decision to accept or reject an inspection lot is reached after inspection of one sample from that inspection lot. In double sampling, three decisions are possible after inspecting the first sample from an inspection lot: accept the inspection lot, reject the inspection lot, or take a second sample, but in any event a decision to accept or reject the inspection lot is reached after not more than two samples from that inspection lot have been inspected. In sequential sampling, one, two, three, or more samples may need to be inspected before a decision to accept or reject an inspection lot is reached.

The three types of sampling plan are illustrated in Table 9.2. The method of using each type of sampling plan is shown on the following pages in terms of these illustrative plans.

TABLE 9.2
SPECIMENS OF THE THREE TYPES OF SAMPLING PLAN

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	225	225	14	15
Double	First	150	150	9	24
	Second	300	450	23	24
Sequential	First	50	50	1	6
	Second	50	100	3	9
	Third	50	150	7	13
	Fourth	50	200	10	16
	Fifth	50	250	13	19
	Sixth	50	300	16	22
	Seventh	50	350	19	25
	Eighth	50	400	24	25

All three specimen sampling plans given above have an AQL of about 4 percent defective (AQL class 3.2 percent to 4.4 percent defective). All three plans reject approximately 5 out of 100 inspection lots containing 4 percent defective items. Furthermore, all three plans afford practically the same protection against the acceptance of inspection lots whose quality is worse than 4 percent defective. This is shown by the curves in Fig. 9.1.

THE SPECIMEN SINGLE-SAMPLING PLAN

Inspect a sample of 225 items

If the sample contains 14 or fewer defective items, accept the inspection lot.

If the sample contains 15 or more defective items, reject the inspection lot.

THE SPECIMEN DOUBLE-SAMPLING PLAN

Inspect a first sample of 150 items

If the first sample contains

9 or fewer defective items, accept the inspection lot.

Between 9 and 24 defective items,

24 or more defective items, reject the inspection lot.

Inspect a second sample of 300 additional items

If the first and second samples *combined* contain

23 or fewer defective items, accept the inspection lot.

24 or more defective items, reject the inspection lot.

Acceptance

Rejection

THE SPECIMEN SEQUENTIAL-SAMPLING PLAN

Inspect a first sample of 50 items

If the first sample contains

1 or fewer defective items, accept the inspection lot.	Between 1 and 6 defective items,	6 or more defective items, reject the inspection lot.

Inspect a second sample of 50 additional items

If first and second samples *combined* contain

3 or fewer defective items, accept the inspection lot.	Between 3 and 9 defective items,	9 or more defective items, reject the inspection lot.

Inspect a third sample of 50 additional items

and so on, through 4 more samples. Then, if the inspection lot has still not been accepted or rejected,

Inspect a last (eighth) sample of 50 additional items

If all samples *combined* contain

24 or fewer defective items, accept the inspection lot.	25 or more defective items, reject the inspection lot.

Acceptance

Rejection

These OC curves show how often, on the average, each of the above sampling plans will accept inspection lots of any given quality. The horizontal scale represents the quality of submitted inspection lots and runs, in this example, from 0 to 12 percent defective. The vertical scale shows the percentage of submitted inspection lots that will be

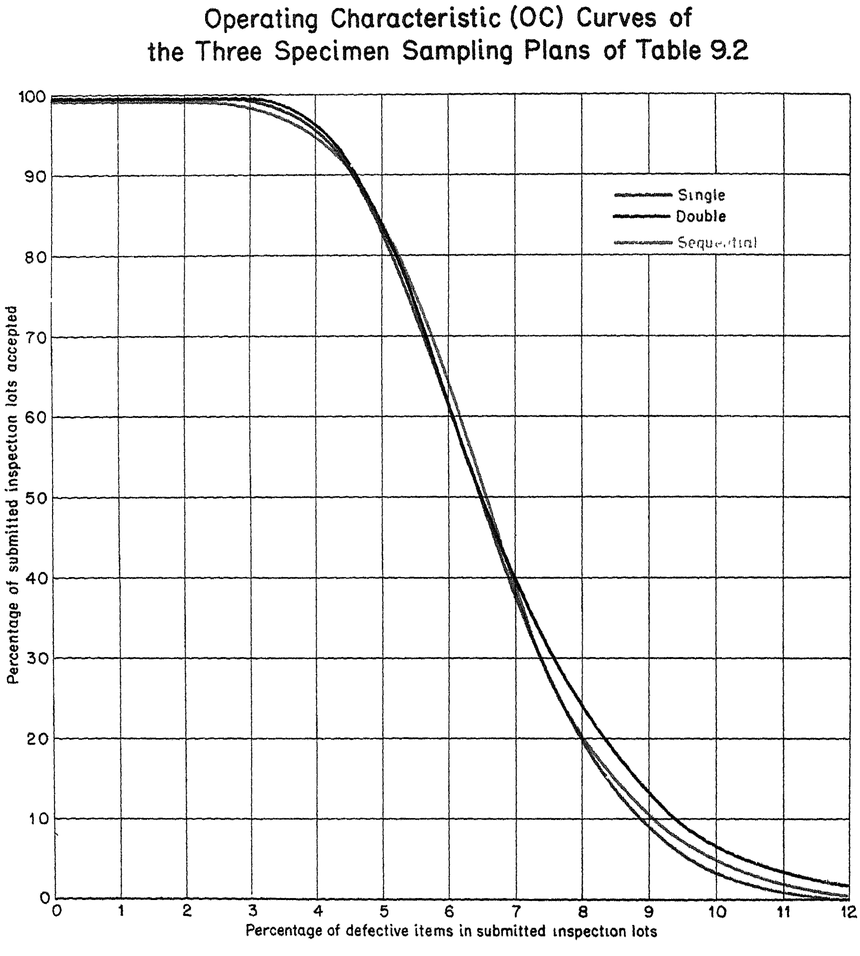


FIG. 9.1.

accepted. The red curve, for example, shows that the sequential-sampling plan accepts inspection lots containing 7.5 percent defective items only 31 times out of 100; and the green and blue curves show that the single- and double-sampling plans accept practically the same number of such inspection lots. Thus, one important feature of these three par-

Average Amount of Inspection for Four Sets of Single, Double, and Sequential Sampling Plans Having Almost the Same Operating Characteristic (OC) Curves

(For single sampling, entire samples inspected; for double and sequential sampling, entire first sample inspected, inspection of later samples curtailed as soon as decision can be reached.)

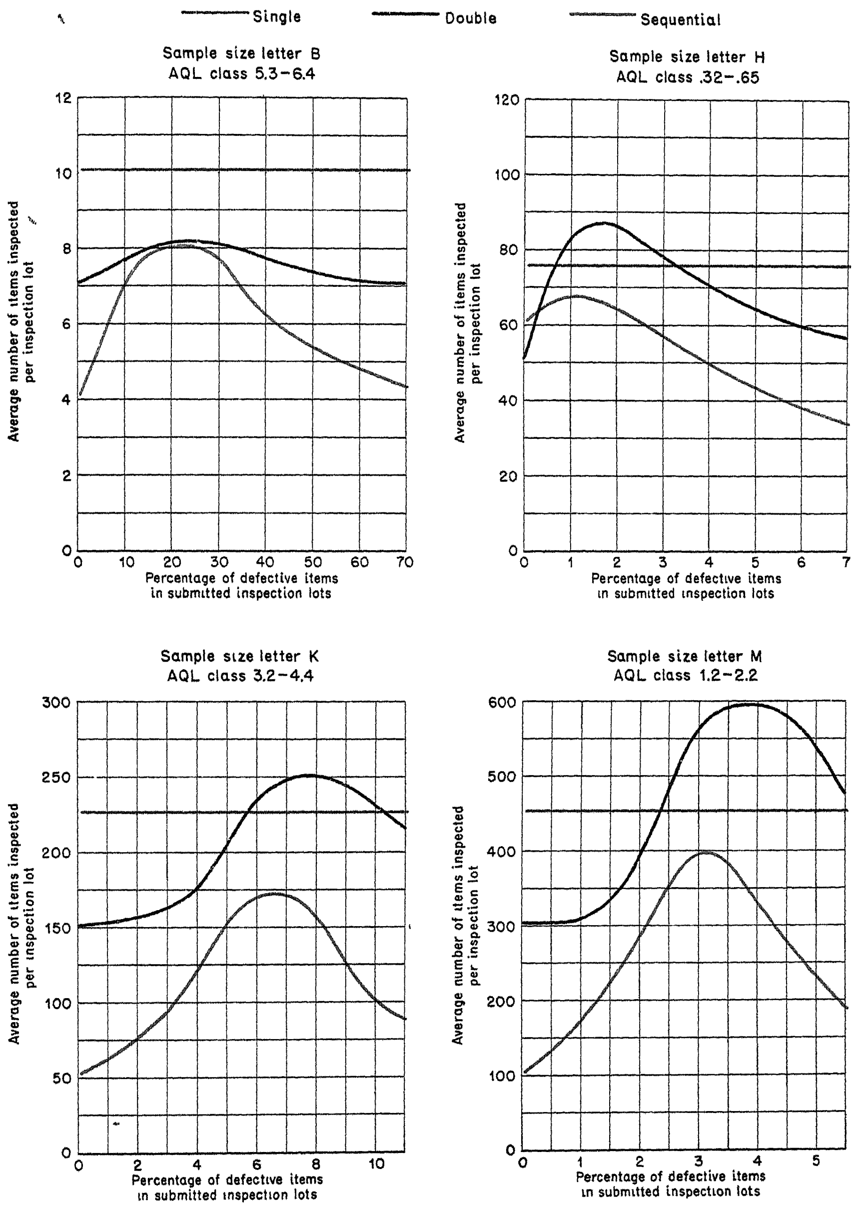


FIG. 9.2.

ticular plans is that their OC curves are practically the same; consequently, these three plans provide about the same amount of protection.

Such an arrangement of sampling plans in sets of three—one single, one double, and one sequential—all having practically the same OC curve, is followed in Part V. Consequently, no account need be taken of the protection provided by the three types of sampling in choosing among plans in any one set. The choice can be based entirely on other factors. Prime consideration can be given, for example, to the average amount of inspection required per inspection lot, sequential sampling requiring least; double sampling, more; and single sampling, most.

The exact amount of inspection saved by using double or sequential sampling in place of single sampling cannot be stated simply, however, since it depends on the quality of inspection lots submitted and on the particular sampling plans compared. Figure 9.2 shows the average number of sample items per inspection lot required by each type of sampling for four of the sets of sampling plans in Part V, including the set of three sampling plans used for illustrative purposes above (sample-size letter K, AQL class 3.2 to 4.4 percent defective). As can be seen from Fig. 9.2, the average saving from the use of double or sequential sampling is greatest for product of very high or very low quality and least for

TABLE 9.3
FACTORS INFLUENCING CHOICE OF TYPE OF SAMPLING

Factor	Single sampling	Double sampling	Sequential sampling
Protection against rejection of good inspection lots and acceptance of bad inspection lots		Practically equal	
Average number of items inspected per inspection lot	Most	Intermediate	Least
Variability in number of inspected items from inspection lot to inspection lot	None	Some	Some
Sampling costs when samples can be drawn as needed	Most expensive	Intermediate	Least expensive
Sampling costs when all samples must be drawn at once	Least expensive	Most expensive	Intermediate
Estimation of average inspection-lot quality (if estimate is based on large number of inspection lots, differences from one type of sampling to another may not matter)	Most precise	Intermediate	Least precise
Training inspectors to use plans	Easiest	Intermediate	Hardest
Psychological: "give inspection lot more than one chance"	Worst	Intermediate	Best

product of intermediate quality. In fact, for product of intermediate quality, double sampling often requires a larger average amount of inspection than single sampling. The quality of most product inspected tends to be equal to or better than the AQL, since, if product regularly submitted does not meet the AQL, excessive rejections occur and action must be taken to improve quality. For product whose quality is equal to or better than the AQL, double sampling frequently requires one-fourth to one-third less inspection on the average than single sampling, and sequential sampling one-third to one-half less on the average than single sampling.

The factors that affect the choice of type of sampling are summarized in Table 9.3. Not all these factors need be considered for a given product. For some products certain factors are decisive; for other products other factors are decisive. The factors are listed in approximate order of importance.

3. EXTRACTION OF SAMPLING PLANS

Once the type of sampling has been chosen, the next step is to extract the particular sampling plan or plans to be used by the inspector in determining whether to accept or reject each inspection lot. The extraction of sampling plans will be explained by the use of an illustrative example.

Given: The following standards for a certain product:

AQL, major	2 percent
AQL, minor	4 percent
Inspection level	III
Usual inspection-lot size	700
Type of sampling to be used	Single

To find: Sampling plans, one for major and one for minor defects, satisfying the above requirements.

The first step in extracting a sampling plan is to determine the sample-size letter from Table 1 (in Part V). This letter, which stands for a particular sample size (or, in double and sequential sampling, for a sequence of sample sizes), is then used in entering Table 2 (in Part V) to obtain a sampling plan. Since the usual inspection-lot size given above is 700, we enter Table 1 on the line for an inspection-lot size of 500 to 800. On this line, in the column for inspection level III, appears the sample-size letter H.

Having determined the sample-size letter, we are now ready to enter Table 2. Table 2 is divided into three parts, Table 2*a* for single sampling, Table 2*b* for double, and Table 2*c* for sequential. Since in this example we are interested in single sampling we turn to Table 2*a*.

The first column of Table 2*a* lists the various sample-size letters. Entering the line for sample-size letter H, we find in the second column of the table the sample size, 75. Moving to the right along this line to the column for an AQL of 1.2 to 2.2 percent (wherein falls our AQL, major, 2 percent), we find acceptance number, 3, and rejection number, 4. Moving along the same line to the column for an AQL of 3.2 to 4.4 percent (wherein falls our AQL, minor, 4 percent), we find acceptance number, 6, and rejection number, 7.

This information may be assembled in the following manner:

SAMPLING PLAN FOR MAJORS

Sample size	Acceptance number	Rejection number
75	3	4

SAMPLING PLAN FOR MINORS

Sample size	Acceptance number	Rejection number
75	6	7

A similar procedure would be followed if it were desired to obtain a double- or a sequential-sampling plan, except that Table 2*b* or Table 2*c* would be used instead of Table 2*a*.

Table 2 may sometimes give sampling plans with different sample sizes for majors than for minors. For example, suppose that the AQL for majors is 0.10 percent and the AQL for minors, 0.75 percent, that the inspection-lot size is between 1,300 and 3,200, and that inspection level III and single sampling are to be used. Table 1 gives the sample-size letter J. For the AQL, major, 0.10 percent, however, Table 2*a* gives no acceptance and rejection numbers on the J-line. Instead, there is a downward pointing arrow which indicates that one must use the sample size and acceptance and rejection numbers given on the M-line (sample size of 450). For the AQL, minor, 0.75 percent, on the other hand, sample size and acceptance and rejection numbers *do* appear for sample-size letter J (sample size of 150). Thus, Table 2*a* requires a sample of 450 for majors but only 150 for minors.

Since different amounts of inspection for different classes of defects might complicate the work of the line inspector, the following rule should be observed whenever feasible: whenever the sample sizes obtained for majors and minors are not the same, use for minors the same sample size as that required for majors. In the above example, therefore, a sample size of 450 should be used for both majors and minors. Naturally, acceptance and rejection numbers for minors would also be obtained from the line of the table corresponding to a sample size of 450 and not from the line for 150, which was the sample size originally indicated for minors.

The sampling plans given in Table 2 are also presented in a different way in Table 4. Each complete page of Table 4 contains three sampling plans (one of each type: single, double, and sequential) for a particular sample-size letter and AQL class, together with their OC curves. The three sampling plans on any one page of Table 4 have approximately the same OC curves. The pages in Table 4 are arranged primarily by sample-size letter and secondarily by AQL. Tables 2 and 4 may, of course, be used interchangeably.

The OC curves on the pages of Table 4 are computed on the assumption that inspection lots are large compared with the samples selected from them. This will usually be so; when it is not, the OC curves shown in Table 4 are only approximate.

The actual AQL of each sampling plan can be determined directly from its OC curve by reading off the percentage of defective items in submitted inspection lots for which 95 percent of submitted inspection lots are accepted. The plans have been so chosen that the AQL determined in this way is almost always within the indicated AQL class.*

* Because of the limited number of plans that can be constructed for each sample size or sequence of sample sizes, a few plans have had to be assigned to AQL classes other than those in which the true AQL's of the plans fall.

CHAPTER 10

OPERATION OF THE PROCEDURE

1. SELECTION OF SAMPLES

1.1. Introduction

The first step taken by the line inspector in operating the standard sampling-inspection procedure on each submitted inspection lot is to draw sample items from the submitted inspection lot.

1.2. Sample Size

The number of sample items to be drawn from an inspection lot is determined from the sample-size column of the particular sampling plan being used. This was illustrated in Sec. 3 of Chap. 9.

In double and sequential sampling it is usually desirable to draw each sample separately. However, for administrative reasons, it is sometimes desirable to draw at one time the maximum number of items that might be needed and then draw the separate samples from these items. If this seems desirable, it will frequently be worth reconsidering the choice of type of sampling since, when all samples are drawn at once, sampling costs tend to be lower under single sampling than under either double or sequential sampling. This advantage of single sampling may, however, be more than offset if the cost of inspecting an item is large compared to the cost of drawing an item.

1.3. Rules for Drawing Sample Items

The more accurately samples represent the quality of the inspection lots from which they are drawn, the more satisfactory is the operation of sampling inspection. There is no method of drawing sample items that guarantees that every sample is representative. Experience has shown, however, that if certain rules are observed in drawing sample items, a representative picture of the inspection lot is obtained more often than if these rules are ignored. It is especially important to follow these rules if it is not feasible to take into account in the formation of inspection lots all the considerations listed in Sec.1.4 of Chap. 9. The rules are

Rule I. Draw proportional samples.
Rule II. Draw sample items from all parts of each sublot.
Rule III. Draw sample items blind.

The first rule, "Draw proportional samples," applies only when the inspection lot is broken into two or more sublots. Under such circumstances, the sample should be drawn so that it gives representation to each sublot in proportion to the size of the sublot. To illustrate: an inspection lot of overcoats is presented for inspection divided into three sublots that comprise the output of three shifts. The composition of the inspection lot is as follows:

Morning shift sublot........	1,000 overcoats, or 50 percent of the total inspection lot
Afternoon shift sublot.....	600 overcoats, or 30 percent of the total inspection lot
Night shift sublot...... ...	400 overcoats, or 20 percent of the total inspection lot
Size of inspection lot	2,000 overcoats

The sampling plan requires a single sample of 150 overcoats. The sample should be drawn as follows:

From morning shift sublot..	75 overcoats, or 50 percent of the total sample
From afternoon shift sublot........	45 overcoats, or 30 percent of the total sample
From night shift sublot...	30 overcoats, or 20 percent of the total sample
Size of sample.............	150 overcoats

The second rule, "Draw sample items from all parts of each sublot," is intended to assure all items in the inspection lot the same chance of being included in the sample. The ability of the inspector to abide by this rule depends on the manner in which the inspection lot is presented for inspection. If the items are presented in cartons or in bins, the inspector is likely to draw predominantly from the top, giving inadequate representation to the items at the middle and bottom of each container. It is highly desirable, therefore, that the inspector be authorized to refuse to begin inspection of any inspection lot until the inspection lot has been arranged so that he can draw sample items from all parts of each of its sublots.

The third rule, "Draw sample items blind," means that sample items should be selected without regard to their quality; the inspector should not make a conscious effort either to draw or to avoid drawing defective items. The purpose of this rule, like that of the second rule, is to assure all items in the inspection lot the same chance of being included in the sample. If some of the items in the inspection lot are obviously defective and the inspector notices them, he should have such items removed from the inspection lot before drawing his sample. In general, the inspector should neither look for nor try to avoid defective items.

2. INSPECTION OF SAMPLE ITEMS

The second step taken by the line inspector in operating the standard sampling-inspection procedure on each submitted inspection lot is to inspect the items in the sample or samples for all classes of defects.

The *technical* aspects of inspecting items of any particular product are outside the scope of this book. There are, however, a few general principles that should be observed when sampling plans are being used.

First, the OC curves given in this book assume that every sample item is subjected to inspection for the same defects in the same way with the same care. Second, there must be a clear distinction between what is a nondefective and what is a defective item of product—established by the use of a list of defects, by photographs, or by actual specimens of the product. The fact that this line may not be sharp in the use to which the product is put should not affect the sharpness of the distinction made by inspectors.

3. ACCEPTANCE OR REJECTION OF AN INSPECTION LOT

3.1. Introduction

Whether to accept or reject an inspection lot is determined by reference to the sampling plans being used. The manner in which sampling plans are employed for this purpose will be explained in detail for sequential sampling since single and double sampling may be considered special cases of sequential sampling.

3.2. Procedure for Major Defects

1. Select a first sample of the size indicated on the first line of the sampling plan for major defects.

2. Inspect the first sample and count the number of major defectives found, that is, the number of items containing major defects.

  a. If the number of major defectives in the first sample is equal to or less than the acceptance number for the first sample, accept the inspection lot with respect to major defects.
  b. If the number of major defectives in the first sample is equal to or greater than the rejection number for the first sample, reject the inspection lot.
  c. If the number of major defectives in the first sample is between the acceptance and the rejection numbers for the first sample, draw a second sample of such size as to bring the size of the *combined* sample up to that on the second line of the sampling plan.

3. If a second sample is drawn, inspect the second sample, count the number of major defectives it contains and determine the number of major defectives in first and second samples *combined.*

a. If the number of major defectives in the first and second samples combined is equal to or less than the acceptance number for the first and second samples combined, accept the inspection lot with respect to major defects.
b. If the number of major defectives in the first and second samples combined is equal to or greater than the rejection number for the first and second samples combined, reject the inspection lot.
c. If the number of major defectives in the first and second samples combined is between the acceptance and rejection numbers, select a third sample of such size as to bring the size of the combined sample up to that on the third line of the sampling plan.

4. Continue the above process until the inspection lot is accepted with respect to major defects or is rejected.

3.3. Procedure for Minor Defects

The procedure for minor defects is the same as for major defects, except that the sampling plan for minor defects is used. Inspection for minor defects is usually conducted at the same time as inspection for major defects.

3.4. When to Accept or Reject an Inspection Lot

An inspection lot is accepted only if it is accepted with respect to *both* major and minor defects. If an inspection lot is accepted with respect to one class of defects, inspection should be continued for the other class of defects until an independent decision is reached with respect to it; inspection (or at least recording) can be discontinued for the class of defects for which a decision to accept is reached first.

An inspection lot is rejected if it is rejected for *either* major defects or minor defects. Inspection might therefore be discontinued as soon as an inspection lot has been rejected for one class of defects. Administrative considerations suggest, however, that inspection should continue until an independent decision is reached for both major and minor defects, thus increasing the amount of information on which to base the administration decision about disposal of a rejected lot (see Sec. 4 of this chapter).

3.5. Curtailing Inspection

3.5.1. First Sample.—It is sometimes possible to know whether an inspection lot will be accepted or rejected for a class of defects before all items in the first sample have been inspected for that class of defects. For example, suppose the sampling plan for major defectives has a rejection number of 3 for a first sample of 40 and that the line inspector

finds 3 major defectives among the first 9 items he inspects. The inspection lot is certain to be rejected even if the remaining 31 items in the first sample contain no major defects. It would therefore be possible to reject the inspection lot after inspecting only 9 items. This should not, however, be done for the first sample, unless the conditions given in Chap. 3, Sec. 4.2.2 are met and the formulas given in the footnote of Chap. 11, Sec. 2.2 (page 117) are used for computing the process average.

3.5.2. Later Samples.—In a later sample, as in the first sample, it is sometimes possible to know whether an inspection lot will be accepted or rejected for a class of defects before all items in that sample have been inspected for that class of defects. For later samples, only administrative and cost considerations need be taken into account in deciding whether to curtail inspection. In order to know when to curtail inspection (within a sample), the inspector must tally each defective separately or keep mental track of the number of defectives found. Either procedure may be burdensome and subject to error, particularly if inspection is for more than one class of defects. The advantage of curtailing is, of course, that it reduces the amount of inspection, and it is therefore desirable to curtail inspection of later samples whenever feasible.

3.6. Use of Sampling Plans When There Is Only One Class of Defects

When there is only one class of defects, the inspector is given one sampling plan instead of two. In that event, the procedure given in Sec. 3.2 is followed, and Secs. 3.3 and 3.4 are ignored.

3.7. Examples of the Use of Sampling Plans

3.7.1. First Example.—Suppose that the following sampling plans are to be used on an inspection lot of 1,400 spark plugs:

SAMPLING PLAN FOR MAJOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number
40	*	3
80	1	4
120	2	4
160	4	6
200	5	7
240	6	8
280	8	9

SAMPLING PLAN FOR MINOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number
40	1	5
80	2	8
120	5	11
160	8	14
200	11	17
240	14	19
280	18	19

* Acceptance not permitted until two samples have been inspected.

The line inspector has three boxes, each large enough to hold about a dozen spark plugs. One box is labelled "majors," the second "minors,"

and the third "mixed."* The inspector begins by drawing from the inspection lot an initial sample of 40 spark plugs. He inspects the 40 spark plugs; he disposes as follows of the defective spark plugs that he finds:

a. If a defective spark plug contains one or more major defects, but no minor defects, he places it in the box marked "majors."

b: If the defective spark plug contains one or more minor defects, but no major defects, he places it in the box marked "minors."

c. If the defective spark plug contains both major and minor defects, he places it in the box marked "mixed."

This procedure is followed for each defective spark plug found among the spark plugs inspected. When inspection of the first sample is completed the inspector counts the number of spark plugs in each of the three boxes, *putting the defective spark plugs back in the boxes.* His results are, say,

Major	Minor	Mixed
No spark plugs	1 spark plug	1 spark plug

The number of major and minor defectives is computed as follows:

0 major	1 minor
1 mixed	1 mixed
1 major defective	2 minor defectives

Note that the mixed defectives are counted both as major defectives and as minor defectives. The results are recorded as shown below:

SAMPLING PLAN FOR MAJOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number	Major defectives found
40	*	3	1
80	1	4	
120	2	4	
160	4	6	
200	5	7	
240	6	8	
280	8	9	

SAMPLING PLAN FOR MINOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number	Minor defectives found
40	1	5	2
80	2	8	
120	5	11	
160	8	14	
200	11	17	
240	14	19	
280	18	19	

* Acceptance not permitted until two samples have been inspected.

* This particular device of boxes will not, of course, be feasible for all products. It is used here to illustrate the desirability of using some device to help the inspector keep track of the total number of defectives of various classes found up to any point. The same purpose can be accomplished by many other devices.

The inspector compares his results with the sampling plans. The number of major defectives, 1, is compared with acceptance and rejection numbers for majors on the first line of the table, * and 3. Since the asterisk means that acceptance cannot be made on the first sample and since 1 is less than 3, no decision can be made for majors. The same is true of minors, since the number of minor defectives, 2, is between the acceptance and rejection numbers for minors, 1 and 5. More evidence is needed on both majors and minors.

The inspector obtains additional evidence by drawing a second sample of 40 spark plugs from the inspection lot; this makes a combined sample size of 80 spark plugs (40 in the first sample and 40 in the second). He inspects all the spark plugs in the second sample, placing defective spark plugs in the three boxes as before. When he finishes inspecting the second sample, the number of spark plugs in the three boxes represents the number of defectives found in the first and second samples combined, *since defectives found in the first sample are still in the boxes.*

The inspector counts the number of defectives in the boxes, as before. His results are, say,

0 major	2 minor
1 mixed	1 mixed
1 major defective	3 minor defectives

These results are recorded as shown below:

SAMPLING PLAN FOR MAJOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number	Major defectives found
40	*	3	1
80	1	4	1✓
120	2	4	
160	4	6	
200	5	7	
240	6	8	
280	8	9	

SAMPLING PLAN FOR MINOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number	Minor defectives found
40	1	5	2
80	2	8	3
120	5	11	
160	8	14	
200	11	17	
240	14	19	
280	18	19	

* Acceptance not permitted until two samples have been inspected.

The new numbers of "defectives found" are compared with acceptance and rejection numbers on the second lines of the plans. The number of major defectives, 1, is now equal to the acceptance number, 1, and the inspection lot is therefore accepted with respect to major defects. This is indicated by a check. The number of minor defectives, 3, is

again between the acceptance and rejection numbers, 2 and 8. Hence, more evidence is needed with regard to minor defects.

The inspector therefore draws and inspects a third sample of 40 spark plugs, which makes a combined sample size of 120 spark plugs. This brings him to the third line of the sampling plan. Let us assume that he finds no more defectives; that is, he obtains once more the cumulative results

0 major	2 minor
1 mixed	1 mixed
1 major defective	3 minor defectives

The result for minors is recorded, but the result for majors is ignored since the inspection lot has already been passed for major defects.*

SAMPLING PLAN FOR MAJOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number	Major defectives found
40	*	3	1
80	1	4	1✓
120	2	4	
160	4	6	
200	5	7	
240	6	8	
280	8	9	

SAMPLING PLAN FOR MINOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number	Minor defectives found
40	1	5	2
80	2	8	3
120	5	11	3✓
160	8	14	
200	11	17	
240	14	19	
280	18	19	

* Acceptance not permitted until two samples have been inspected.

Since the number of minor defectives, 3, is now less than the corresponding acceptance number, 5, the inspection lot is accepted with respect to minor defects. This is also indicated by a check.

Having accepted the inspection lot with respect to both major and minor defects, the inspector accepts the inspection lot. Inspection of the inspection lot is finished.

3.7.2. Second Example.—As an example of an inspection lot that is ultimately rejected, suppose the following results had been obtained:

* Had any major defectives been noted, they would have been placed in the box for majors even though the inspection lot had already been accepted for major defects. The purpose of this is simply to remove as many defectives from the inspection lot as possible. This is not part of the sampling plan for evaluating the quality of the inspection lot, but merely a costless step that improves the quality of product finally accepted.

SAMPLING PLAN FOR MAJOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number	Major defectives found
40	*	3	1
80	1	4	2
120	2	4	2✓
160	4	6	
200	5	7	
240	6	8	
280	8	9	

SAMPLING PLAN FOR MINOR DEFECTIVES

Combined sample size	Acceptance number	Rejection number	Minor defectives found
40	1	5	4
80	2	8	7
120	5	11	12×
160	8	14	
200	11	17	
240	14	19	
280	18	19	

* Acceptance not permitted until two samples have been inspected.

Note that the inspection lot was acceptable with respect to major defects but was nevertheless rejected because it was rejected for minor defects. This is indicated by a cross.

4. DISPOSITION OF UNACCEPTABLE PRODUCT

4.1. Introduction

The disposition of unacceptable product will be considered as follows:

a. Disposition of defective sample items.

b. Disposition of inspection lots rejected on the basis of sampling inspection.

4.2. Disposition of Defective Sample Items

A few defective items may be found in the sample or samples inspected, even if the inspection lot is accepted on the basis of sampling inspection. These defective sample items should, if possible, be replaced in the inspection lot by nondefective items since this results in a slight improvement in the quality of product leaving the inspector. The nondefective items in the sample should, of course, also be returned to the inspection lot unless inspection impairs their quality.* The replacement of defective sample items by nondefective items is not, of course, part of the procedure for evaluating the quality of the inspection lot but simply a device for improving slightly the quality of product accepted.

4.3. Disposition of Rejected Inspection Lots

The action to be taken on a rejected inspection lot varies from product to product. With some products it is uneconomical to salvage a

* For example, a strain test may impair quality even though the items pass the test.

rejected inspection lot (even though it contains many nondefective items) and it may therefore be scrapped. With other products it is customary to screen a rejected inspection lot and rework or discard all defective items. With some products, the rejected inspection lot can be used for a purpose other than that for which it was originally intended and for which the defects it contains are less serious. If the product inspected is being purchased from another concern, rejected inspection lots may sometimes be purchased at a lower price.

4.4. Average Outgoing Quality (AOQ)

If rejected inspection lots are screened, the quality of outgoing product is certain to be better, on the average, than the quality of product originally submitted for inspection, because all defectives are removed from the inspection lots that are screened and a few defectives (those found in the samples inspected) are removed from the inspection lots that are not screened.

When the average percent defective of product orginally submitted for inspection is known, the approximate average outgoing quality (AOQ) under a particular sampling plan can be computed from the following formula:*

$$\text{AOQ} = pL_p$$

where AOQ = average outgoing quality (the average percentage of defective items in product finally accepted; product finally accepted includes inspection lots accepted on the basis of the sampling plan and inspection lots originally rejected on the basis of the sampling plan and finally accepted after removal of all defective items)

p = average percentage of defective items in product originally submitted for inspection

L_p = probability, under the particular sampling plan being used, of accepting inspection lots whose quality is equal to p. This probability is determined from the OC curve (shown in Table 4) of the particular sampling plan involved.

Example: The average percentage of defective items in a certain product when originally submitted for inspection is 9.0, the sample-

* A more exact formula is

$$\text{AOQ} = pL_p\left(1 - \frac{\bar{n}}{N}\right)$$

where $\bar{n}$ = average number of items inspected per inspection lot

N = inspection-lot size (assumed constant from inspection lot to inspection lot)

size letter is G, and the AQL class is 5.3 percent to 6.4 percent. Sequential sampling is being used. The page in Table 4 for this sample-size letter and AQL class shows the OC curve for the required sequential-sampling plan; this OC curve is reproduced in Fig. 10.1. The probability of accepting inspection lots containing 9.0 percent defective items is about 77 out of 100. The AOQ is therefore $0.77 \times 0.090 = 0.06930$ or approximately 6.9 percent defective.

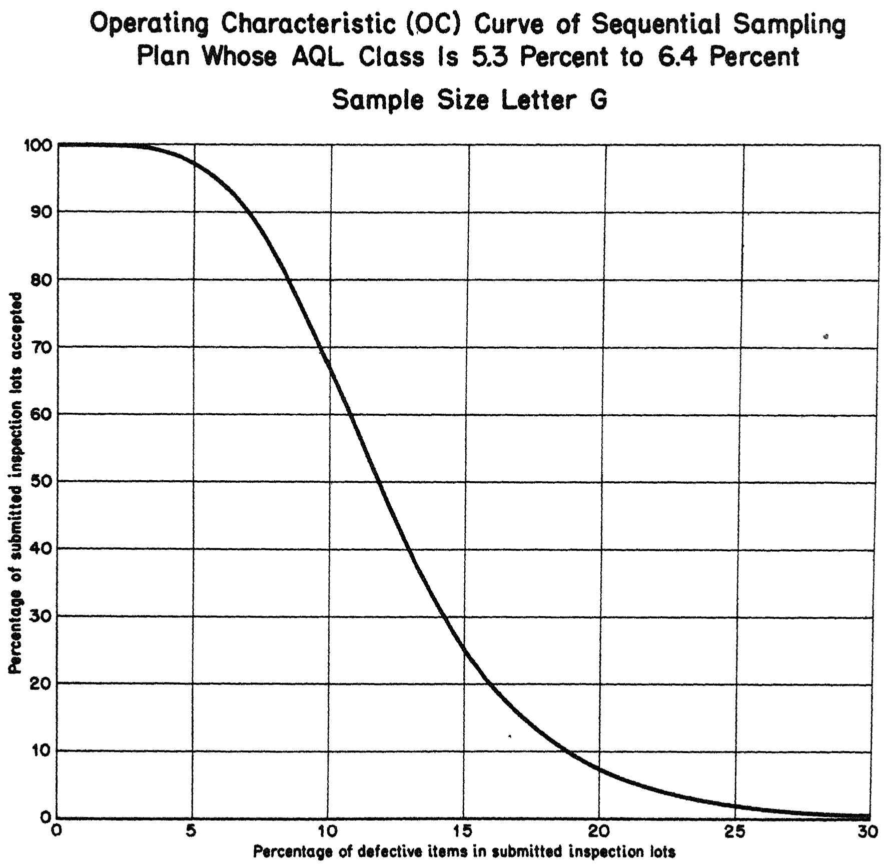


FIG. 10.1.

The outgoing quality that may be expected if inspection lots of any given quality are submitted for inspection under a particular sampling plan can be computed as indicated above from the OC curve of that sampling plan. The results of such computations can be plotted to give an AOQ curve, such as that shown in Fig. 10.2 for the sequential-sampling plan whose OC curve is shown in Fig. 10.1 (sample-size letter G, AQL class 5.3 percent to 6.4 percent). As can be seen, this AOQ curve

reaches its maximum at an AOQ of 6.9 percent defective. This maximum value of average outgoing quality is called the average outgoing-quality limit (AOQL), since it represents the poorest average quality of outgoing product that a particular sampling plan (plus screening) will yield, regardless of the quality of product originally submitted for inspection.

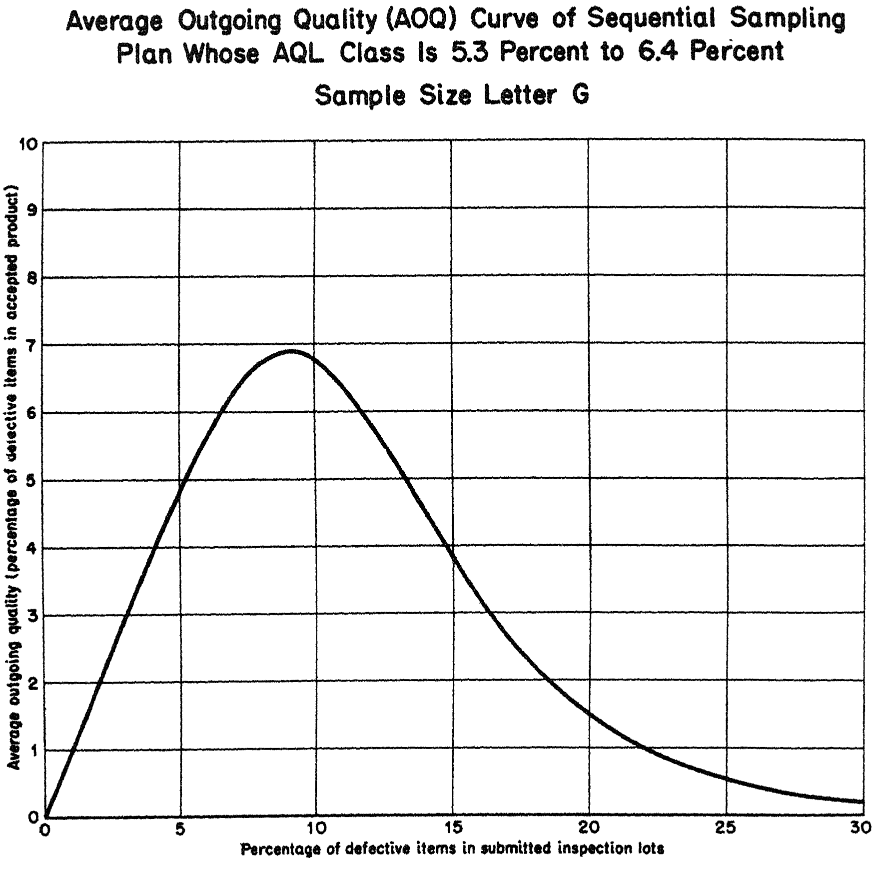


FIG. 10.2.

The AOQL of each sampling plan in Part V of this book is shown, in the form of an AOQL class interval into which it falls, in the upper corner of the appropriate page of Table 4. In addition, the AOQL's have been summarized in Table 3 for speedy reference. It should be borne in mind that the AOQL's in Tables 3 and 4 are computed on the assumption that rejected inspection lots are screened and all defective items in screened inspection lots removed and replaced and that the inspection lot is large compared with the sample. If these conditions are not satisfied, either

because some of the items in rejected inspection lots are not reinspected, or because the items in rejected inspection lots are not properly inspected, or because the inspection lot is not much larger than the sample, the AOQL values given in Tables 3 and 4 are not applicable.*

* The discussion of Sec. 4.4 assumes that inspection covers only one class of defects. When inspection covers more than one class of defects the situation is more complicated, as is illustrated by the fact that a lot which is accepted for, say, majors may nevertheless be screened because it may be rejected for minors, and the screening will ordinarily cover major as well as minor defects.

CHAPTER 11

REVIEW OF PAST RESULTS

1. INTRODUCTION

An important feature of the standard procedure described in this part is the periodic review of past inspection results and the use of such reviews as guides to future action. The purposes of periodic review are

a. To determine the average quality of product submitted for inspection during the recent past.

b. To determine whether to use normal, reduced, or tightened inspection in the future.

c. To predict the approximate percentage of inspection lots that will be rejected by the sampling plan in the future.

2. COMPUTATION OF PROCESS AVERAGE

2.1. Introduction

The first step in reviewing past results is to compute the process average.

If every item in a series of inspection lots submitted for inspection were examined, the percentage of items found to contain defects would be the *true* percentage of defective items in the submitted product. The true percentage of defective items can never be known when inspection is on a sampling basis because only some of the submitted product is examined. It must therefore be estimated. The estimate of the true percentage of defective items is called the "process average."

2.2. Approximate Computation

If inspection lots do not vary much in size and if the first samples are all the same size, the following formula can be used to compute the process average for a series of k inspection lots:

$$\begin{array}{c}\text{Approximate}\\ \text{process average}\\ \text{(in percent defective)}\end{array} = \frac{\begin{array}{c}\text{total number of defective items in the}\\ \text{first samples from the } k \text{ inspection lots}\end{array}}{\begin{array}{c}\text{total number of items in the first samples}\\ \text{from the } k \text{ inspection lots}\end{array}} \times 100$$

If there is more than one class of defects, the process average is computed separately for each class. There may be, for example, a process average, major, and a process average, minor. Only items containing

TABLE 11.1
SPECIMEN COMPUTATION OF PROCESS AVERAGE

Inspection lot number	Inspection lot size N	Size of first sample n	Defectives in first sample: Number	Defectives in first sample: Proportion p	Estimated number of defectives in inspection lot pN
1	2,000	40	2	0.05	100
2	1,800	40	2	0.05	90
3	2,000	40	0	0.00	0
4	2,000	40	4	0.10	200
5	2,000	40	0	0.00	0
6	2,100	40	0	0.00	0
7	2,000	40	0	0.00	0
8	2,000	40	12	0.30	600
9	2,000	40	1	0.025	50
10	2,000	40	0	0.00	0
11	2,000	40	0	0.00	0
12	1,800	40	3	0.075	135
13	1,800	40	1	0.025	45
14	2,000	40	2	0.05	100
15	2,000	40	0	0.00	0
16	2,000	40	0	0.00	0
17	2,000	40	0	0.00	0
18	2,100	40	0	0.00	0
19	2,100	40	0	0.00	0
20	2,000	40	1	0.025	50
21	1,900	40	2	0.05	95
22	2,000	40	0	0.00	0
23	2,000	40	0	0.00	0
24	2,100	40	0	0.00	0
25	2,000	40	1	0.025	50
26	2,100	40	0	0.00	0
	51,800	1040	31		1515

Computation by formula in Sec. 2.2:

$$\text{Process average} = \frac{31}{1{,}040} \times 100 = 2.98\,\%$$

Computation by formula in Sec. 2.3:

$$\text{Process average} = \frac{1{,}515}{51{,}800} \times 100 = 2.92\,\%$$

one or more major defects are counted in computing the process average, major, and only items containing one or more minor defects, in computing the process average, minor.

Use of this formula is illustrated by the specimen computation of process average in Table 11.1. It will be noted from this specimen computation that only the first sample of each inspection lot is included. When single sampling is used, the first sample is, of course, the only sample. When double or sequential sampling is used, more than one sample may be required to determine the acceptance or rejection of each inspection lot. In that event, only the first sample from each inspection lot is used in computing the process average. Inclusion of samples other than the first may bias the process average; that is, the process average so calculated may tend to be higher or to be lower than the true percentage of defective items. The inspection of all items in the first sample, as recommended above, is also designed to avoid bias.*

2.3. Exact Computation

The formula given in Sec. 2.2 yields a good estimate of the process average only if inspection-lot size and first-sample size do not vary much. If either the inspection-lot size or first-sample size varies considerably,

* While the formula given above will yield a biased process average if all items in the first sample are not inspected, unbiased estimates of the quality of an inspection lot and, from these, an unbiased process average can be obtained from the data of a curtailed first sample in the following way:

Denote the sample size by n and the rejection number by r; then curtailing means that we stop inspecting and reject the inspection lot when r defectives are observed or we stop inspecting and accept the inspection lot as soon as $n - r + 1$ nondefectives are observed. When we reject the inspection lot an estimate of the process average is given by

$$\frac{(r - 1) \times 100}{\text{One less than the total number of items inspected}}$$

and, when we accept the inspection lot, by

$$\frac{\text{Number of defectives observed} \times 100}{\text{One less than the total number of items inspected}}$$

Suppose $n = 225$, $r = 15$, and the 15th defective is found at the 35th item inspected. Then we estimate the process average to be $(14 \times 100)/34 = 41.2$ percent defective. We would accept the inspection lot if $n - r + 1 = 225 - 15 + 1 = 211$ nondefectives were observed. Suppose the 211th nondefective occurs at the 215th item inspected. Then we accept the inspection lot and estimate the process average to be

$$(4 \times 100)/214 = 1.9 \text{ percent}$$

For further information on this subject see M. A. Girshick, Frederick Mosteller, and L. J. Savage, "Unbiased Estimates for Certain Binomial Sampling Problems with Applications," *Annals of Mathematical Statistics*, Vol. 17 (1946), pp. 13–23.

the following formula should be used in computing the process average:

$$\begin{array}{c}\text{Process average} \\ \text{(in percent defective)}\end{array} = \frac{p_1N_1 + p_2N_2 + \cdots + p_kN_k}{N_1 + N_2 + \cdots + N_k} \times 100$$

where

$$p_1 = \frac{\text{number of defectives in first sample from the first inspection lot}}{\text{number of items in the first sample from the first inspection lot}}$$
$$p_2 = \frac{\text{number of defectives in first sample from the second inspection lot}}{\text{number of items in the first sample from the second inspection lot}}$$
$$p_k = \frac{\text{number of defectives in first sample from the } k\text{th inspection lot}}{\text{number of items in the first sample from the } k\text{th inspection lot}}$$

N_1 = number of items in the first inspection lot
N_2 = number of items in the second inspection lot
N_k = number of items in the kth inspection lot
k = number of inspection lots

In this formula, as in the preceding one, if there is more than one class of defects, "number of defectives" is to be interpreted as the number of defectives of a particular class and the resulting process average as referring to that class of defectives.

2.4. Exclusion of data

A process average is computed in order to estimate the percentage of defective items in product *normally* submitted; this estimate may then be used to predict the percentage of defective items in product submitted in the near future if conditions remain the same. Therefore, all data compiled from the reinspection of inspection lots resubmitted after having been rejected must be excluded from the process average, since the quality of such inspection lots is likely to be better than the quality of inspection lots submitted for the first time. Furthermore, any data *known* to have arisen under abnormal conditions must also be excluded. To illustrate, in the specimen computation of process average of Table 11.1, the sample from inspection lot 8 contains 12 defective items. Suppose it is known that inspection lot 8 was produced from a faulty batch of raw material and that this is an abnormal condition. Inspection lot 8 should then be excluded from the computations; this results in an approximate process average of 1.90 percent rather than 2.98 percent (formula in Sec. 2.2):

$$\frac{19}{1000} \times 100 = 1.9 \text{ percent}$$

Care must be taken not to exclude data unless it is definitely *known* that the data arose under abnormal circumstances. The mere fact that

figures look unreasonable is not sufficient basis for excluding them; there must also be a sound technical reason.

2.5. Number of Inspection Lots and Number of Items to Include in Computation

The number of inspection lots (and therefore the number of first samples) included in the process average should normally be not less than 20 and, if the AQL is less than 0.22 percent defective, should be nearer 40. If, however, sampling inspection has only recently been introduced, the process average may be computed after as few as 5 or 10 inspection lots have been inspected; thereafter at least 20 inspection lots should be included. If more frequent computation of the process average is desired, it may be computed after every 10 inspection lots, though data from the last 20 or more inspection lots should always be used.

The minimum number of sample items on which the process average should be based depends on the AQL and is given in Table 11.2.

TABLE 11.2

MINIMUM NUMBER OF SAMPLE ITEMS ON WHICH THE PROCESS AVERAGE SHOULD BE BASED

AQL Class (in % Defective)	Minimum Number of Sample Items
0.024 to 0.035	15,000
0.035 to 0.06	10,000
0.06 to 0.12	7,000
0.12 to 0.17	5,000
0.17 to 0.22	3,000
0.22 and over	1,000

In the early stages of inspection when the process average is based on as few as 5 or 10 inspection lots, it may be impossible to conform to Table 11.2.

2.6. Frequency of Computation

How frequently the process average should be computed can only be determined by experience. It should, of course, be computed whenever there is sound reason to suspect a significant change in the quality of submitted product. If a new process average is computed before the number of sample items inspected is equal to the minimum number given in Table 11.2, the new process average should be based on data from the most recent inspection lots combined with as many of the inspection lots included in the preceding process average as are necessary to make up the required number of sample items.

2.7. Separate Computation for Each Class of Defects

When two or more classes of defects are involved, the process average must be computed independently for each class. Thus, there may be a process average, major, and a process average, minor.

3. NORMAL, TIGHTENED, AND REDUCED INSPECTION

3.1. Introduction

As can be seen from the OC curves, no sampling plan gives perfect discrimination between good and bad inspection lots; there is always some risk of accepting an inspection lot whose percent defective is worse than the AQL and some risk of rejecting an inspection lot whose quality is better than the AQL. The amount of protection afforded by a sampling plan against the acceptance of bad inspection lots depends on the AQL, inspection level, and inspection-lot size. Generally speaking, the lower the AQL, the higher the inspection level; or, the larger the inspection-lot size, the greater the protection against the acceptance of bad inspection lots.

Since quality of product submitted fluctuates from time to time, it is desirable to make appropriate changes in the sampling plan. If quality deteriorates, it may be desirable to tighten inspection; if quality improves, it may be desirable to loosen inspection.

3.2. Tightened Inspection

Inspection can be tightened either by raising the inspection level or by employing a smaller AQL. Since raising the inspection level means

TABLE 11.3

ACCEPTABLE-QUALITY LEVEL (AQL) CLASSES FROM WHICH SAMPLING PLANS FOR TIGHTENED INSPECTION SHOULD BE OBTAINED

AQL Used for Normal Inspection (AQL Established for the Product) % Defective	AQL to Be Used for Tightened Inspection % Defective
0.024–0.035	*
0.035–0.06	0.024–0.035
0.06 –0.12	0.024–0.035
0.12 –0.17	0.035–0.06
0.17 –0.22	0.06 –0.12
0.22 –0.32	0.12 –0.17
0.32 –0.65	0.17 –0.22
0.65 –1.2	0.22 –0.32
1.2 –2.2	0.32 –0.65
2.2 –3.2	0.65 –1.2
3.2 –4.4	1.2 –2.2
4.4 –5.3	2.2 –3.2
5.3 –6.4	3.2 –4.4
6.4 –8.5	4.4 –5.3

* If the AQL used for normal inspection is in the class 0.024–0.035, tightened inspection is not available.

an increased amount of inspection, it is generally preferable to tighten inspection by using a sampling plan whose AQL is smaller than that used normally. Table 11.3 shows the AQL classes from which sampling plans for tightened inspection should be obtained.

It will be noted that Table 11.3 provides an AQL class two classes lower (if such a class exists) for tightened inspection than for normal inspection. The sample-size letter is the same for tightened and normal inspection. A tightened inspection plan chosen in accordance with Table 11.3 typically has an AOQL below the AQL used for normal inspection.

3.3. Reduced Inspection

When the quality of product is superior, there are two ways of loosening inspection: by relaxing (raising) the AQL or by reducing the amount of inspection per inspection lot. Obviously the second of these alternatives is the one to use.

Reduction of the amount of inspection is accomplished by using the same AQL as for normal inspection but an inspection level two levels lower (if such a level exists) than the level used for normal inspection. Table 11.4 shows the inspection level to be used for reduced inspection.

TABLE 11.4
INSPECTION LEVEL TO BE USED FOR REDUCED INSPECTION

Inspection Level Used for Normal Inspection (Inspection Level Established for the Product)	Inspection Level to Be Used for Reduced Inspection
I	*
II	I
III	I
IV	II
V	III

* If inspection level I is used for normal inspection, reduced inspection is not available.

3.4. When to Use Tightened or Reduced Inspection

Whether to use tightened or reduced inspection depends primarily on how the process average compares with the AQL. If the process average is about equal to the AQL, normal inspection is used. If the process average is considerably higher than the AQL, tightened inspection is used; if the process average is considerably lower than the AQL, reduced inspection may be used (provided that certain other conditions are also satisfied).

Since the process average is based on samples, it is subject to a certain amount of error. This must be taken into account when comparing the process average with the AQL; that is, the process average must be *considerably* higher than the AQL before tightened inspection is used

TABLE
UPPER AND LOWER LIMITS
(These limits are to be used in determining whether

Number of sample items on which process average is based	Acceptable-quality level (percent defective)													
	0.024–0.035		0 035–0.060		0.060–0 120		0 120–0 170		0 170–0.220		0 22–0 32		0.32–0.65	
	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average
1,000– 1,500	*		*		*		*		*		0.00	0.71	0.01	1.17
1,500– 2,000	*		*		*		*		*		0.00	0 64	0.06	1 08
2,000– 3,000	*		*		*		*		*		0.04	0.58	0 10	1.00
3,000– 4,000	*		*		*		*		0.035	0 402	0.06	0.54	0.13	0.94
4,000– 5,000	*		*		*		*		0 050	0.378	0 08	0.51	0.16	0.91
5,000– 7,000	*		*		*		0.033	0.291	0.066	0 355	0 10	0.48	0.18	0.85
7,000–10,000	*		*		0.007	0.205	0.047	0 268	0.083	0.331	0.12	0.45	0.20	0.82
10,000–15,000	*		0.003	0.111	0 017	0.189	0.059	0.250	0.098	0.310	0.14	0.42	0.22	0.79
15,000–20,000	0.001	0.068	0.008	0.102	0.024	0.177	0 069	0.236	0.109	0.289	0.15	0.40	0.24	0.77
20,000–30,000	0.005	0.062	0.012	0.094	0.029	0 167	0.077	0.221	0.119	0.278	0.16	0.39	0.25	0.75
30,000–40,000	0.008	0.057	0.015	0.088	0.034	0.156	0.084	0.213	0.127	0.269	0.17	0 38	0.26	0.73
40,000–50,000	0.010	0.054	0.018	0.085	0 037	0.152	0.088	0.208	0 132	0.263	0.18	0 37	0.27	0.72
50,000 and over	0.010	0.053	0.019	0.083	0.038	0.150	0 090	0.206	0.134	0.261	0.18	0.37	0.27	0.72

* More inspection results required.

Procedure for Determining Appropriate Upper and Lower Limits on Process Average

1. For process average, major:

   a. Determine from inspection records the number of sample items on which the process average, major, is based.
   b. In the first column of Table 11.5, find the line containing this number.
   c. On this line, the appropriate upper and lower limits on the process average, major, are given in the columns for the acceptable-quality level class containing the acceptable-quality level, major.

2. For other classes of defectives (e.g., minor): Substitute "minor" for "major" in point 1.

Conditions under Which Reduced Inspection Is Used

3. Process average for each class of defectives is equal to or less than the appropriate lower limit in Table 11.5.
4. No inspection lot among the last 20 inspection lots submitted by the supplier has been rejected on the basis of the sampling plan.
5. The last 20 inspection lots have been produced without any serious interruptions in production.
6. Conditons 3, 4, and 5 must *all* be met.

Conditions under Which Tightened Inspection Is Used

7. For major defectives: Process average, major, is equal to or greater than the appropriate upper limit in Table 11.5.
8. For other classes of defectives (e.g., minor): Substitute "minor" for "major" in point 7.
9. A *separate* decision regarding tightened inspection is made for each class of defectives.

11.5
ON PROCESS AVERAGE
reduced or tightened inspection is required.)

Number of sample items on which process average is based	Acceptable-quality level (percent defective)											
	0 65–1 2		1 2–2 2		2 2–3.2		3.2–4.4		4 4–5.3		5.3–6 4	
	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average	Lower limit on process average	Upper limit on process average
1,000– 1,500	0.21	1.9	0 6	3.0	1.4	4 2	2.2	5 5	3.3	6 5	4.1	7.8
1,500– 2,000	0 27	1.8	0 7	2 9	1 5	4 0	2 4	5.4	3.4	6 3	4 3	7.5
2,000– 3,000	0 33	1 7	0 8	2 8	1.6	3 9	2 5	5 2	3.6	6.2	4 4	7 4
3,000– 4,000	0.38	1 6	0 8	2 7	1.7	3 8	2.6	5.1	3.7	6 0	4 6	7.2
4,000– 5,000	0.41	1.5	0.9	2.6	1.8	3 7	2 7	5 0	3 8	6 0	4.6	7 1
5,000– 7,000	0.45	1.5	0 9	2 6	1.8	3 6	2.8	4.9	3 9	5.9	4.7	7.0
7,000–10,000	0.48	1.4	1.0	2 5	1.9	3 6	2 8	4 8	4 0	5.8	4.8	6.9
10,000–15,000	0 51	1 4	1.0	2.5	1.9	3.5	2.9	4 8	4 0	5 7	4.9	6.8
15,000–20,000	0.53	1 4	1 0	2 4	2 0	3 5	2 9	4.7	4 1	5.6	5 0	6.8
20,000–30,000	0.55	1 3	1.1	2 4	2 0	3 4	3.0	4 7	4 1	5 6	5.0	6.7
30,000–40,000	0.57	1 3	1.1	2 4	2.0	3 4	3.0	4.6	4 2	5 5	5.1	6.7
40,000–50,000	0.58	1 3	1.1	2 3	2 1	3 4	3 0	4 6	4 2	5 5	5 1	6.6
50,000 and over	0 58	1 3	1.1	2.3	2 1	3 4	3 0	4 6	4 2	5 5	5.1	6.6

Conditions under Which Normal Inspection Is Resumed

10. If reduced inspection is being used, resume normal inspection when any one of the following occurs:

a. The process average for any class of defectives is equal to or greater than the larger of the two values defining the acceptable-equality level class for that class of defectives.
b. An inspection lot is rejected on the basis of the sampling plan.
c. Production is interrupted.

11. If tightened inspection is being used for any class of defectives, resume normal inspection for that class of defectives if the process average for that class of defectives is equal to or less than the larger of the two values defining the acceptable-quality level class for that class of defectives.

Notes

12. The process average for any class of defectives is the average percentage of defective items of that class in the first samples from a group of consecutive inspection lots. Resubmitted inspection lots are not included in computing the process average.

and likewise the process average must be *considerably* lower than the AQL before reduced inspection may be used.

A numerical definition of "considerably," that is, of the amount of leeway that should be permitted above and below the AQL, is given in Table 11.5. The use of Table 11.5 is illustrated by the following example.

Given: The AQL for a certain product is 3 percent. Separate inspection lots are formed from the output of each of three production lines whose process averages are as follows:

Production line	Process average (in percent defective)	Number of sample items on which process average is based
X	3.34	1,100
Y	5.22	1,800
Z	1.31	1,200

To determine: Whether to use normal, tightened, or reduced inspection for each production line.

Method: Table 11.5 is entered in the vertical column for AQL 2.2 percent to 3.2 percent since the AQL for this product is 3 percent. For production line X upper and lower limits on the process average are extracted from the line for 1,000 to 1,500 sample items, since the process average for production line X is based on 1,100 items. A similar procedure is followed for the other two production lines. The limits so determined are

Production line	Lower limit on process average	Upper limit on process average
X	1.4	4.2
Y	1.5	4.0
Z	1.4	4.2

Conclusions: (*a*) The process average of production line X, 3.34 percent, is between the limits of 1.4 percent and 4.2 percent. Normal inspection should be used on the output of this production line. (*b*) The process average of production line Y, 5.22 percent, is above the upper limit of 4.0 percent. Tightened inspection should be used on the output of this production line. (*c*) The process average of production line Z, 1.31 percent, is below the lower limit of 1.4 percent. Reduced inspection may be used on the output of this production line (provided that certain other conditions are also satisfied.)

3.5. Rules for Determining Which Type of Inspection to Use

The rules for determining which of the three types of inspection to use are summarized below:

Action to Be Taken	*Conditions under Which Action Is Taken*
Use normal inspection	As long as process average is between upper and lower limits given in Table 11.5.
Use tightened inspection	When process average is equal to or greater than the upper limit given in Table 11.5.
Discontinue tightened inspection and return to normal inspection	When process average improves, becoming equal to or less than the larger of the two values defining the AQL class in which the AQL established for the product falls.
Use reduced inspection	If *all* the following requirements are met: *a.* The last 20 inspection lots have been subjected to normal inspection and none has been rejected. *b.* The process average computed from the last 20 inspection lots is equal to or less than the lower limit in Table 11.5. *c.* Production is at a steady rate.
Discontinue reduced inspection and return to normal inspection on the next inspection lot	If *any* of the following conditions obtain: *a.* An inspection lot is rejected under reduced inspection. *b.* The process average under reduced inspection exceeds the larger of the two values defining the AQL class in which the AQL established for the product falls. *c.* Production suffers an abnormally long interruption.

3.6. Classes of Defects and Types of Inspection

When two or more classes of defects are involved, the following procedure is usually the most practical:

a. Make a *separate* decision regarding tightened inspection for each class of defects. Thus, tightened inspection may be used for majors and normal for minors; or minors may be placed under tightened inspection while majors remain under normal; or, finally, both majors and minors may be placed under tightened inspection at the same time.

b. Make a *combined* judgment regarding reduced inspection. Use reduced inspection only if eligibility requirements are met for both majors and minors. Discontinue reduced inspection, on the other hand, if eligibility is lost for either major or minor defects.

4. PREDICTING FREQUENCY OF REJECTION

4.1. The Operating-characteristic (OC) Curve

The third step in reviewing past results is to predict from them approximately what percentage of future inspection lots will be rejected

by the sampling plan. This step is not absolutely necessary but the results can be of great value.

If product of any given percent defective (estimated by the process average) is submitted, the percentage of inspection lots that may be expected to be rejected by a particular sampling plan is found from the OC curve of that sampling plan. To illustrate: a certain product has a process average of 8 percent defective. The sample-size letter for the product is H and the AQL class, 3.2 percent to 4.4 percent. Sequential sampling is being used. From the OC curve for this sequential-sampling plan it is found that inspection lots whose quality is 8 percent defective will be accepted 56 times out of 100 (or that such inspection lots will be rejected 44 times out of 100).

4.2. Errors of Prediction

In using OC curves to predict the frequency of rejection, the following possible errors should be kept in mind:

a. The expected frequency with which inspection lots containing any given percentage of defective items are accepted (determined by plotting the process average on the OC curve) is based on the assumption that the quality of the product is in statistical control; that is, that, except for chance variations, every inspection lot has essentially the same percentage of defective items as every other inspection lot. In practical terms, this means that the process average must represent a series of inspection lots of similar quality and not a series whose quality varies considerably from inspection lot to inspection lot. If the latter condition prevails, plotting the process average on the OC curve does not yield an accurate prediction of future rejections.

b. Since the process average is an estimate obtained from samples, it is subject to error. Hence, any prediction of future rejections based on this estimate of past quality is subject to error.

c. The prediction of future rejections is based on the assumption that the percentage of defective items will be the same in the future as in the immediate past. If the percentage of defective items changes, the prediction of future rejections is not valid.

Of the sources of error mentioned above, the most important is the third. Under ordinary circumstances the first two sources of error are sufficiently unimportant to be ignored.

4.3. Use of the Prediction

Prediction of the percentage of inspection lots that may be expected to be rejected is useful as a basis for reevaluating the AQL. If the process average is considerably, and consistently, worse than the AQL, and the expected frequency of rejection consequently high, it may be that the

AQL is too exacting for the production process to which it is applied. If this is true, it may be advisable to liberalize the AQL in order to prevent the rejection of an excessive percentage of inspection lots. By periodically considering the expected frequency of rejection, furthermore, one can keep the AQL current with changes in production schedules. When schedules are high, the AQL can be made higher; and, when schedules are low, lower, thereby striking a balance between quality and quantity of production.

Changing the AQL to affect the percentage of inspection lots accepted is not an attractive course of action, since the AQL should reflect, so far as possible, the quality that it is desired to attain, as well as the quality currently being produced. Frequently, however, the alternative method of getting the desired output is to waive rejections of individual inspection lots. Waivers of individual inspection lots tend to nullify the entire sampling-inspection program; they substitute essentially arbitrary and subjective judgments for consistent, objective criteria. Under such circumstances, it is far better to raise the AQL or to alter the classification of defects, thereby getting the desired output and at the same time retaining a systematic device for accepting or rejecting individual inspection lots.

CHAPTER 12

SUGGESTED FORMS

1. SAMPLING-INSPECTION RECORD

A suggested form for use in recording the results of lot-by-lot inspection is the specimen Sampling-inspection Record shown in Table 12.1. This form has space for six inspection lots and is used in accordance with the following instructions:

1. Name of product. Enter the name of the product being inspected.

2. Style or model number. Enter style number or model number, or any other information that would serve to identify the product.

3. Point of inspection. Describe the point at which inspection is being performed.

4. Inspection performed in accordance with. Enter the number of the list of defects or other document on which inspection of individual items is based.

5. Inspection lots submitted by. Identify the plant or shop whose product is being inspected.

6. Date of inspection. Enter date of inspection of each inspection lot.

7. Inspection-lot number. Enter serial number of inspection lot.

8. Inspection-lot size. Enter number of items in inspection lot.

9–12. These entries apply to major defects.

9. Sampling table for majors. Fill in sampling table for major defects.

10. Major defectives. After each sample has been examined, enter the total number of items found with major defects thus far (cumulative number of major defectives). An item counts as *one* major defective no matter how many major defects it contains. When an inspection lot is accepted for majors, make a check next to the sample size on which acceptance occurred; then stop filling in this column. When an inspection lot is rejected for majors, make a cross next to the sample size on which rejection occurred, and stop inspection of the inspection lot.

11. Types of major defects found. As inspection proceeds, list on a separate line the nature of each different major defect found.

12. Major defects. As each major defective is found, tally *once*

TABLE 12.1
SPECIMEN SAMPLING-INSPECTION RECORD

1. Name of Product						
2 Style or Model Number						
3. Point of Inspection						
4 Inspection Performed in Accordance with						
5. Inspection Lots Submitted by						
6. Date of Inspection						
7 Inspection Lot Number						
8. Inspection Lot Size (No. of Items in Inspection Lot)						

Major Defects	9 Sampling Table for Majors			10. Major Defectives					
	Sample Size	Acc. No.	Rej No.	Number	Number	Number	Number	Number	Number
	11. Types of Major Defects Found			12. Major Defects					
				Number	Number	Number	Number	Number	Number

Minor Defects	13 Sampling Table for Minors			14. Minor Defectives					
	Sample Size	Acc. No.	Rej. No.	Number	Number	Number	Number	Number	Number
	15. Types of Minor Defects Found			16. Minor Defects					
				Number	Number	Number	Number	Number	Number

17. Inspection Lot Accepted or Inspection Lot Rejected (Check One)						
	Acc.	Acc.	Acc.	Acc.	Acc.	Acc.
	Rej.	Rej.	Rej	Rej.	Rej.	Rej.

18. Signature of Inspector

for each major defect it contains. Thus, if one item contains three *different* major defects, it should be tallied three times. On the other hand, if an item contains three major defects *of the same type*, it should be tallied only once.

13–16. These entries apply to minor defects.

13. Same as 9 but for minors.
14. Same as 10 but for minors.
15. Same as 11 but for minors.
16. Same as 12 but for minors.
17. Inspection lot accepted or inspection lot rejected. Check one.

Note: If any inspection lot has an abnormally high number of defectives, indicate the cause (if known) at the bottom of the form. If there is not enough room, use the reverse side of the form.

2. QUALITY-HISTORY RECORD

A suggested form for use in summarizing past inspection results is the specimen Quality-history Record shown in Table 12.2. Entries 1 to 5 are duplicates of entries on the Sampling-inspection Record. The remaining entries are filled in as follows:

6. Sampling plan applied. This is a record of the sampling plans applied, that is, the AQL's, the inspection level, and the type of sampling. Enter a new line each time any of these are changed. Under "Type of Inspection," enter *N*, *T*, or *R*, for normal, tightened, or reduced, respectively.

7–8. Period. Enter date of beginning and end of period for which inspection records are being summarized.

9–10. Production accepted. Enter number of inspection lots accepted and number of items accepted during period.

11–12. Production rejected. Enter number of inspection lots rejected and number of items rejected during period.

13–15. Number of items in first samples.

13. Total. Enter total number of items in first samples only.

14. Excluded. Enter number of sample items to be excluded from computation of process average because their production took place under known abnormal conditions.

15. Included. Enter number of sample items to be included in process average.

16–17. Major defectives.

16. Number. Enter number of major defectives found in samples included in process average.

TABLE 12.2
QUALITY-HISTORY RECORD

QUALITY-HISTORY RECORD

ame of Product
Cooking Spoon

2. Style or Model No.
Z-2437X

oint of Inspection
After Polishing Operation

ıspection Performed in Accordance with
Defects Chart No. 16

ıspection Lots Submitted by
Shop No. 5, Plant C

6. Sampling Plan Applied

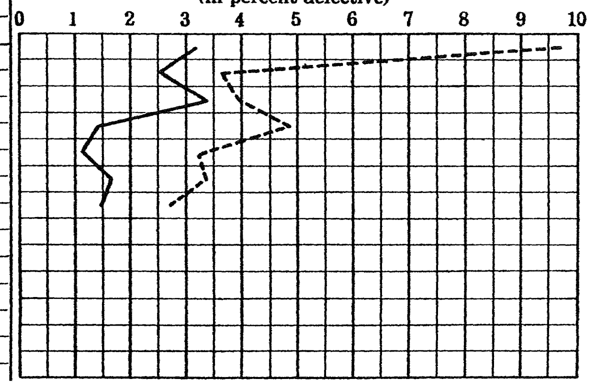
Date	AQL Maj.	AQL Min.	Inspection Level	Type Inspection Maj.	Type Inspection Min.
10/8/45	3%	6%	III	N	T
11/5/45	3%	6%	III	N	N

Original Inspection Lots

Period m	Period To	Production Accepted Lots	Production Accepted Items	Production Rejected Lots	Production Rejected Items	Items in First Samples Total	Items in First Samples Ex-cluded	Items in First Samples In-cluded	Major Defectives Number	Major Defectives Proc-ess Aver-age	Minor Defectives Number	Minor Defectives Proc-ess Aver-age
	8	9	10	11	12	13	14	15	16	17	18	19
/8	10/18	32	19,200	19	11,400	1,020	20	1,000	32	3.2	197	9.7
'19	11/4	46	23,000	4	2,000	1,000	0	1,000	26	2.6	36	3.6
/5	11/10	51	35,700	3	2,100	1,080	60	1,020	37	3.4	43	4.0
/10	11/19	50	30,000	0	0	1,000	0	1,000	14	1.4	49	4.9
/20	11/30	45	27,900	0	0	900	0	900	11	1.2	30	3.3
/1	12/10	50	31,000	0	0	1,000	0	1,000	17	1.7	34	3.4
/11	12/20	60	38,270	0	0	1,200	0	1,200	18	1.5	33	2.8

20. Quality-control Chart
—— Process Average, Major
.... Process Average, Minor

Process Average
(in percent defective)



17. Process average. Enter process average, major. This is expressed in percent and is computed as follows:

$$\frac{\text{Entry 16}}{\text{Entry 15}} \times 100$$

18–19. Minor defectives.

18. Number. Enter number of minor defectives found in samples included in process average.

19. Process average. Enter process average, minor. This is expressed in percent and is computed as follows:

$$\frac{\text{Entry 18}}{\text{Entry 15}} \times 100$$

20. Quality-control chart. Plot process averages if desired. Use solid line for majors, dotted line for minors.

CHAPTER 13

APPLICATION OF THE STANDARD PROCEDURE TO CONTROL SAMPLING

1. PURPOSE OF THIS CHAPTER

The preceding chapters of Part III describe a standard sampling-inspection procedure for use in acceptance sampling. It is the purpose of this chapter to indicate how this procedure should be modified so that it may apply to control sampling.

2. DETERMINING THE ACCEPTABLE-QUALITY LEVEL (AQL) FOR CONTROL SAMPLING

In control sampling, the AQL will sometimes be set equal to the process average. A sampling plan with an AQL equal to the process average will indicate that corrective action on the process is needed when in fact it is not (that is, when the product submitted is of process-average quality), 1 time in 20 (5 percent of the time). This is the translation into terms of control sampling of the definition of AQL used in acceptance sampling. If, for a particular application, this is too large or small a risk of searching for trouble when there is none, the procedure for selecting sampling plans described above cannot be used. As mentioned in Chap. 6, Sec. 9.1, an alternative is to set the AQL equal to the largest percent defective at which it is economical to look for trouble only 5 percent of the time. If the sample size is predetermined, as it often is in control sampling, turn to the appropriate sample-size letter and AQL class to get the acceptance and rejection numbers. If the sample size is not predetermined, pick the percentage defective at which it will be economical to hunt trouble as often as 90 percent of the time; this is the LTPD. Then examine the OC curves of the different sample-size letters having the chosen AQL class until the one that has the most satisfactory LTPD is found. When this curve is discovered a sampling plan appearing on the same page will be used. Generally the single-sampling plan will be used, though when control sampling is used on a continuous production line it will often be feasible and economical to use double or sequential plans.

In control sampling each time a change is observed in the process average a corresponding change is made in the AQL.

3. USE OF OPERATING-CHARACTERISTIC (OC) CURVES FOR CONTROL SAMPLING

OC curves are applicable to control as well as to acceptance sampling with the following modification. The vertical scale (ordinate) of the curve should read "percentage of time process is accepted" instead of "percentage of submitted inspection lots accepted." In other words, the height of the curve represents, for a given percentage of defective items in submitted inspection lots, the frequency with which the sampling plan indicates that no corrective action should be taken on the production process. The vertical distance from the curve to the top of the graph represents, for a given percentage of defective items in submitted inspection lots, the frequency with which the sampling plan indicates that corrective action should be taken on the production process.

4. ACTION TAKEN WHEN THE SAMPLING PLAN INDICATES REJECTION

When sampling is performed for control purposes only, no action is taken on the submitted inspection lot. Instead, when the sampling plan indicates rejection, the production process is investigated and corrective action is taken to eliminate the causes of trouble. From the viewpoint of control sampling, the instructions in Chap. 8 through 12 that refer to accepting or rejecting inspection lots may be interpreted to refer to accepting or rejecting the quality of the production process.

5. USE OF THE CONTROL-CHART TECHNIQUE

Quality control by the control-chart technique was described in Chap. 6. It was also explained in Chap. 6 how the sampling plans in this book can be used for control sampling. The reader is therefore referred to Chap. 6 for further information on this subject.

It should be borne in mind that control sampling gives protection against bad quality in future product and not against bad quality in product already produced. For this reason, control sampling must often be supplemented by a program of acceptance sampling, such as described in the earlier chapters of Part II and in Part III.

Part IV

CONSTRUCTION OF SAMPLING TABLES AND STANDARD PROCEDURE

CHAPTER 14

INTRODUCTION

1. PURPOSE OF PART IV

The tables in Part V, together with the text of Part III, provide (*a*) a collection of more than 350 sampling plans for inspection by attributes (Tables 2 and 4 of Part V) and (*b*) a procedure for selecting a sampling plan from this collection for use in a particular inspection problem (Table 11.5 of Chap. 11, Table 1 of Part V, and text of Part III).

The purpose of Part IV is to describe in detail how this collection of sampling plans was constructed and how the procedure was devised. Both the collection of sampling plans and the standard procedure are an application of the general principles of sampling inspection described in Part II. Familiarity with these principles, as well as with the tables and standard procedure, is therefore assumed in this part.

2. RELATION OF THE COLLECTION OF SAMPLING PLANS TO THE STANDARD PROCEDURE

A collection of sampling plans can serve two somewhat different purposes. First, it can serve simply as a catalogue of sampling plans from which a particular plan can be selected for each inspection problem on the basis of a special analysis of that problem. Second, it can serve as the basis of an integrated inspection program involving the systematic, coordinated selection of sampling plans for several products, several suppliers of the same product, or several departments or plants of the same firm.

A collection of sampling plans is adequate for the first purpose if it (*a*) is fairly comprehensive—that is, contains a large number of plans appropriate for a wide variety of inspection problems, (*b*) is arranged so that a particular sampling plan can be found easily, (*c*) gives as much relevant information as possible about each plan. Such a collection might, however, be entirely inadequate for an integrated inspection program. Such a program requires not only standardization and the use of ready-made rather than custom-built plans but also the use of a standard procedure for selecting a sampling plan and a separation of responsibilities for different phases of the inspection program. To meet these needs, the collection of sampling plans must not only be comprehensive enough to provide satisfactory plans for most problems but must be so

classified as to permit the steps in the procedure for selecting a plan to correspond with the division of responsibility in the organization using the collection of plans. For example, a classification of single-sampling plans by sample size and acceptance number might be convenient for finding a plan and hence might be entirely adequate for a simple catalogue of sampling plans. It would be entirely inadequate for an integrated inspection program, since the only separation of responsibilities it would permit would be the choice of the sample size by one person or group and of the acceptance number by another. But the acceptance number alone has no significance without the sample size, and conversely.

The collection of sampling tables in this book can be used merely as a catalogue. It was not, however, designed for that purpose but rather to serve as the basis of an integrated inspection program. The collection of sampling plans is therefore closely related to the procedure for selecting a plan, and both the collection and procedure are, in turn, shaped by the kind of separation of responsibilities conceived as necessary in an integrated inspection program.

The separation of responsibilities used in preparing the collection of sampling plans and the procedure in this book involves at least two main levels of decisions: decisions about quality requirements and relative amount of inspection applying to several sampling plans, and decisions about administrative features of the individual sampling plan. For example, a firm may buy the same product from several suppliers, the product submitted by each supplier being inspected separately for acceptance. It is ordinarily desirable to apply the same quality requirements to all suppliers, and the decision about the quality requirements is ordinarily made by an executive who has general responsibility. Someone who has immediate responsibility for inspection will then have to adjust the sampling plan for each supplier to the particular circumstances under which the product is submitted—the frequency and size of shipments, the form of packing, and so forth. As another example, consider a firm inspecting product at intermediate stages of production to determine whether the product should be passed for further processing. It is ordinarily desirable to impose the same, or at any event consistent, quality requirements on different departments and, subject to these quality requirements, to adjust the particular sampling plan for each department to the particular circumstances of that department.

For simplicity, the discussion that follows speaks in terms of the first example—that is, in terms of decisions for a product and decisions for a particular supplier of that product. It should be understood, however, that the same considerations apply to a much wider variety of inspection programs—inspection of the output of the same supplier at different times, of the output of several departments or plants of the same firm, of

the product submitted by suppliers of different products, of the final product of a firm before shipment, and so forth.

3. RELATION OF PLANS AND PROCEDURE TO PRIOR WORK

The collection of sampling plans and the procedure for selecting a sampling plan in this book are a reformulation and extension of much prior work on similar problems. The excellent tables and procedures of the Army Ordnance Department* are the immediate precursor of the tables in this book, which, while it differs in many respects from the Ordnance tables and procedures, adopts the same fundamental approaches. Extensive use has also been made of the experience of the Army Quartermaster Corps, particularly in the widespread use of sequential-sampling plans. And all sampling tables and procedures owe a great debt to the pioneering work of the Bell Telephone Laboratories, the fundamentals of which have now become so widely known and accepted that their origin tends to be obscured.

The Army Ordnance tables classify a large number of single- and double-sampling plans by amount of inspection and AQL.† Their procedure for selecting a plan for normal inspection is to specify an AQL and size of inspection lot. The amount of inspection is then determined uniquely by the inspection-lot size. Provision is made for both tightened and reduced inspection.

The collection of sampling plans in this book was based on Army Ordnance tables for several reasons. First, of the tables in existence, they were the most extensive and best suited to the needs of an integrated inspection program. In general, they are well constructed and incorporate the latest developments in single- and double-sampling plans.‡ Second, it was desired to avoid any further proliferation of tables except in so far as real differences were introduced; hence, it seemed desirable to base a new collection of sampling plans on a collection already in existence and to have the two collections overlap so far as possible. Third, double sampling is included in the present collection of tables primarily because double-sampling plans—principally Army Ordnance plans—have

* Army Service Forces, Ordnance Department, Industrial Service: *Ordnance Inspection Handbook on Standard Inspection Procedures, Quality Control*, Ord-M608-8.

† The Army Ordnance definition of AQL differs from that used in this book. See Sec. 2.2.1.2 of Chap. 15.

‡ One important qualification to this statement is that their double-sampling plans all have the second sample twice the size of the first and the same rejection numbers for the two samples. This particular relation between the sample sizes may not be the best, and the use of the same rejection number for both samples almost always leads to more inspection on the average than is necessary to obtain the associated OC curve. Neither feature is changed in the tables in this book since, as mentioned below, double sampling was included only to conform with existing practice.

been used fairly widely and the advantage of using a sequential-sampling plan instead of a double-sampling plan may not be sufficient to compensate for the cost of changing. (Sequential sampling had not been developed when the Army Ordnance tables were constructed.) This objective required close conformity between the double-sampling plans in the present collection and those in the Army Ordnance collection.

The major respects in which the present collection of sampling plans differs from the Army Ordnance collection are as follows: (*a*) it includes sequential-sampling plans as well as single- and double-sampling plans; (*b*) it includes plans for smaller sample sizes—for example, the single-sampling plans in the main Army Ordnance single-sampling table (Table C) have sample sizes varying from 75 to 1,500; in Table 2 of this book, the sample sizes vary from 5 to 1,500; (*c*) it uses a different concept of AQL; (*d*) all the plans are contained in a single table (Table 2), whereas Army Ordnance uses supplementary tables for reduced and tightened inspection and for inspection for incidental defects.

The procedure for selecting a plan in this book differs from the Army Ordnance procedure mainly in that it provides greater flexibility through the introduction of the concept of inspection level.

CHAPTER 15

THE COLLECTION OF SAMPLING PLANS

1. PROBLEMS IN PREPARING A COLLECTION OF SAMPLING PLANS

There are three major problems in preparing a collection of sampling plans: (*a*) determining the criteria by which the plans are to be classified, (*b*) selecting the specific classification to be used for each of the selected criteria, and (*c*) deriving the specific plans to go into the cells determined by this classification. The first two of these problems are discussed in Section 2, the third in Section 3.

2. CRITERIA FOR CLASSIFYING PLANS; PROTECTION AND COST

2.1. Introduction

The protection provided by a sampling plan and the cost of using it are the major considerations affecting its choice, and hence the major factors to be taken into account in classifying sampling plans.

2.2. Protection

2.2.1. Methods of Specifying Protection

2.2.1.1. INTRODUCTION.—The protection provided by a sampling plan is described completely by its OC curve. The two most important features of the OC curve are its steepness and its location. The steepness of the curve measures the power of the sampling plan to discriminate between inspection lots of different quality and is closely related to the amount and hence to the cost of inspection. The location of the curve measures the quality of inspection lots among which the plan discriminates, that is, what quality inspection lots it accepts and what quality it rejects. The location of the OC curve is a feature of protection that can generally be considered apart from the amount of inspection and can be used as a separate basis of classifying sampling plans.

The location of an OC curve is a vague concept that must be defined more precisely if it is to be used to classify sampling plans.* One way to define location is by means of a single point on the OC curve. The

* Note that only one classification can be used in addition to amount of inspection and type of sampling, since single-sampling attribute plans are a two-parameter family, that is, a single-sampling plan is completely determined by sample size and acceptance number.

ordinates of an OC curve run from 0 percent to 100 percent for all sampling plans. A specific value between 0 percent and 100 percent can therefore be selected and the location of the OC curve defined by the percentage of defective items for which the ordinate has this value. Examples are AQL and LTPD. The problem is what value to select. The considerations involved in choosing it can be covered fairly adequately by considering three possibilities: (*a*) a value near 100 percent, (*b*) a value around 50 percent, (*c*) a value near 0 percent. Another way to define location is by means of some measure computed from the OC curve. One such measure—the only one that has been widely used—is (*d*) the AOQL.

2.2.1.2. ACCEPTABLE-QUALITY LEVEL (AQL).—The quality of product corresponding to an ordinate of the OC curve close to 100 percent is commonly called the *acceptable-quality level*, abbreviated to AQL. There is not universal agreement on the numerical value that should be assigned to "close to 100 percent" (indeed, as indicated below, sometimes no single numerical value is assigned); the value assigned, somewhat arbitrarily, in this book is 95 percent. As the term is used in this book, then, the AQL of a sampling plan is defined as the percentage of defective items in an inspection lot such that the sampling plan will result in the acceptance of 95 percent of submitted inspection lots containing that percentage of defective items.

Two features of this definition should be noted.* First, it makes the AQL a technical property of a sampling plan. This is not always done. For example, Army Ordnance, in its inspection procedure, defines the AQL as "the maximum percent-defective which a facility can be permitted continually to present for acceptance."† In the Ordnance tables, which classify sampling plans by AQL and amount of inspection, the plans are so arranged that if the percentage of defective items in submitted inspection lots is equal to the stated AQL, the percentage of inspection lots accepted rises steadily from 95 percent to about 99 percent as the amount of inspection increases. The AQL so defined is therefore not a property of a sampling plan but an integral part of the procedure for selecting a plan.

We have defined the AQL as a property of a sampling plan for two main reasons. (*a*) It facilitates separation of responsibilities. The location of the OC curve is frequently chosen by one person or activity, the amount of inspection by another. The activity choosing the location can do so best if it is choosing something that has the same meaning and

* These features and the discussion of them that follows apply also to the definitions of indifference quality and lot tolerance percent defective.

† *Ordnance Inspection Handbook on Standard Inspection Procedures, Quality Control,* p. 3.

consequences no matter what choices are made by others. (*b*) It facilitates a separation of the technical from the planning phases of a procedure. If sampling plans are classified by their statistical properties, decisions about how to combine protection and cost—which are essentially decisions about policy—can be embodied in the instructions for selecting a plan from the tables. Thus, the plans can remain fixed despite changes of policy in their use.

The second feature of the definition of AQL that requires discussion is the choice of 95 as a percentage representing "close to 100." This choice is essentially arbitrary. A limited set of tables cannot include a plan for each possible AQL value. Approximate agreement between the AQL desired and the AQL of one of the available plans has to suffice. It is therefore desirable that the AQL not be defined in a manner that suggests extreme precision. The figure 95 has the advantage of being close to 100, yet a round figure. Further, use of the 95 percent point is quite common.

2.2.1.3. INDIFFERENCE QUALITY.—The quality of product corresponding to the ordinate of the OC curve equal to 50 percent may be called the indifference quality. Inspection lots of higher quality are accepted more frequently than they are rejected; inspection lots of lower quality are rejected more frequently than they are accepted. The quality that is accepted 50 percent of the time may be interpreted as the quality separating inspection lots that it would be desired to accept all the time if perfect discrimination could be achieved from inspection lots that it would be desired to reject all the time.

2.2.1.4. LOT TOLERANCE PERCENT DEFECTIVE (LTPD).—Symmetry with the definition of the AQL suggests selecting the point at which 95 percent of inspection lots are rejected as a point on the OC curve at which almost all inspection lots are rejected and calling the quality at this point the unacceptable-quality level. Symmetry was abandoned, however, in favor of custom. Tables of sampling plans classified by a point on the OC curve at which almost all inspection lots are rejected typically use a point at which 90 percent of inspection lots are rejected, that is, 10 percent are accepted, and call the quality at this point the *lot tolerance percent defective* (LTPD).

2.2.1.5. AVERAGE OUTGOING-QUALITY LIMIT (AOQL).—The average outgoing quality of product submitted to inspection under a particular sampling plan is approximately equal to the abscissa of the OC curve of that sampling plan times the ordinate; it is small for very good submitted quality and also for very poor submitted quality, and reaches a maximum for some intermediate quality. This maximum of the average outgoing quality is called the *average outgoing-quality limit* (AOQL).

The AOQL is valid only if inspection lots rejected on the basis of the

sampling plan are inspected completely and if this inspection is sufficiently accurate to discover all, or practically all, defective items. Since these conditions are not always satisfied and sometimes cannot be satisfied (for example, they cannot be satisfied if the quality of an item can be determined only by a destructive test), a classification of sampling plans by AOQL cannot meet all needs and, if used, must be supplemented by some other classification.

2.2.2. Reason for Selecting Acceptable-quality Level (AQL) as Basis of Classification.—Suppose there are a number of suppliers of the same product and that the sampling plans to be used for these suppliers are likely to call for different amounts of inspection (that is, to involve OC curves differing in steepness). Is it preferable for the sampling plans for the different suppliers to have the same AQL, or the same indifference quality, or the same LTPD, or the same AOQL? This question poses the central issue in choosing among these four methods of defining the location of an OC curve.*

The effect of using a sampling plan for acceptance inspection is in general to force the supplier to submit product of such a quality that a small percentage of submitted inspection lots is rejected. If he submits product of very low quality, many inspection lots are rejected, and he will be forced to correct them, scrap them, or sell them elsewhere. If he submits product of extremely high quality, he may be incurring higher production costs than are necessary in order that the great bulk of his product be accepted. Just what quality it pays him to submit depends, of course, on his circumstances. If the rejection of inspection lots involves him in considerable expense, while the maintenance of a low percentage of defective items in submitted product involves little added expense, it pays him to submit product so high in quality that rejections are extremely low. On the other hand, if rejections involve little expense, while the maintenance of a low percentage of defectives is very costly, it may pay him to submit product inferior in quality and to accept substantial rejections.

If the AQL is the same for different suppliers of the same product, they are all likely to operate at much the same quality level, since the AQL defines a quality at which rejections are low (5 percent) and since suppliers of the same product are likely to be industrially similar. The pressures on the suppliers depend to some extent on the amount of inspection per inspection lot. If the amount of inspection per inspection lot is small, the supplier will find that a slight decrease in the quality of

* As indicated in Sec. 2 of Chap. 14, the example of different suppliers of the same product is used to exemplify the kind of problem that arises in many different inspection programs—inspection of the product of the same supplier at different times, of the output of different parts of the same plant, and so forth.

product submitted will increase rejections only moderately; whereas, if the amount of inspection per inspection lot is large, the same decrease in quality will increase rejections sharply. Nonetheless the dominant effect is much the same: only by supplying product as good as or superior to the AQL can rejections be kept consistently small, regardless of the amount of inspection.

There are decided advantages in an acceptance program that makes the desirable region of operation the same for different suppliers of the same product—particularly when the sampling plans used are chosen by the purchaser of the product and when it is important to the purchaser that as much acceptable product as possible be submitted. If the purchaser chooses sampling plans for different suppliers in such a way as to keep the AQL the same, all suppliers, in effect, are being asked to maintain the same quality level. Once the suppliers know the AQL, they have the major piece of information they need to plan their production and inspection and to set prices in their bids for contracts. Since other features of the sampling plan do not affect them significantly, the purchaser is relatively free to make any other adjustments in the sampling plans he deems suitable, either in advance or in the course of inspection. By imposing the same quality requirements on suppliers, the purchaser is likely to broaden the available sources of supply. If he were to select plans for different suppliers in some other way, say by keeping the LTPD the same, he would be in the position of indirectly imposing stiffer requirements on some suppliers than on others (since, if the LTPD is the same, the AQL is lower—that is, represents higher quality—the smaller the amount of inspection per inspection lot), thereby imposing extra costs on the former and reducing their willingness to submit product in competition with the suppliers on whom the requirements are easier. Such differences in quality requirements would not reflect any difference in quality needed—since the plans under consideration are to be used for the same product produced by different suppliers—but solely an unwillingness to do as much inspection per inspection lot for some suppliers as for others. The potential output cut off by varying the quality requirements would be output as acceptable to the purchaser as other output he is purchasing.

The chief disadvantage of making the AQL the same for all suppliers of the same product is that it leads to variation in the risk of accepting product that is not satisfactory to the purchaser, if such product is offered—that is, it leads to variation in LTPD or AOQL. While the continued use of a sampling plan for acceptance inspection has the general effect of leading the supplier to submit product whose quality is at least as good as the AQL of the sampling plan, some product whose quality is worse than the AQL will be submitted: accidental fluctuations in quality

may lead to some poor product being submitted; the supplier may be temporarily or permanently unable to keep the quality of his product sufficiently high; or the cost to the supplier of rejections may be so low and the cost of improving quality so high that it pays him to submit relatively low-quality product. If the AQL is kept the same for different suppliers of the same product, the chance that such unsatisfactory product will be accepted is higher (that is, the LTPD or AOQL corresponds to poorer product) for those suppliers for whom the amount of inspection per inspection lot is small than for those for whom the amount of inspection per inspection lot is large.

The relative importance of the advantages and disadvantages of making the AQL the same for different suppliers depends in considerable measure on the setting in which inspection is being done. The advantages of fixing the AQL and thereby leading suppliers to operate at much the same quality level are likely to be greatest when a product is being purchased and inspected continuously from a number of suppliers, so that a running check can be kept on the quality of product being submitted and the quality of different suppliers can be compared;* when the costs of rejection to the supplier are relatively high, so that even small increases in rejections will force him to improve quality; and when there is an urgent need for the maximum output of acceptable-quality product. The advantages of fixing the LTPD or AOQL and thereby avoiding variation in the risk of accepting unsatisfactory product are likely to be greatest when product is purchased at irregular intervals or over a short period or when the product to be purchased has already been produced, so that there is little possibility for quality to be changed in response to the rate of rejections; when the costs of rejection to the supplier are relatively small; and when it is more important to avoid accepting unsatisfactory product than to accept as much satisfactory product as possible.

The AQL was selected as the method of classifying sampling plans by the location of the OC curve because it was judged that most integrated inspection programs would be of a kind for which the advantages of fixing the AQL more than counterbalance the disadvantages. To provide for other programs, the AOQL of each plan is given (in the form of a class interval) in Table 4; a supplementary table (Table 3) is also provided for use when sampling plans are chosen on the basis of AOQL.

2.2.3. Selection of Acceptable-quality Level (AQL) Classes.—Having decided that the AQL is to be used as a basis of classification, there remains the problem of determining the specific classes to be used—the number of classes, the range of AQL values they should cover, and the method of designating them.

* When production is continuous, the disadvantages of fixing AQL can be reduced considerably by the use of tightened inspection.

The number of classes and the range of AQL values covered largely follow the Army Ordnance tables. The difference in definition of AQL necessitated some reclassification of the Army Ordnance plans, but this reclassification did not change the number of classes or the approximate range of AQL values covered.

The Army Ordnance tables have 14 AQL classes running from AQL class (their AQL major) 0.005–0.010 percent defective to 4.1–5.0 percent defective. The lowest class was dropped because it corresponds to so low a percentage of defectives that extremely large samples would be needed to provide discrimination between acceptable and unacceptable inspection lots. The remaining 13 classes were relabeled, as described below, and a 14th class added, covering larger percentages of defectives, to yield the AQL classes in the tables of Part V.

Two methods of labeling each AQL class were considered: (*a*) designating each class by a single AQL value corresponding as closely as possible to the AQL's of the sampling plans in the class—for example, the plans in the first AQL class could have been described as having an AQL of 0.03, in the next class, of 0.05, etc.; (*b*) designating each class by a range of AQL values, the range for each class starting where the range for the preceding class leaves off; this procedure is the one adopted, the first class being designated 0.024 to 0.035, the next 0.035 to 0.06, and so forth.

The advantage of a single value is that each plan actually has only a single AQL, that is, a single quality at which 95 percent of submitted inspection lots are accepted. The disadvantage is that, in actual use, a plan may be desired with an AQL value other than one of those used to designate an AQL class; to select a plan, the closest AQL value given must be used; but the closest AQL value is likely to be taken as the one numerically closest, whereas the relevant criterion is the closeness of the probability of acceptance. For example, suppose that a plan giving an AQL of 4.5 percent is desired, that there are classes designated 4.2 and 5.0, and that two plans corresponding to these classes give OC curves like those in Figure 15.1. The AQL value 4.5 is numerically closer to 4.2 than to 5.0. Yet for the 4.2 plan in the figure, the probability of accepting product containing 4.5 percent defectives is only 90 percent, while it is 96 percent for the 5.0 plan. Since 96 percent is considerably closer to the desired probability, 95 percent, than is 90 percent, the plan whose AQL is 5.0 is preferable to the plan whose AQL is 4.2. The disadvantage of single values is, therefore, that the wrong criterion is almost certain to be used in rounding other values to them.

The use of AQL ranges avoids this disadvantage; in effect, it is a device for rounding in advance by the correct criterion instead of placing this burden on the user of the plans. The disadvantage is that it may give the erroneous impression that the sampling plans in a specified AQL class give a probability of acceptance of 95 percent for every value in the

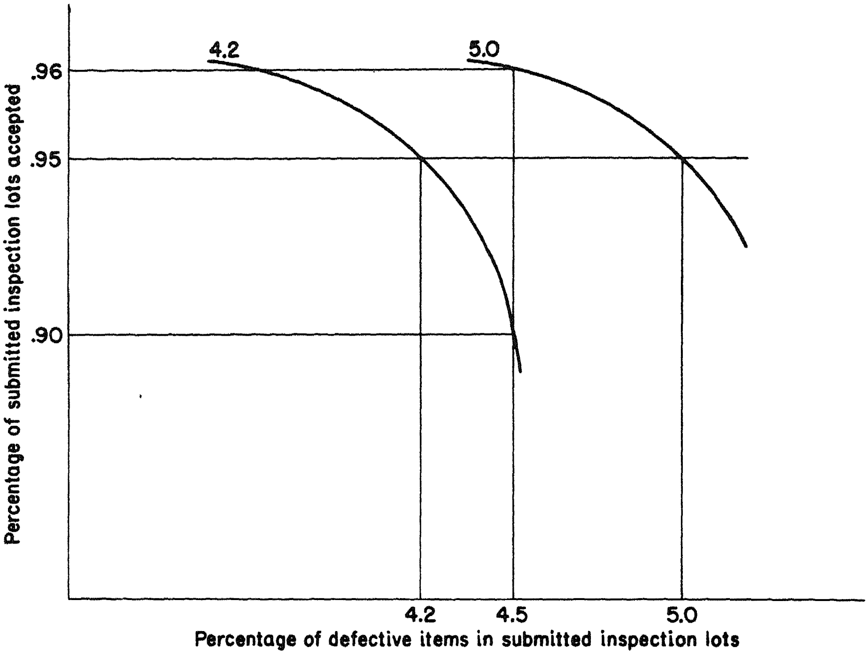


FIG. 15.1.

class, though a glance at the OC curve of any sampling plan would dispel this impression and give the actual AQL of the plan.

The discussion of rounding indicates the principle used to choose the ranges, namely, to choose the ranges so that, if the percentage defective is equal to the upper limit of the class, the percentage of inspection lots accepted is at least 92 percent for every sampling plan in the class. Subject to this requirement, the range was intended to include the exact AQL values of all plans in the class. It was not feasible to satisfy these requirements in every case; for a few plans, the probability of acceptance at the upper limit of the AQL class in which they are placed is slightly less than 0.92; for a somewhat larger number of plans (about 60 of the more than 340 plans in Table 2) the exact AQL value is outside the AQL class—usually above it. The percentage of inspection lots accepted if the percentage of defectives is equal to the lower limit of the class is usually well above 95, in some cases as high as 99 or 99.5.

2.3. Cost of Inspection

2.3.1. Factors on Which the Cost of Inspection Depends.—The total cost of inspection per item of output depends on (*a*) the type of sampling (single, double, or sequential); (*b*) the steepness of the OC curve for the selected sampling plan; (*c*) the size of the inspection lot.

The type of sampling affects both the sampling-inspection costs (the costs of selecting and inspecting sample items) that must be incurred per inspection lot to attain a given OC curve and the costs of administering inspection. The steepness of the OC curve determines the amount of inspection per inspection lot required for any given type of sampling. The size of the inspection lot determines the cost per item of output, given the cost per inspection lot.

The first two factors affecting cost are statistical characteristics of sampling plans and must therefore be considered in classifying sampling plans by characteristics affecting cost. The third, size of the inspection lot, is not a characteristic of sampling plans but rather of the product inspected. Since, further, the statistical characteristics of sampling plans do not, in general, depend on the size of the inspection lot,* the size of the inspection lot is neglected in this chapter; it will be considered in Chap. 16 in the discussion of the procedure for selecting a sampling plan.

2.3.2. Type of Sampling.—The type of sampling can in general be selected independently of any other decisions made in selecting a plan. Approximately the same protection—that is, the same OC curve—can be attained with any of the three types of sampling—single, double, or sequential. And the relative merits of the three types of sampling are not much affected by the particular OC curve they are to be used to attain.†

The decision on type of sampling should be made by the person or activity responsible for administering a specific sampling plan since the relative merits of the three types of sampling depend on the specific conditions under which inspection is conducted. The simplest way to set up a classification of sampling plans so that this is possible is to construct sets of sampling plans differing in type of inspection but having the same OC curve. The different types of sampling then enter as alternative methods of obtaining the same result.

This is the method adopted in the tables in this book. Each page of Table 4 (except for some incomplete pages) contains a matched trio of sampling plans—single-, double-, and sequential-sampling plans having approximately the same OC curves. It is not possible, of course, to get exactly the same OC curve with the different types of sampling, but Table 4 shows that it is possible to get OC curves close enough so that the differences are of little or no practical significance.

* They do so only if the sample is large compared with the inspection lot.

† There is some dependence, since the average amount of inspection saved by using sequential instead of double, or double instead of single, sampling depends to some extent on the OC curve that is to be attained. For the most part, however, this is an unimportant effect except when the desired OC curve can be attained by a single-sampling plan with zero acceptance number. See Sec. 3.2.3 below.

2.3.3. Amount of Inspection per Inspection Lot.—For any given type of sampling, the steepness of the OC curve depends on the amount of inspection per inspection lot. The simplest way, therefore, to classify plans by the steepness of the OC curve is to classify them by amount of inspection per inspection lot.

2.3.3.1. SINGLE-SAMPLING PLANS.—The amount of inspection under a single-sampling plan depends on whether inspection is or is not curtailed as soon as a decision can be reached. If the entire sample is inspected, the number of items inspected per inspection lot is equal to the size of the single sample and is the same no matter what quality product is submitted. If inspection is curtailed, the number of items inspected per inspection lot depends on the quality of the inspection lots submitted and there is no way to describe the amount of inspection by a single number or a few numbers. The size of the sample serves, however, as a rough index of the amount of inspection when inspection is curtailed. If the quality of the submitted inspection lots is in the neighborhood of the AQL —and most submitted inspection lots may be expected to be of this quality—practically all will be accepted, and hence the average amount of inspection per inspection lot will seldom differ from the sample size by more than the acceptance number.

Sample size therefore seems the appropriate basis for classifying single-sampling plans by the amount of inspection per inspection lot. The only problem that arises is whether to use the same set of sample sizes for all AQL classes. It is highly desirable that the plans in the same AQL class have as nearly the same AQL as possible. It would be very much easier to accomplish this if the set of sample sizes varied, even if only slightly, among the AQL classes. If this were done, single-sampling plans would be arrayed in a two-way table; the columns, let us say, would correspond to different AQL classes; the rows would correspond to different amounts of inspection; but the sample size in each row would vary somewhat from AQL class to AQL class.

Such variation in sample size from column to column of the table would be a serious disadvantage when inspection covers more than one class of defects and more than one AQL is used. In such cases, a separate sampling plan must be used for each class of defects, these plans, in general, being chosen from the same row of the table. It simplifies inspection greatly if these plans all call for the same sample size, since if they do one sample can be selected and the items in the sample inspected for each defect. If the plans for different classes of defects call for different sample sizes, difficulties are encountered in drawing different size samples and in making sure that the proper number of items is inspected for each class of defects. For this reason, the same set of sample sizes has been used for all AQL classes.

The exact set of sample sizes used is, of course, rather arbitrary. The sample sizes from 75 up are the same as those in the Army Ordnance tables. To provide for smaller amounts of inspection, the sample sizes 5, 10, 15, 20, 30, 40, and 55 were added to those in the Army Ordnance tables.

2.3.3.2. DOUBLE- AND SEQUENTIAL-SAMPLING PLANS.—The amount of inspection per inspection lot required by a double- or sequential-sampling plan depends on the quality of product being submitted and cannot be described even roughly by the size of samples to be selected. This is particularly true for sequential sampling, since the same OC curve can be attained with a large number of different sample sizes. However, the decision to treat type of sampling by constructing sets of single, double, and sequential plans giving about the same OC curve means that the classification of single-sampling plans by amount of inspection determines the corresponding classification of double- and sequential-sampling plans. These plans are to be chosen to give the same OC curve as the corresponding single-sampling plan; in so far as there are alternative double- or sequential-sampling plans, those should be chosen that require the smallest amount of inspection in the neighborhood of the AQL (since it may be expected that most submitted product will be of AQL quality or better); the amount of inspection is then automatically determined.

The sample sizes to be used for the double and sequential plans are somewhat independent of the amount of inspection. The considerations cited above in discussing single sampling indicate the desirability of using a single set of sample sizes for each amount of inspection class, these sample sizes applying to all AQL classes.

The sample sizes for the double-sampling plans corresponding to single-sample sizes of 75 and up (sample-size letters H through O) were taken from the Army Ordnance tables. These tables uniformly make the first sample of the double-sampling plan two-thirds the matching single sample; and the second sample, twice the first (see the last footnote on page 141). Accordingly, these relations were used to derive the sample sizes for the double-sampling plans corresponding to the single-sample sizes less than 75 (sample-size letters A through G).

There is considerable leeway in the choice of the number and size of the samples for the sequential-sampling plans, since essentially the same OC curve can be attained with a wide variety of sample sizes with little loss in amount of inspection. The set of sample sizes that would require the least inspection on the average would be the one that would make the grouped sequential plans of the kind used in this book essentially equivalent to the corresponding item-by-item sequential plans.* But this

* See Statistical Research Group, Columbia University, *Sequential Analysis of Statistical Data: Applications*, Sec. 2, Second Revision, Columbia University Press, New York, 1945.

would require a different set of sample sizes for each AQL class, which is ruled out by the desirability of using the same sample sizes for all AQL classes.

Administrative considerations suggested the desirability of making the successive samples equal in size, of having each sample moderate in size, and of setting a fairly low limit on the maximum number of samples that might be required.

The advantage of having the successive samples equal in size is that the inspector then has to remember only one sample size. Equal sample sizes do, however, have one disadvantage. The process average is ordinarily based solely on the results for the first sample; there is some justification, therefore, for making the first sample larger than the others in order to get more extensive data on which to base the process average. This consideration is most important, of course, for the lower sample-size letters.*

The reasons for having each sample moderate in size are, first, to facilitate the selection of samples, second, to provide sufficient information for computing the process average. In practice, it is usually desirable to segregate the inspection lot into portions that can be expected to be fairly homogeneous and to select the sample proportionately from these portions. This type of sampling becomes extremely difficult if a very small sample is to be selected.

The desirability of a fairly low limit on the maximum number of samples is in part a corollary of the desirability of having each sample moderate in size. An additional reason is that inspection costs depend to some extent on the number of samples. More or less arbitrarily it was decided that nine was about the largest number of samples that should be used.

Superimposed on these administrative considerations is the desirability of having the maximum number of items that might have to be inspected (the truncation point) sufficiently high to get the major advan-

* For sample-size letter A, the sample sizes are not equal, the first sample consisting of three items, the second and third of two. The reason for this departure is that there are only two sampling plans for this sample-size letter, and it turned out to be easier to get the desired properties for these sampling plans by using sample sizes 3, 2, and 2 than by using equal sample sizes.

It should be noted that in many of the sampling plans for low sample-size letters, acceptance is not possible on the first sample. In these cases, the process average to be used in determining eligibility for reduced sampling could be based on results through the earliest sample at which acceptance is possible, provided that no more than 20 inspection lots are used in computing the process average. This process average would not be satisfactory if any of the inspection ots had been rejected with samples smaller than the minimum size for acceptance; in that case, however, reduced inspection would in any event not be permitted, since one of the requirements for reduced inspection is that no inspection lot out of the last 20 submitted be rejected.

tages of sequential analysis. It has been shown that little is lost in truncating a sequential plan if the truncation occurs at or beyond something like twice the maximum average number of items required by the item-by-item sequential plan.* The maximum average number of items required by an item-by-item sequential plan was therefore computed, for each cell of Table 2.† These numbers differed, of course, from AQL class to AQL class, in general being higher for the low AQL classes. Maximum numbers of items were then chosen for each row of Table 2 that would be at least twice the maximum average number of items for most cells and not less than the maximum average number of items for any cell.

These maximum numbers turned out to be in general in the neighborhood of the maximum number of items for the corresponding double-sampling plans. They were therefore adjusted so as to be uniformly below the maximum number for the double-sampling plans and then rounded to convenient figures. For the highest sample-size letters, they were divided into nine samples and then into successively smaller numbers of samples for the lower sample-size letters in order to keep each sample sufficiently large. The precise number of samples for each sample-size letter and the precise size of each sample are clearly somewhat arbitrary. The choice involved balancing the advantage of having many samples against the disadvantage of having each sample too small and, in addition, took account of the desirability of having each sample a convenient size. For example, it may seem somewhat illogical to have 8 samples for sample-size letter K and only 7 for sample-size letter L; but this enabled each sample size to be 50 for sample-size letter K and 75 for sample-size letter L, which seemed more convenient than, say, 60 and 75, or 50 and 65.

2.3.3.3. DESIGNATION OF AMOUNT OF INSPECTION CLASSES.—Each amount of inspection class includes single-, double-, and sequential-sampling plans, each with its own sample size or set of sample szies. The concept of *sample-size letter* was therefore introduced to provide a convenient method of designating the amount of inspection class. The class requiring the smallest amount of inspection was labeled A; the next class B; and so on, the final class being labeled O. The sample-size letter A therefore means a sample of 5 items for single sampling, a first sample of 3 and a second sample of 6 for double sampling, and a first sample of 3 and two additional samples of 2 each for sequential sampling; and the other sample-size letters have similar meanings.

* See *Sequential Analysis of Statistical Data: Applications* Appendices.

† These plans were determined by taking p_1 as the desired AQL in the cell, p_2 as the desired LTPD, α as 0.05, and β as 0.10. The desired AQL's and LTPD's were determined from the single- and double-sampling plans in the corresponding cells since, at this stage of the work, the sequential plans had not, of course, been derived. See *Ibid.*, Sec. 2, Second Revision, pp. 2.12, 2.15, for the meaning of p_1, p_2, α, and β.

3. PREPARATION OF TABLE 2

3.1. General Procedure

As indicated in Sec. 3 of Chap. 14, the sampling plans in the Army Ordnance tables formed the basis for the collection of sampling plans in Table 2. The Army Ordnance tables provided single- and double-sampling plans for the lower part of Tables 2*a* and 2*b*, that is, for sample-size letters H through O, and for all except the highest AQL class (6.4 to 8.5 percent defective). The difference between the concept of AQL employed in the tables of this book and the concept employed by Army Ordnance meant, however, that these plans could not be inserted directly into Table 2. As the first step in the preparation of Table 2, therefore, the AQL, as defined in this book, was computed for each of the Army Ordnance plans, and the plans were reclassified and, where necessary, minor adjustments made in the acceptance and rejection numbers, so that the set of plans assigned to each AQL class would have as nearly as possible the same AQL. The second step was the addition of tentative single- and double-sampling plans for the smaller sample sizes, that is, for sample-size letters A through H, and for the added AQL class. The third step was the addition of single-sampling plans with zero acceptance number, since these are not included in the main Ordnance table but only in a supplementary table. The fourth step was the addition of tentative sequential-sampling plans to cells containing single- and double-sampling plans, cells containing only double-sampling plans, and some cells containing neither single- nor double-sampling plans (sequential sampling had not been developed when the Army Ordnance tables were constructed). These steps yielded a tentative Table 2, completely filled in. The single-, double-, and sequential-sampling plans in each cell were, however, only tentative, since the agreement among their OC curves had been checked only at two points. As the final step in the preparation of Table 2, therefore, the OC curves of all the plans in the tentative table were computed, the OC curves of plans in the same cell were compared, and adjustments were made in any plans for which the agreement among the OC curves did not seem satisfactory.

The following sections describe each of these steps in greater detail, except that the first step is omitted because it requires no further discussion.

3.2. Single- and Double-sampling Plans for Small Sample Sizes and for the Added Acceptable-quality Level (AQL) Class

The procedure followed in deriving single- and double-sampling plans for the smaller sample sizes and the added AQL class was to fill in single-sampling plans first and then to add matching double-sampling plans.

The reason for filling in the single-sampling plan first is that a single-sampling plan has fewer parameters than a double-sampling plan and hence it is easier to match a double-sampling plan to a prescribed single-sampling plan than to do the reverse.

3.2.1. Single-sampling Plans.—A single-sampling plan is determined entirely by two parameters: the sample size and acceptance number. Given the sample size, the acceptance number is determined by the requirement that the plan have the desired AQL. For the smaller sample sizes, there was little problem of what acceptance numbers to use, since it was generally necessary to use every acceptance number from 0 to some upper limit and, even so, it was not possible to get plans for all AQL classes, since low AQL values cannot be attained with small samples.*

3.2.2. Double-sampling Plans.—For double-sampling plans, there are three free parameters even after the sample sizes are specified: (*a*) the acceptance number for the first sample, (*b*) the acceptance number for the second sample, (*c*) the rejection number for the first sample.† However, Army Ordnance plans, which we adopted, use the same rejection number for the first and second samples. This leaves only two free parameters: the two acceptance numbers. Two conditions can therefore be imposed in selecting a double-sampling plan to match a prescribed single-sampling plan, although, because the acceptance numbers must be integers, such conditions cannot be met exactly. The two conditions imposed in the derivation of the tentative plans were that the single- and double-sampling plans have the same AQL and LTPD.

The determination of the appropriate double-sampling plans was considerably simplified by the use of tables computed at the Ballistic Research Laboratory, Aberdeen Proving Ground, though these tables could not be used for the low sample-size letters.‡

*More generally, if the sample size is n; the acceptance number, a; and the proportion of defects in the inspection lot, p; the probability of acceptance is the sum of the first $a + 1$ terms (that is, the terms giving the probability of 0, 1, 2, . . . , a defectives) of the binomial expansion

$$[(1 - p) + p)]^n$$

For p fairly small and n fairly large, the binomial can be approximated sufficiently accurately by the Poisson exponential; and for n sufficiently large, by the normal distribution. If the AQL is to be p', the acceptance number, a, is determined by the requirement that the sum of the first $a + 1$ terms of the binomial expansion with p replaced by p' or of the approximating Poisson, or the corresponding integral under the approximating normal curve, be 0.95.

† The rejection number for the second sample is fixed by the requirement that it be one unit larger than the acceptance number for the second sample.

‡ L. W. Shaw and A. Stein, *Tables of Acceptance Probabilities for Double Sampling Plans with* $n_2 = 2n_1$, Aberdeen Ballistic Research Laboratory Memorandum Report No. 264.

3.2.3. Single-sampling Plans with Zero Acceptance Numbers.—Army Ordnance Table C, which contains the single-sampling plans, contains no plans with zero acceptance numbers, though such plans are included in a supplementary Army Ordnance Table VI. It was desired in the tables in this volume to avoid, so far as possible, supplementary tables and to cover as wide a range of AQL's as possible. Accordingly, plans with zero acceptance numbers were included in Table 2 in the appropriate AQL classes. For cells containing such plans, no matching double or sequential plans are included; instead, the user of the table is instructed to use single sampling. The reason for this is that if the OC curve of a single-sampling plan with acceptance number zero is satisfactory, no double or sequential plan is as economical.*

3.3. Sequential-sampling Plans

3.3.1. Reasons for Using Grouped and Truncated Sequential Plans.—The inclusion of sequential-sampling plans in the tables posed a problem of the type of sequential plan to use. Sequential analysis applies strictly when one item is inspected at a time, a decision to accept the inspection lot, reject the inspection lot, or continue inspection is made after every item, and no definite upper limit is set on the number of items inspected.† For the problem of attribute inspection, the item-by-item sequential plan that minimizes the average amount of inspection at two values of the proportion of defective items, say p_1 and p_2, is completely determined by specifying the desired probabilities of acceptance at p_1 and p_2.

Item-by-item sequential plans do not, however, seem desirable or feasible for a large-scale inspection program. They are likely to be difficult to administer and they require that each item to be inspected be selected strictly at random from the entire inspection lot, which seems hard to assure. It is preferable to use a *grouped* sequential, that is, a multiple-sampling, plan in which a group of items is selected at random and inspected; on the basis of the results for this group, the inspection lot is accepted or rejected or an additional group of items is selected and inspected, and so forth; and also to *truncate* the sequential plan, that is, set a definite upper limit on the number of items inspected. The substitution of a grouped and truncated sequential plan for an item-by-item sequential plan requires the inspection of a somewhat larger average

* The mathematical proof of this, which is due to Milton Friedman, is contained in an unpublished memorandum, "Uniformly Best Tests for a Special Class of Problems," dated 12 June 1945.

† See A. Wald, "Sequential Tests of Statistical Hypotheses," *Annals of Mathematical Statistics*, Vol. 16 (1945), pp. 117–186; Statistical Research Group, *Sequential Analysis of Statistical Data: Applications*, Columbia University Press, New York, 1945.

number of items per inspection lot to attain the same OC curve. In return for this loss, it reduces the variability of the amount of inspection from inspection lot to inspection lot. Even more important, the success of the Army Quartermaster Corps, which used such grouped and truncated sequential plans in its extensive sampling-inspection program, had demonstrated that such plans were feasible in a large-scale inspection program.

The decision to use grouped and truncated instead of item-by-item sequential plans made it necessary to decide (*a*) how large to make each group or sample and how many samples to have, that is, where to truncate, (*b*) what acceptance and rejection numbers to use. The decisions reached about the number and size of samples have already been described. We turn therefore to the choice of acceptance and rejection numbers.

3.3.2. Choice of Acceptance and Rejection Numbers.—The item-by-item sequential plan that minimizes the amount of inspection at two proportions defective, say p_1 and p_2, was taken as the basis for the determination of the acceptance and rejection numbers for the grouped and truncated sequential plans. For each cell of Table 2 containing both single- and double-sampling plans, there was available from the prior work the AQL and LTPD of these plans. The single- and double-sampling plans did not, of course, have exactly the same AQL or the same LTPD. A value was chosen for p_1 that was about midway between the AQL's for the single- and double-sampling plans and a value for p_2 about midway between the LTPD's. The parameters of an item-by-item sequential plan giving an AQL equal to p_1 and an LTPD equal to p_2 and minimizing the amount of inspection at these two points were then determined.* These parameters in effect define acceptance and rejection numbers for an inspection plan in which each sample consists of one item. No exact method is known of deriving, from such acceptance and rejection numbers, acceptance and rejection numbers of a grouped and truncated sequential plan having essentially the same OC curve. A trial-and-error process to accomplish this is described below (Chap. 17). Its most important feature, for the present purpose, is that the probability of acceptance at p_1, computed exactly for the grouped and truncated

* An alternative procedure would be to determine for each AQL class and all sample-size letters a single value of p_1 equal to the average of the AQL values for the single- and double-sampling plans in that AQL class; to plot for each AQL class the LTPD values as a function of the single-sample size; to pass a smooth curve through these LTPD values; and to determine p_2 for each sample-size letter from this curve. This would mean somewhat less close matching of OC curves within each cell but somewhat more consistency from cell to cell.

This alternative procedure was followed in constructing the sequential plans for cells containing no single- or double-sampling plans.

sequential plan, is approximately 0.95 and at p_2, approximately 0.10. While, therefore, grouping and truncating sacrifice some of the efficiency of the item-by-item sequential plan, they are consistent with the retention of the OC curve.

A similar procedure was used to derive sequential plans for cells containing only double-sampling plans, except that the AQL and LTPD of the double-sampling plans were taken as p_1 and p_2. In addition, sequential plans were added to some cells containing neither single- nor double-sampling plans. This was possible because the greater flexibility of sequential plans makes it possible to attain AQL's that cannot be attained with the other types of sampling. For these cells, the values of p_1 and p_2 were obtained by extrapolation from the AQL and LTPD of plans in neighboring cells (see footnote on page 159).

3.4. Matching Operating-characteristic (OC) Curves

For the sample sizes and AQL classes covered by the Army Ordnance tables, the single and double plans initially placed in each cell were simply those that corresponded to one another in the Army Ordnance tables, except for minor modifications to attain uniformity of AQL values. Sequential plans were initially matched to these, and single, double, and sequential plans were initially matched for other sample sizes and the added AQL class, by the crude procedure of getting rough agreement at two points, the AQL and the LTPD. After this had been done, OC curves were computed for all the plans tentatively selected, and the OC curves for single, double, and sequential plans in the same cell were plotted on the same graph. Examination of these graphs revealed close agreement of the entire OC curves for most cells, rather bad agreement for some. For the latter cells, revisions were made in one or more of the plans to achieve closer agreement. The degree of success that attended the effort to match OC curves can be judged from the pages of Table 4, which give the OC curves of the final plans.

4. AVERAGE OUTGOING-QUALITY LIMIT (AOQL) CLASSES FOR SAMPLING PLANS

Table 4 gives for each sampling plan an AOQL class, and Table 3 classifies all sampling plans by AOQL class and AQL class to facilitate the use of these plans in an inspection program based on AOQL instead of AQL. Table 4 also includes a few sampling plans not included in Table 2. These were added in order to give sampling plans for all AOQL classes. This section describes how the AOQL classes shown in Tables 3 and 4 were derived.

The AOQL of each plan in Table 2 was computed on the assumption that the size of the inspection lot is large relative to the sample. The

plans were then tentatively classified by AOQL. On the basis of this tentative classification, AOQL class intervals were chosen so that (*a*) there would be as few gaps as possible in the table, to avoid the necessity of including additional plans; (*b*) there would be as few sampling plans as possible with AOQL values outside the AOQL classes in which they would have to be included to avoid gaps; (*c*) there would be as many AOQL classes as possible.

It was possible to come fairly close to satisfying these requirements. The only additional plans included are for the AOQL class 7.0 to 11.0, sample-size letters J and K. For these sample-size letters, none of the plans in Table 2 has an AOQL above 7.0 percent. A few of the plans in Table 4 have AOQL values a trifle outside the AOQL classes in which they are listed; and a few others had to be revised to avoid even larger discrepancies.

The AOQL values according to which the plans are classified are correct only if the inspection lot is large compared with the sample, because these values do not allow for the elimination of defective items from inspection lots accepted on the basis of sampling inspection. If the sample is not a negligible fraction of the inspection lot, the actual AOQL will be lower since all defective items will be removed from the samples inspected. The supplement to Table 3 shows, for certain single-sampling plans, the actual AOQL for various inspection-lot sizes.

5. SUPPLEMENTARY DATA ON SAMPLING PLANS

Some data on the sampling plans in Table 4 are available that are not given in that table. These data are summarized in Tables 15.1 to 15.7.

Table 15.1.—As already explained, the OC curves in Table 4 for sequential plans are only approximate, the OC curve for each grouped and truncated sequential (multiple-sampling) plan being approximated by the OC curve for the item-by-item sequential plan from which the multiple-sampling plan was derived. Table 15.1 gives two ordinates of the exact OC curve for the multiple-sampling plans—the ordinate at the intended AQL (L_{p_1}) and at the intended LTPD (L_{p_2}). The first ordinate is approximated in Table 4 by 0.95; the second, by 0.10. The difference between the values of L_{p_1} in Table 15.1 and 0.95, and between the values of L_{p_2} and 0.10, therefore gives an indication of the possible error in the approximate OC curves in Table 4.

Tables 15.2 *and* 15.3.—These tables give the AQL and LTPD of each sampling plan. The values given were read from graphs of the OC curves.

Table 15.4.—This table gives the AOQL of each sampling plan. For a few of the single-sampling plans the values were obtained mathematically. The expression for the average outgoing quality was written

TABLE 15.1

PROBABILITIES OF ACCEPTANCE FOR MULTIPLE-SAMPLING PLANS AT p_1 [INTENDED ACCEPTABLE-QUALITY LEVEL (AQL)] AND p_2 [INTENDED LOT TOLERANCE PERCENT DEFECTIVE (LTPD)]

L_{p_1} = probability of acceptance at p_1

L_{p_2} = probability of acceptance at p_2

Acceptable-quality level (AQL) class (in % defective)

Sample-size letter	0 024-0.035		0 035-0 06		0 06-0.12		0 12-0.17		0 17-0 22		0 22-0 32		0.32-0 65		0 65-1 2		1 2-2 2		2 2-3 2		3 2-4 4		4 4-5 3		5 3-6 4		6 4-8.5		8 5-11.0	
	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}	L_{p_1}	L_{p_2}
A																									0 948	0 097	0 955	0 110		
B																					0 954	0 095	0 956	0 102	0 952	0 099	0 942	0 100		
C																			0 957	0 110	0.942	0 131	0.948	0 116	0 946	0 111	0.946	0 103		
D																	0.947	0 108	0 947	0 132	0 945	0 098	0 938	0.114	0 941	0 108	0 945	0 100		
E															0 962	0 111	0 956	0 098	0.960	0 101	0 944	0 107	0.946	0 103	0 951	0 098	0 952	0 099		
F															0 952	0 098	0 949	0 101	0 944	0.105	0 949	0 102	0 941	0 110	0 944	0 098	0 948	0.100		
G													0 969	0 091	0 953	0.091	0 952	0 109	0.945	0 102	0 944	0 096	0 950	0.103	0.950	0 097	0 947	0 101		
H											0 954	0 096	0 957	0 107	0 950	0 099	0 946	0 100	0 952	0 093	0 952	0 097	0.951	0 098	0.951	0.101	0 951	0.097		
I									0 964	0 095	0 948	0 102	0 944	0 098	0 949	0 097	0 949	0 102	0 947	0.100	0 950	0.097	0 950	0 098	0.940	0 106	0 950	0 098		
J							0 964	0 095	0.958	0.095	0 953	0 096	0 949	0 089	0 954	0 091	0 952	0 103	0.950	0 102	0 949	0 098	0.950	0 100	0 949	0 095	0 951	0 095	0 950	0.099
K					0 953	0 089	0 950	0 124	0 955	0 092	0 952	0.099	0 953	0.100	0 949	0 102	0 953	0.099	0 953	0 098	0 949	0 099	0 950	0 095	0 949	0 090	0 947	0 102	0 947	0.101
L			0 954	0 115	0 952	0 108	0 951	0 103	0 950	0 101	0 949	0.089	0.950	0.101	0.960	0 105	0 950	0 105	0 947	0 110	0.949	0.105	0 951	0 099	0 952	0 094				
M	0.955	0.091	0 944	0 108	0 950	0 101	0.950	0 102	0.953	0 099	0 952	0.107	0 954	0 105	0.949	0.105	0 945	0 110	0 942	0 108	0 945	0 105								
N	0 942	0 127	0 948	0 117	0 949	0 102	0 951	0 099	0 950	0 102	0 950	0 104	0.948	0 107	0 957	0.106	0 947	0 100												
O	0 949	0 121	0 948	0 106	0.945	0 104	0.957	0.104	0 955	0 104	0 948	0 107	0 941	0 106	0 947	0 100														

TABLE 15.2

ACCEPTABLE-QUALITY LEVEL (AQL) OF SAMPLING PLANS*

Sample-size letter	Type of sampling	AQL of sampling plan in AQL class (in % defective)														
		0.024–0.035	0.035–0.06	0.06–0.12	0.12–0.17	0.17–0.22	0.22–0.32	0.32–0.65	0.65–1.2	1.2–2.2	2.2–3.2	3.2–4.4	4.4–5.3	5.3–6.4	6.4–8.5	8.5–11.0
A	Single								1.0					7.2	7.2	
	Double													5.5	5.5	
	Sequential													6.5	7.5	
B	Single							0.50				3.6	3.6	9.0	9.0	
	Double											4.3	4.3	6.5	8.5	
	Sequential											4.0	5.0	7.5	8.5	
C	Single							0.50			2.5	2.5	5.6	5.6	9.6	
	Double										3.0	3.0	4.7	6.2	9.8	
	Sequential										2.8	3.5	5.0	5.8	8.5	
D	Single						0.50			1.8	1.8	4.1	4.1	7.2	7.2	
	Double									2.4	2.4	5.2	5.2	7.2	7.2	
	Sequential									2.0	2.7	4.0	5.0	7.0	7.5	
E	Single					0.20			1.1	2.6	2.6	4.6	4.6	6.5	9.1	
	Double								0.80	2.3	2.9	3.8	4.8	6.4	8.6	
	Sequential								1.2	2.0	2.6	4.2	4.8	6.5	8.5	
F	Single				0.20				0.90	2.1	3.4	3.4	5.1	6.7	8.4	
	Double								1.3	2.3	3.0	3.8	4.4	6.8	8.6	
	Sequential								1.1	2.2	3.1	3.8	4.8	6.3	8.5	
G	Single			0.10				0.65	1.5	2.5	2.5	3.7	4.7	6.0	8.2	
	Double							0.40	1.3	2.2	2.6	3.7	4.9	5.9	8.1	
	Sequential							0.50	1.4	2.0	2.6	3.3	4.9	5.8	8.2	
H	Single		0.05					0.40	1.1	2.0	2.6	4.3	5.3	6.2	8.2	
	Double							0.60	1.2	1.9	2.6	4.1	5.0	6.3	8.2	
	Sequential						0.30	0.50	1.1	2.0	2.6	4.0	5.0	6.0	8.2	
I	Single							0.70	1.2	2.3	2.9	4.0	5.4	6.0	8.0	
	Double						0.20	0.55	1.2	1.9	2.9	3.9	5.0	6.0	8.0	
	Sequential					0.20	0.30	0.60	1.2	2.0	3.0	4.0	5.0	6.0	8.0	
J	Single	0.04					0.22	0.52	1.3	2.2	3.0	4.1	5.1	6.2	8.3	9.3
	Double					0.18	0.30	0.60	1.1	2.0	3.0	4.1	5.2	6.4	8.1	9.0
	Sequential				0.15	0.20	0.27	0.60	1.1	2.0	3.0	4.1	5.1	6.1	8.0	9.2
K	Single					0.14	0.35	0.60	1.2	2.1	3.0	4.0	5.2	6.5	8.1	9.4
	Double				0.20	0.20	0.30	0.65	1.1	2.2	3.1	4.0	5.2	6.5	8.2	9.8
	Sequential			0.10	0.15	0.20	0.30	0.60	1.2	2.1	3.1	4.1	5.2	6.2	8.0	9.6
L	Single				0.11	0.26	0.26	0.65	1.3	2.0	3.0	4.1	5.2	6.6		
	Double			0.07	0.16	0.20	0.30	0.65	1.2	2.0	3.2	4.2	5.2	6.6		
	Sequential		0.05	0.10	0.15	0.23	0.30	0.65	1.1	2.1	3.2	4.2	5.2	6.2		
M	Single			0.06	0.18	0.18	0.32	0.58	1.2	2.0	3.0	4.2				
	Double		0.06	0.10	0.15	0.20	0.32	0.60	1.2	2.1	3.0	4.3				
	Sequential	0.03	0.05	0.10	0.18	0.20	0.30	0.60	1.1	2.1	3.2	4.2				
N	Single		0.04	0.10	0.19	0.19	0.35	0.62	1.1	2.2						
	Double	0.03	0.06	0.11	0.19	0.19	0.32	0.61	1.2	2.2						
	Sequential	0.03	0.05	0.10	0.15	0.20	0.30	0.60	1.1	2.2						
O	Single	0.02	0.06	0.09	0.14	0.22	0.30	0.56	1.2							
	Double	0.03	0.06	0.10	0.15	0.21	0.30	0.62	1.2							
	Sequential	0.03	0.06	0.10	0.15	0.20	0.30	0.60	1.1							

* The figures in Table 15.2 are graph readings and are not necessarily accurate to as many figures as given. They are stated as percentages defective.

TABLE 15.3
LOT TOLERANCE PERCENT DEFECTIVE (LTPD) OF SAMPLING PLANS*

Sample-size letter	Type of sampling	LTPD (in % defective) of sampling plan in AQL class (in % defective)														
		0 024–0.035	0 035–0 06	0 06–0.12	0 12–0 17	0 17–0 22	0 22–0 32	0.32–0 65	0 65–1 2	1 2–2 2	2.2–3 2	3 2–4 4	4 4–5 3	5 3–6.4	6 4–8 5	8 5–11 0
A	Single								36 0					58 0	58 0	
	Double													54 2	54 2	
	Sequential													56 0	58.0	
B	Single							20 5				33 4	33 4	45 0	45 0	
	Double											30 0	30 0	46.0	45 0	
	Sequential											31 7	31 7	45.0	45 0	
C	Single							14 0			23 6	23 6	31 9	31 9	39 5	
	Double										21 6	21.6	33 6	33 2	35.0	
	Sequential										22.5	22 5	32.5	33 0	39 5	
D	Single						10 8			18 0	18.0	24 6	24 6	30 6	30 6	
	Double									17 1	17 1	21.2	21.2	27 7	30 6	
	Sequential									17 8	17 8	23 0	23 0	29 0	30 6	
E	Single					7 2			12 3	16 9	16 9	21 1	21.1	24 7	28 6	
	Double								11 0	18 1	18 0	18 5	18 5	24 4	30.5	
	Sequential								11 8	17.2	17.5	19 5	19.8	24 5	28 6	
F	Single				5 6				9 4	12 8	15 8	15 8	18 7	22.5	24 8	
	Double								9 4	14 3	14 6	15 2	20 0	20 4	21 6	
	Sequential								9 4	13 6	15 5	15 5	19 2	21 2	24.8	
G	Single			4.2				7.1	9 7	12.1	12.1	14 5	16 8	19 2	23 6	
	Double							6 4	10 5	10 7	11 0	14 5	14 9	18 2	22 8	
	Sequential							6 7	10.2	11 4	11 5	14 5	16 0	18.8	23 2	
H	Single		3 1					5.2	7 0	8 9	10 7	13 6	15 1	16 6	20.4	
	Double							4 8	7 8	8 0	10 7	13.3	16.0	18 5	21.1	
	Sequential						3 5	5 0	7 6	8 5	10 7	13 8	16 0	18 0	20 8	
I	Single							4 7	5 8	8 1	9.1	11.3	13.3	14 4	17 4	
	Double						3 1	5 2	5.4	8.8	10 7	12 3	14 1	14 0	17.2	
	Sequential					2.5	3.0	5.0	5.6	8 2	10 0	11 8	13.8	13.6	17.3	
J	Single	1 5					2 6	3.6	5 4	7 1	8 6	10 1	11 5	13 1	16 5	17.9
	Double					2.3	2.4	3 9	5.4	6.7	9.3	10 5	13 0	13.1	16 7	17.7
	Sequential				1.9	2 3	2.5	4 2	5.6	7.0	8.6	10 2	12 0	14.0	16 5	17.8
K	Single					1 7	2 4	3.0	4 1	5 7	7 4	9 0	10 6	12 6	14.6	16.5
	Double				1.6	1.6	2 6	3 6	4.4	6 2	7 8	9.5	11 0	13.5	14,8	16.0
	Sequential			1 3	1 5	1 9	2 6	3 6	4.3	6.2	7 6	9.2	11 0	13.0	14.6	16 3
L	Single				1.3	1.8	1 8	2.6	3 9	5 1	6.6	8.2	9.8	11 6		
	Double			1.2	1 2	2.0	2 0	2.6	4.0	5 3	7 0	8 3	10.2	12 4		
	Sequential		0 90	1 1	1.3	1 9	1 9	2.7	4 0	5.2	6.6	8 2	10 0	12.0		
M	Single			0.86	1 2	1.2	1.5	2 1	3.2	4.5	6 0	7.6				
	Double		0 75	0 80	1 3	1.3	1.3	2 2	3 1	4 4	6.0	7 6				
	Sequential	0 55	0 70	0.90	1.2	1.3	1.4	2 1	3 2	4 3	5 7	7 2				
N	Single		0.51	0 71	0 89	0.90	1.2	1 7	2.5	4.1						
	Double	0.46	0 48	0 79	0.81	1 1	1 3	1 8	2.4	4.0						
	Sequential	0.40	0 50	0.65	0 85	1 0	1 4	1 8	2 5	4 0						
O	Single	0.26	0 36	0 42	0.54	0.70	0.87	1.3	2.1							
	Double	0 24	0.39	0.40	0 54	0.67	0.93	1 2	2.0							
	Sequential	0.25	0.37	0 40	0.55	0.70	0 90	1 2	2.0							

* The figures in Table 15.3 are graph readings and are not necessarily accurate to as many figures as given. They are stated as percentages defective.

TABLE 15.4
AVERAGE OUTGOING-QUALITY LIMIT (AOQL) OF SAMPLING PLANS*

Sample-size letter	Type of sampling	AOQL (in % defective) of sampling plan in AQL class (in % defective)														
		0 024–0 035	0 035–0 06	0.06–0.12	0.12–0.17	0.17–0.22	0 22–0 32	0.32–0 65	0 65–1.2	1.2–2 2	2 2–3.2	3.2–4.4	4 4–5 3	5 3–6 4	6.4–8.5	8.5–11.0
A	Single								6.7					16.0	16.0	
	Double													12.5	12.5	
	Sequential													14.6	15.7	
B	Single							3.5				8 2	8 2	13 6	13.6	
	Double											7.5	7.5	11.8	12 7	
	Sequential											8.0	8 6	12.8	13.4	
C	Single							2.4			5 5	5 5	9.1	9.1	13.1	
	Double										5.3	5.3	8 3	9 0	12.0	
	Sequential										5.5	6.0	8.8	9.4	12.4	
D	Single						1.8			4.1	4.1	6 8	6.8	9 7	9 7	
	Double									4 1	4.1	6 8	6.8	9.1	9 1	
	Sequential									4.2	4.7	6.4	7.1	9.4	10.0	
E	Single					1 2			2.8	4 6	4 6	6 5	6.5	8.5	10 7	
	Double								2.1	4.2	4.6	5.1	5.8	7 8	10.5	
	Sequential								2 6	4.1	4 5	5.9	6.4	8.3	10 3	
F	Single				0.91				2.1	3.4	4 9	4.9	6 4	8 0	9 6	
	Double								2.2	3 6	4 1	4 7	5.8	7 6	9.2	
	Sequential								2.2	3.6	4.5	5.0	6.3	7.6	9 7	
G	Single			0.66				1 5	2.5	3.5	3.5	4.6	5.8	6.9	9.5	
	Double							1.2	2 4	3.0	3 4	4.5	5.4	6 5	8 6	
	Sequential							1.4	2 5	3.1	3.6	4 5	5.8	6.9	9.3	
H	Single		0 49					1.1	1 8	2 6	3 4	5.1	6 0	6.9	8.7	
	Double							1 1	1.8	2 4	3 2	4.6	5.5	6.7	8 4	
	Sequential						0.74	1 1	1.9	2.7	3.4	4 9	5.9	6.9	8.9	
I	Single							1 2	1.7	2.8	3.3	4.5	5 7	6.3	8.2	
	Double						0 56	1.1	1.6	2.6	3.4	4.4	5.3	6 0	7.9	
	Sequential					0 52	0 66	1.2	1.7	2 6	3 6	4.6	5.6	6.2	8.2	
J	Single	0.25					0.56	0.91	1.7	2.5	3 4	4.4	5.3	6.3	8 2	9.3
	Double					0.42	0.54	0 92	1.5	2 3	3.3	4.2	5.4	6 3	8.1	8.8
	Sequential				0.39	0.49	0 57	1.1	1.6	2 4	3 4	4.4	5.3	6.3	8.1	9 2
K	Single					0.37	0.61	0 86	1 4	2 3	3 2	4.2	5 2	6.5	7.9	9.3
	Double				0.36	0.36	0.57	0.93	1.4	2.3	3.2	4 2	5 2	6.4	8.0	9.5
	Sequential			0.27	0.33	0.43	0.60	0.96	1 4	2.4	3.3	4 2	5 3	6.3	7 9	9.3
L	Single				0.28	0.46	0.46	0.85	1 5	2 2	3 1	4.1	5 1	6 4		
	Double			0 21	0.27	0.43	0 46	0 79	1.3	2.1	3.2	4.2	5.0	6.3		
	Sequential		0.17	0 24	0.30	0.44	0.49	0.85	1 4	2.2	3 2	4 2	5 2	6.2		
M	Single			0.19	0 30	0 30	0.43	0.70	1.3	2.1	3.1	4.1				
	Double		0 14	0.18	0.28	0.31	0.40	0 72	1 3	2.1	3 0	4.1				
	Sequential	0.10	0.14	0 20	0.31	0 33	0.42	0.73	1.2	2.1	3 1	4 0				
N	Single		0 11	0.18	0.26	0 26	0.42	0.69	1.2	2 2						
	Double	0.084	0.11	0 18	0 24	0.28	0.41	0.67	1.1	2 1						
	Sequential	0.081	0.11	0.17	0 23	0 29	0.42	0.68	1 1	2.2						
O	Single	0.056	0.091	0.13	0.17	0.25	0 34	0 58	1.1							
	Double	0.054	0 092	0 12	0 17	0.23	0.33	0.61	1 1							
	Sequential	0.058	0.094	0.13	0 19	0.24	0 34	0.60	1 1							

* The figures in Table 15.4 are mostly graph readings and are not necessarily accurate to as many figures as given. They are stated as percentages defective.

TAB

VALUES OF p_1 AND p_2 USED IN DERIVING ITEM-BY-ITEM SEQUENTIAL

Probability is $\alpha = 0.05$ of rejecting an inspection lot
Probability is $\beta = 0.10$ of accepting an inspection lot

Sample-size letter	Acceptable-quality level													
	0 024-0 035		0 035-0.06		0 06-0 12		0.12-0.17		0 17-0 22		0 22-0 32		0 32-0 65	
	p_1	p_2	p_1	p_2	p_1	p_2	p_1	p_2	p_1	p_2	p_1	p_2	p_1	p_2
A														
B														
C														
D														
E														
F														
G													0.005	0 067
H											0 003	0 035	0 005	0 05
I									0 002	0.025	0 003	0 030	0 006	0 05
J							0.0015	0.01875	0 002	0 023	0.0027	0 025	0 006	0.042
K					0 001	0 013	0 0015	0.015	0 0020	0 019	0.003	0 026	0 006	0 036
L			0.0005	0 0090	0 001	0 011	0.0015	0.013	0 0023	0 0188	0.003	0 0186	0.0065	0 027
M	0.0003	0.0055	0.0005	0 0070	0.001	0.0090	0 00175	0 0125	0 0020	0 013	0 003	0 014	0.006	0.021
N	0 0003	0 0040	0.0005	0 0050	0 001	0 0065	0 0015	0.0085	0.002	0 010	0 003	0.014	0.006	0 018
O	0 0003	0 0025	0.00055	0 0037	0.001	0.0040	0.0015	0.0055	0 002	0.0070	0 003	0 009	0 006	0 012

as a function of the fraction defective and differentiated. The derivative was set equal to zero and the resulting equation solved. For the remaining plans, the AOQL was read from a graph of the average outgoing quality as a function of fraction defective. Throughout, the inspection lot was assumed indefinitely large; that is, the average outgoing quality was taken equal to pL_p, where L_p is the probability of accepting an inspection lot with fraction defective p.

Table 15.5.—This table gives the intended AQL and LTPD of each multiple-sampling plan. These values were used in deriving the item-by-item sequential plans on which the grouped and truncated sequential plans are based.

Table 15.6.—This table gives the parameters of item-by-item sequential plans for which p_1, p_2, α, and β are as shown in Table 15.5, where α

LE 15.5

PLANS ON WHICH MULTIPLE-SAMPLING PLANS IN PART V ARE BASED*

with proportion defective p_1
with proportion defective p_2

(AQL) class (in % defective)

0.65-1 2		1.2-2.2		2.2-3 2		3.2-4.4		4.4-5.3		5.3-6.4		6.4-8.5		8.5-11.0	
p_1	p_2	p_1	p_2	p_1	p_2	p_1	p_2	p_1	p_2	p_1	p_2	p_1	p_2	p_1	p_2
										0.065	0.560	0.075	0.58		
						0.04	0.317	0.05	0.317	0 075	0.450	0 085	0.45		
				0.028	0 225	0.035	0.225	0.05	0.325	0.058	0.33	0.085	0.395		
		0 02	0.178	0.027	0.178	0.04	0.23	0.05	0.23	0.07	0.29	0.075	0.306		
0.0118	0 1175	0 020	0 172	0.0255	0 175	0.042	0.195	0.0475	0.1975	0.065	0.245	0.085	0.286		
0.011	0.094	0.022	0.136	0 031	0.155	0.0375	0.155	0.048	0.192	0.063	0.2125	0.085	0.248		
0.014	0 102	0.020	0.114	0.026	0.115	0.033	0.145	0.049	0.16	0 0575	0.1875	0.082	0.232		
0.011	0.076	0.02	0.085	0.026	0.107	0.04	0.138	0.05	0.16	0.06	0.18	0.082	0.2075		
0.012	0.056	0 02	0.082	0.03	0.100	0.04	0.118	0.05	0.138	0.06	0.136	0.08	0.173		
0.011	0 056	0.02	0 070	0.03	0.086	0.041	0.102	0.051	0.12	0.061	0.14	0,080	0.165	0.092	0.178
0.0115	0.043	0.021	0.062	0 031	0.076	0.041	0.092	0 052	0.11	0.062	0.13	0,080	0.146	0.096	0.163
0 0110	0 040	0.021	0.052	0 032	0.066	0 042	0.082	0.052	0.10	0.062	0.12				
0.0110	0.032	0.021	0.043	0.032	0.057	0 042	0.072								
0.0110	0.025	0.022	0.040												
0.0110	0.020														

* The figures in Table 15.5 are stated as *proportions*, whereas the AQL classes are stated as *percentages*, defective.

is the probability of rejecting an inspection lot containing a fraction defective p_1 and β is the probability of accepting on inspection lot containing a fraction defective p_2.

Table 15.7.—A by-product of the computations necessary to determine the acceptance and rejection numbers for the multiple-sampling plans is the probability of reaching a decision on each sample if the proportion defective in submitted product is p_1 or p_2. From these probabilities it is simple to compute the average amount of inspection if each sample is inspected completely (that is, inspection is not curtailed). The results of these computations are given in this table. The average amount of inspection when inspection is curtailed is, of course, less than the values in this table, though never by more than the group size.

TABLE 15.6

PARAMETERS OF ITEM-BY-ITEM SEQUENTIAL PLANS ON WHICH MULTIPLE-SAMPLING PLANS IN PART V ARE BASED

h_1 = negative of intercept of acceptance line of item-by-item sequential plan

h_2 = intercept of rejection line of item-by-item sequential plan

s = common slope of acceptance and rejection lines

Sample-size letter	Parameter	Acceptable-quality level (AQL) class (in % defective)														
		0.024–0.035	0.035–0.06	0.06–0.12	0.12–0.17	0.17–0.22	0.22–0.32	0.32–0.65	0.65–1.2	1 2–2 2	2.2–3.2	3.2–4.4	4.4–5.3	5 3–6 4	6.4–8.5	8 5–11.0
A	h_1													0.7744	0 7941	
	h_2													0 9942	1.0195	
	s													0.2593	0.2785	
B	h_1											0.9340	1 0342	0.9739	1.0348	
	h_2											1.1991	1.3278	1.2504	1.3285	
	s											0.1412	0 1516	0.2249	0 2340	
C	h_1										0 9744	1.0823	1.0170	1.0827	1 1545	
	h_2										1 2510	1.3896	1.3058	1 3900	1 4823	
	s										0.0980	0.1054	0.1544	0 1639	0.2122	
D	h_1									0.9532	1.0957	1.1429	1.2967	1.3311	1 3294	
	h_2									1.2238	1.4068	1 4674	1.6648	1 7090	1 7068	
	s									0.0744	0.0821	0.1120	0.1210	0.1596	0.1697	
E	h_1								0 9336	0.9703	1.0758	1.3171	1.4103	1 4612	1 5405	
	h_2								1.1986	1.2457	1.3812	1.6909	1 8106	1 8760	1.9778	
	s								0 0469	0.0726	0.0796	0.1018	0 1073	0.1388	0.1697	
F	h_1								1.0082	1.1571	1.2891	1.4531	1.4522	1:6201	1.7769	
	h_2								1 2944	1.4856	1 6551	1.8656	1.8644	2 0800	2 2813	
	s								0.0393	0.0637	0.0784	0.0840	0.1058	0 1251	0.1548	

G	h_1							0.8465	1.0827	1.2227	1.4225	1.4041	1 7219	1.6922	1.8477	
	h_2							1.0868	1.3900	1 5697	1.8263	1.8027	2.2107	2.1725	2.3722	
	s							0.0242	0.0450	0.0548	0.0605	0 0768	0 0949	0 1116	0 1464	
H	h_1						0.9044	0.9585	1 1252	1.4855	1 4993	1 6725	1.7503	1 8226	2.0934	
	h_2						1.1611	1 2305	1.4446	1.9071	1.9249	2 1473	2.2472	2 3400	2 6877	
	s						0.0131	0.0197	0 0340	0.0453	0.0578	0.0800	0 0957	0.1106	0.1367	
I	h_1					0.8832	0.9662	1 0396	1.4195	1 5249	1.7604	1 9299	2 0237	2 4942	2 5646	
	h_2					1.1339	1.2405	1.3347	1 8224	1.9578	2.2601	2 4777	2.5982	3.2022	3.2926	
	s					0.0091	0.0118	0 0209	0 0287	0.0443	0.0586	0.0726	0.0874	0 0934	0.1214	
J	h_1				0.8852	0.9138	1.0014	1.1354	1.3448	1.7250	2.0234	2.3040	2 4177	2.4506	2 7426	2 9642
	h_2				1 1365	1 1732	1.2856	1.4577	1 7266	2 2146	2 5978	2 9580	3 1041	3.1463	3.5211	3 8056
	s				0.0069	0.0086	0.0101	0.0186	0 0278	0 0401	0.0534	0.0673	0 0811	0 0957	0 1181	0.1310
K	h_1			0.8736	0.9720	0.9924	1.0314	1.2353	1 6661	2 0004	2.3841	2 6091	2 7713	2 7601	3.3302	3 7125
	h_2			1.1216	1 2479	1.2741	1.3241	1 5860	2.1391	2 5683	3.0609	3.3497	3 5580	3.5436	4.2756	4.7664
	s			0.0047	0.0059	0.0076	0.0107	0.0168	0.0240	0 0380	0.0504	0.0633	0.0777	0.0923	0.1101	0.1270
L	h_1		0 7766	0.9349	1 0370	1.0631	1 2233	1.5581	1.7046	2.3978	2.9635	3.1633	3.1893	3.1087		
	h_2		0 9971	1.2003	1.3313	1.3649	1.5706	2 0004	2 1884	3 0785	3.8048	4.0612	4.0947	3.9912		
	s		0.0029	0.0042	0 0053	0.0079	0 0086	0.0144	0 0255	0.0343	0.0471	0.0599	0.0736	0.0881		
M	h_1	0 7726	0.8510	1 0209	1.1385	1 1957	1.4510	1.7755	2 0667	3.0447	3 7305	3 9440				
	h_2	0.9919	1 0925	1 3107	1.4617	1 5351	1 8629	2.2795	2 6534	3 9090	4.7895	5.0636				
	s	0 0018	0.0025	0 0036	0 0055	0.0059	0 0072	0.0120	0.0197	0.0307	0.0434	0.0557				
N	h_1	0 8679	0 9758	1.1992	1.2926	1.3918	1 4510	2 0268	2 6954	3.6522						
	h_2	1 1143	1.2528	1.5396	1.6596	1 7870	1.8629	2.6022	3 4605	4.6890						
	s	0 0014	0 0020	0.0029	0 0040	0 0050	0.0072	0.0109	0.0171	0 0301						
O	h_1	1 0607	1 1791	1.6204	1 7274	1 7899	2.0380	3.2198	3.7090							
	h_2	1.3618	1 5138	2.0804	2.2177	2 2980	2.6166	4.1338	4 7619							
	s	0 0010	0 0017	0.0022	0.0031	0.0040	0.0055	0.0087	0 0151							

TABLE 15.7

AVERAGE AMOUNT OF INSPECTION AT p_1 AND p_2 FOR MULTIPLE-SAMPLING PLANS IF EACH SAMPLE IS COMPLETELY INSPECTED

$\bar{N}_{p_1}$ = average number of items inspected per inspection lot if fraction defective is p_1

$\bar{N}_{p_2}$ = average number of items inspected per inspection lot if fraction defective is p_2

Acceptable-quality level (AQL) class (in % defective)

Sample-size letter	0.024–0.035		0.035–0 06		0.06–0 12		0 12–0.17		0.17–0 22		0.22–0 32		0.32–0 65		0.65–1 2	
	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$
A																
B																
C																
D																
E															26	19
F															35	25
G													47	34	34	29
H											90	62	65	47	49	40
I									131	89	102	76	68	53	85	73
J							174	119	135	100	139	98	97	74	91	77
K					230	164	190	143	175	127	125	100	123	98	131	116
L			306	186	262	200	263	191	188	148	214	169	202	170	143	147
M	518	306	413	265	352	264	281	235	286	235	324	279	290	264	223	219
N	631	416	573	431	575	434	487	391	429	375	348	307	392	361	431	442
O	1215	930	1032	776	1324	1060	997	932	872	793	783	722	1131	1121	891	904

Sample-size letter	1.2–2.2		2.2–3 2		3.2–4.4		4 4–5.3		5 3–6 4		6 4–8.5		8.5–11.0	
	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$	$\bar{N}_{p_1}$	$\bar{N}_{p_2}$
A									4	4	4	4		
B					9	8	10	9	6	6	7	6		
C			12	11	13	11	11	8	11	9	9	9		
D	15	13	17	14	15	12	16	13	15	13	14	13		
E	18	15	19	17	22	18	23	20	20	18	17	17		
F	25	22	26	21	29	27	24	22	26	23	24	24		
G	35	28	40	34	33	29	38	33	30	30	29	27		
H	59	52	49	42	42	41	39	39	38	38	39	41		
I	70	57	64	61	60	59	54	56	73	76	64	69		
J	84	82	88	85	92	90	81	83	73	79	77	79	77	90
K	119	113	127	128	123	123	116	118	96	104	107	113	121	136
L	179	175	191	199	181	194	160	172	128	139				
M	300	306	316	335	285	311								
N	446	452												
O														

CHAPTER 16

THE STANDARD PROCEDURE FOR SELECTING A SAMPLING PLAN

1. INTRODUCTION

There are two somewhat separate problems involved in setting up a procedure for selecting a sampling plan from Table 2 (or Table 4); first, the selection of a plan to be used when inspection is first begun—what may be called the "normal sampling plan"; second, provision for changing the plan in the light of the quality of product submitted. The plan that is appropriate for any inspection problem depends on the quality of product submitted for inspection. This is frequently not known in advance; even if past quality is known, the introduction of sampling inspection is likely to have a significant effect on the quality submitted; and, even though this effect could be estimated, unexpected changes in quality can occur and provision should be made for them.

2. SELECTION OF THE NORMAL SAMPLING PLAN

The selection of a plan from Table 2 involves the choice of (*a*) AQL, (*b*) type of sampling, (*c*) sample-size letter. The procedure for determining the AQL and type of sampling is relatively straightforward and requires no special machinery: the person or activity responsible for quality requirements on the product selects an AQL for the product that seems to provide the best compromise between quality desired and quality attainable with present production methods; the person or activity responsible for administering inspection chooses the type of sampling that seems best suited to the particular product and particular supplier. The proper procedure for determining the sample-size letter is by no means so obvious or straightforward.

2.1. Selection of the Sample-size Letter

2.1.1. General Considerations.—The selection of the sample-size letter determines directly the amount of inspection *per inspection lot* and hence the steepness of the OC curve. Together with the size of the inspection lot, it determines the amount of inspection *per item of output* and hence an important part of the costs of inspection. The selection of a sample-size letter therefore means deciding how much discrimination

(that is, steepness of OC curve) is worth buying, given its cost, and introduces a new variable—the size of the inspection lot.

The advantages of sharp discrimination are twofold: first, given the distribution by quality of product submitted, the sharper the discrimination, the better is likely to be the quality of product accepted, since unacceptable inspection lots are more likely to be rejected when offered; second, the sharper the discrimination, the greater the pressure on the supplier to submit high-quality product, since any given deterioration in quality of product submitted will lead to a larger increase in rejections. For any given size of inspection lot, the amount it is worth paying—or is necessary to pay—for these advantages will clearly vary from product to product. If the product is cheap to inspect and the inclusion of an undue percentage of defective items in accepted product is serious, it is worth doing more inspection than if the product is expensive to inspect and the inclusion of additional defective items in accepted product is of no great moment. If the cost of rejections to suppliers is low and the cost of improving quality is high, it is necessary to do more inspection in order to give the same incentive to improve quality than if the cost of rejections is high and the cost of improving quality is low.

These factors all depend on the nature of the product and the use to be made of it; decisions based on them should therefore be made by the person or activity responsible for quality requirements on the product. These factors alone, however, do not suffice to determine the sample-size letter, since so far no account has been taken of the size of the inspection lot.

The discrimination attained depends primarily on the number of items inspected per inspection lot; the cost of inspection, on the other hand, depends on the number of items inspected per item submitted, that is, on the percentage of items inspected.* If the same number of items were inspected for all size inspection lots, the discrimination would also be about the same for all size inspection lots; the cost of inspection per item submitted would, however, vary widely, being very high for small inspection lots and very low for large inspection lots. Suppose a particular level of discrimination and cost seems correct for some size inspection lot; that is, for an inspection lot containing, say, N items the extra discrimination attained by adding an additional item to the sample is just worth its cost. For a smaller inspection lot, an additional sample item would add the same amount to discrimination; it would, however, add more to cost (per item of output) than for an inspection lot of size N; it would therefore not be worth while, and it would be desirable to use a smaller sample than for an inspection lot of size N.

* This statement and the whole of this section assume that costs of reinspection of rejected inspection lots are neglected.

For inspection lots containing more than N items, an additional sample item would add the same amount to discrimination; it would, however, add less to cost than for an inspection lot of size N; it would therefore be worth while to use a larger sample than for an inspection lot of size N. It follows that *the larger the inspection lot, the larger, other things the same, should be the amount of inspection per inspection lot.*

It is not possible, of course, to say in general how much the amount of inspection should increase with the size of the inspection lot. It seems plausible, however, that it should not increase as rapidly as the size of the inspection lot; that is, the percentage of items inspected should decrease as the size of the inspection lot increases. This limit is suggested by the fact that if the percentage of items inspected were kept constant, the cost per item submitted would be the same for all size inspection lots, but the discrimination would vary widely, being very poor for small inspection lots and very sharp for large inspection lots. It seems reasonable that the appropriate solution is a compromise between constant cost and constant discrimination.

2.1.2. Mathematical Reformulation of General Considerations.—The considerations of the preceding section can be expressed more formally.

Let n be sample size (we assume single sampling for simplicity),
- N be size of inspection lot,
- $V(n,N) = V(n)$ be the total value (per item of product submitted) of inspecting n items out of N; that is, the value of the resulting OC curve. If N is large relative to n, the OC curve is independent of N and hence so is V.
- $C(n,N)$ be the cost (per item of product submitted) of inspecting n items out of N.

The costs of inspection per item of product submitted can be regarded as consisting of two parts: (*a*) costs that are independent of the percentage of submitted items inspected, for example, costs of handling the submitted product, storing it, providing inspection facilities and gauges, and, to some extent, of administering inspection; (*b*) costs that depend directly on the percentage of submitted items inspected, for example, costs of inspecting each item, of selecting samples, and of recording results. Assume that, to a first approximation, the second group of costs is independent of any of the factors affecting the first and that it depends linearly on the percentage of submitted items inspected. Then we can write

$$C(n,N) = f(N,L, \cdots) + \gamma \frac{n}{N} \tag{1}$$

where $f(N,L, \ldots)$ describes the first group of costs and may depend on such variables as the number of items in each inspection lot (N), the

number of inspection lots submitted per day (L), and so forth, but not on n, and γ is the cost of selecting and inspecting an item, recording results on the item, and so forth.

The value function cannot be written down explicitly, but it is reasonable to suppose that (a) $\frac{dV}{dn} > 0$ and (b) $\frac{d^2V}{dn^2} < 0$. Condition (a) states that the value of inspection increases with the size of sample, that is, that the steeper the OC curve the more valuable it is. Condition (b) states that the larger the sample, the less valuable is the increase in steepness resulting from adding one item to the sample. This must be true for sufficiently large samples, since the maximum value of inspection is finite, and it seems plausible that it is true for all sample sizes.

The net value of inspection is

$$V(n) - C(n,N) \tag{2}$$

and for this to be a maximum with respect to n, it is necessary that

$$\frac{dV}{dn} - \frac{\partial C}{\partial n} = 0 \tag{3}$$

or, from (1), that

$$\frac{dV}{dn} - \frac{\gamma}{N} = 0 \tag{4}$$

Equation (4) defines the optimum value of n as an implicit function of N, say $\phi(n,N) = 0$. Then

$$\frac{dn}{dN} = -\frac{\frac{\partial \phi}{\partial N}}{\frac{\partial \phi}{\partial n}} = -\frac{\gamma}{N^2 \frac{d^2V}{dn^2}} \tag{5}$$

Since γ is positive, the requirement that $\frac{d^2V}{dn^2}$ be negative suffices to assure that $\frac{dn}{dN} > 0$; that is, that the optimum sample size increase with the size of the inspection lot.

The condition that the value function must satisfy in order for the optimum percentage of inspection to decrease as the size of inspection lot increases can be derived as follows.

It is desired to determine the condition on $V(n)$ in order that

$$\frac{d\left(\frac{n}{N}\right)}{dN} < 0 \tag{6}$$

where n is understood to be the optimum defined implicitly by (4). Now

$$\frac{d\left(\frac{n}{N}\right)}{dN} = \frac{N\frac{dn}{dN} - n}{N^2} \tag{7}$$

So, for (6) to be satisfied,

$$\frac{dn}{dN} < \frac{n}{N} \tag{8}$$

Substituting from (5), (8) becomes

$$\frac{-\gamma}{N^2\frac{d^2V}{dn^2}} < \frac{n}{N} \tag{9}$$

or

$$-\frac{d^2V}{dn^2} > \frac{\gamma}{nN} \tag{10}$$

But from (4), $\frac{\gamma}{N} = \frac{dV}{dn}$; hence (10) becomes

$$-\frac{d^2V}{dn^2} > \frac{1}{n}\frac{dV}{dn} \tag{11}$$

and this is the condition that the value function must satisfy in order for the optimum percentage of inspection to decrease as the size of inspection lot increases.

We have been unable to find any general line of reasoning to show that all reasonable value functions would satisfy (11). However, a few special cases have been investigated for which the value function could be written explicitly, and (11) is satisfied for these.

2.1.3. Procedures Adopted.—The discussion of the two preceding sections indicates that the appropriate sample-size letter depends on (*a*) the product and (*b*) the size of the inspection lot. For each product there exists a function relating the appropriate amount of inspection to the size of the inspection lot. This function might differ from product to product both in height (that is, the amount of inspection for a given size inspection lot) and in shape or slope (that is, the rate at which the amount of inspection increases with the size of the inspection lot). To take account of all such differences, the activity responsible for quality requirements on the product would have to specify the function to be used for each product separately. However, such a procedure would be unwieldly and complex; no method of determining the appropriate function has ever been worked out in detail, and, even if it had, the information about the product required to apply it would seldom be available. A much rougher and simpler procedure is therefore desirable on administrative grounds and is about all that can be justified by the present state of knowledge.

The procedure adopted in this book is to assume that only the height of the function relating sample size to inspection-lot size need vary from product to product and that the shape of the function can be settled once and for all for all products. Table 1 provides five functions relating sample size to inspection-lot size. These functions are essentially multiples of one another: the amount of inspection for one function is approximately the same multiple of the amount of inspection for another, for all inspection-lot sizes. We have adopted the term *inspection level* to designate the height of the function and used the Roman numbers I, II, III, IV, and V to designate the five inspection levels corresponding to the five functions.

With this simplification, the procedure is for the person or activity responsible for quality requirements to choose an inspection level appropriate to the product. This choice fixes the functional relation between amount of inspection and inspection-lot size. The sample-size letter is then uniquely determined by the size of the inspection lot. The functional relation between amount of inspection and inspection-lot size satisfies the requirements listed above: the amount of inspection increases with inspection-lot size but less rapidly than the inspection-lot size.

This procedure is a compromise in two respects: (*a*) It assumes that the amount of inspection should bear the same relation to inspection-lot size for all products; (*b*) it assumes that the same steepness of OC curve is appropriate for a given inspection-lot size regardless of type of sampling, which is not strictly correct, since costs will vary with type of sampling and hence different OC curves may be appropriate for different types of sampling.

2.2. Preparation of Table 1

2.2.1. Relation between Sample Size and Inspection-lot Size.—The relation between sample size and inspection-lot size incorporated in Table 1 is fairly arbitrary. About the best that can be said for it is that it is not unreasonable and that it provides a consistent and systematic basis for selecting plans.

The particular relation is essentially that used in the Army Ordnance tables. These tables specify the sample size to be used for each inspection-lot size for single-sample sizes of 75 and over and for inspection-lot sizes of 500 and over. The inspection-lot size intervals in Table 1 for inspection lots of 500 and over are identical with the Army Ordnance intervals; the sample-size letters for these intervals for inspection level III correspond to the single- and double-sample sizes specified for these intervals by Army Ordnance. This relation was extrapolated back to smaller inspection lots rather roughly to complete Table 1 for inspection level III.

2.2.2. Adjustment to Different Inspection Levels.—The sample-size letters for inspection levels other than III were determined so that the average amount of inspection would be multiplied by a factor as nearly constant as possible for all inspection-lot sizes. The particular multiplying factors chosen are relatively arbitrary; a 4 to 1 ratio between the highest and lowest inspection levels seemed to cover a wide enough span to meet most needs, and five inspection levels seemed sufficient in view of the rough basis on which inspection level would necessarily have to be chosen. Consequently, the sample-size letters for inspection level I imply about half as much inspection for each inspection-lot size as do those for inspection level III; for II, about three-fourths as much; for IV, one and one-half times as much; and for V, twice as much. Because of the limited number of sample sizes available, the desired constant relationship could not be achieved throughout, as can be seen from an inspection of Table 1.

3. ADJUSTMENT OF SAMPLING PLAN

3.1. Reduced and Tightened Inspection

The procedure for selecting the normal sampling plan takes little account of the quality of product actually submitted. The only point at which the quality of product enters is in the selection of the AQL, and here what enters is the level of quality that is thought attainable by all suppliers of the product. Yet the quality of product actually submitted by each supplier is clearly relevant.

If the product submitted is of generally high quality, inspection performs the functions of giving information on quality, protecting against the acceptance of the few poor inspection lots that may be submitted, and providing some assurance that a sudden and unexpected deterioration in quality will be detected and the associated low-quality inspection lots rejected. The functions of sorting good inspection lots from bad and of giving an incentive to the supplier to improve quality are of minor importance. Bad inspection lots are seldom offered; and the supplier is already producing high-quality product. A relatively small amount of inspection will suffice to perform the functions that remain.

If product of generally low quality is submitted under the operation of a normal sampling plan, most inspection lots will be rejected, and many of those that are accepted will be of low quality. It is clear that the sampling plan does not give sufficient incentive to the supplier to improve the quality of product submitted. If he is one of many suppliers and if the quality requirements (AQL's) have been properly set for all suppliers of the product, this supplier is out of line. Either his product must be entirely rejected and purchases from him discontinued, or stricter stand-

ards must be imposed, both to assure that only the few good inspection lots are accepted and to induce the supplier to improve quality.

The procedure adopted in this book is to use two variations from the normal sampling plan—one when quality is high, the other when it is low. The use of two variations, instead of a larger number, is desirable on practical grounds: frequent modification of the sampling plan is undesirable, since it complicates administration; the quality of product submitted can be estimated only rather poorly for short intervals, so the information required for frequent change is not available. Reduced inspection is provided when quality submitted is very good and tightened inspection when quality submitted is very poor.

3.2. Selection of Sampling Plans for Reduced and Tightened Inspection

3.2.1. Introduction.—For reduced inspection, it seems reasonable to retain the same AQL but to use a lower inspection level than for normal inspection. A sampling plan for reduced inspection is therefore selected in the same manner as a sampling plan for normal inspection except that an inspection level two levels lower than that specified for normal inspection is used. A sampling plan so selected will require approximately half as much inspection on the average as the normal sampling plan.

For tightened inspection, two possibilities were considered: (*a*) raising the inspection level, (*b*) using a plan with a lower (that is, stiffer) AQL. Either would accomplish the desired result of increasing the percentage of poor inspection lots rejected. The second was adopted primarily on the grounds that the submission of low-quality product represented a failure by the supplier—hence the supplier should be penalized by an increase in rejections rather than the receiver's being penalized by being required to do more inspection.

3.2.2. Criteria for Reduced, Normal, or Tightened Inspection

3.2.2.1. GENERAL PRINCIPLES.—Evidence for determining when the quality of product submitted is sufficiently high to justify reduced inspection or sufficiently low to demand tightened inspection is provided by the sampling-inspection records from prior inspection. This evidence is in the form of the number of defective items found in samples and the number of inspection lots accepted or rejected.

The general principle followed in specifying criteria for determining from this evidence the type of inspection to use is that, in general, normal inspection is to be used when the average percentage of defective items in submitted product is within the AQL class specified for the product; reduced inspection is to be used when the average percentage of defective items is clearly below the smaller of the two values defining the AQL class; and tightened inspection is to be used when the average

percentage of defective items is clearly above the larger of the two values defining the AQL class.

3.2.2.2. REDUCED INSPECTION.—For reduced inspection, three requirements are imposed: first, that no inspection lot among the preceding 20 has been rejected; second, that no serious interruptions in production have occurred; third, that the average percentage of defective items in the samples inspected (the process average) is sufficiently low to give reasonable assurance that the true percentage is definitely below the desired level. The reason for the first two requirements is to ensure that the quality of product is uniform. The rejection of some submitted inspection lots is likely to mean that there is considerable variation in quality and hence that the function of sorting good from bad inspection lots is still important. An interruption of production may be followed by a change in quality and hence make results of past inspection an uncertain guide for the future. The value below which the process average must fall to meet the third requirement is given in Table 11.5, where it is entitled "lower limit of process average." This value depends on the AQL class for the product, because the AQL class defines the quality expected, and on the number of sample items from which the process average is computed, because the process average is less subject to error and therefore a better guide to the true percentage of defective items the larger the number of sample items on which it is based.

3.2.2.3. TIGHTENED INSPECTION.—There is sufficient justification for tightened inspection if the average percentage of defective items is high. A high percentage of defective items is likely to mean that many inspection lots containing too many defectives are being submitted. Even though some containing few defective items are also being submitted, it is desirable to put more pressure on the supplier to improve quality. Accordingly, the only requirement imposed for tightened inspection is that the process average be above the value in Table 11.5 entitled "upper limit of process average." This value, like the lower limit and for the same reasons, depends on the AQL class for the product and the number of sample items from which the process average is computed.

3.2.2.4. RESUMPTION OF NORMAL INSPECTION.—If reduced inspection is in use, it seems clear that normal inspection should be resumed if an inspection lot is rejected, production is interrupted, or the process average becomes too high. If tightened inspection is in use, normal inspection should be resumed if the process average becomes sufficiently low. The only problem requiring further attention is the interpretation to be placed on "too high" and "sufficiently low."

It is clearly desirable, on administrative grounds, to avoid frequent shifts in type of inspection. It is therefore desirable to allow some leeway

before normal inspection is resumed. For example, if any process average above the lower limit of the process average given in Table 11.5 were considered "too high" for a continuation of reduced inspection, a supplier producing product containing on the average just this percentage of defectives would be likely to be shifted back and forth frequently, simply because of sampling fluctuations in the process average.

In order to give the desired leeway, it is provided that "too high" be interpreted as above, and "sufficiently low" as below, the larger of the two values defining the AQL class. This is a considerably more liberal interpretation for reduced than for tightened inspection: much greater variation is permitted in the process average when reduced inspection is being used than when tightened inspection is being used. The reason for this difference is that the process average is the only criterion for determining when to shift from tightened to normal inspection, whereas there are other criteria for shifting from reduced to normal inspection. These other criteria—in particular, the requirement that normal inspection be resumed if any inspection lot is rejected—add important safeguards and make it possible to be more liberal in the process-average requirement.

4. PREPARATION OF TABLE 11.5

4.1. Contents of Table 11.5

4.1.1. Summary.—Table 11.5 gives (*a*) the "lower limit on process average" below which the process average must lie for reduced inspection to be permitted; (*b*) the "upper limit on process average" above which the process average must lie for tightened inspection to be required; (*c*) implicitly, as a function of the AQL class, the minimum number of items on which the process average must be based.

4.1.2. Limits on Process Average.—The limits on the process average depend on the AQL class because the AQL class defines the quality of product that is acceptable. The limits depend on the number of sample items from which the process average is computed to allow for sampling fluctuations. Process averages based on few items vary much more widely from the true percentage of defective items than do process averages based on many items. A fairly low process average based on a few items may therefore give little confidence that the true percentage of defective items is below the AQL class, whereas a process average of the same numerical value may give sufficient confidence if it is based on many items.

It is not feasible, of course, to give limits for every possible number of sample items. For simplicity, 13 classes by number of items are used. The limits in Table 11.5 are computed for the mid-points of these classes.

Each lower limit in Table 11.5 is so determined that a process average equal to the lower limit and based on a number of observations equal to

the mid-point of the corresponding class gives 97.5 percent confidence that the true percentage defective is below the smaller of the two values defining the AQL class. Stated differently, if the true percentage defective were equal to the smaller of the two values defining the AQL class, the observed process average would be equal to or less than the lower limit only 1 time in 40. There is therefore at most 1 chance in 40 that this criterion for reduced inspection will be satisfied when reduced inspection should not be used. Similarly, each upper limit is so determined that a process average equal to the upper limit and based on the appropriate number of items gives 97.5 percent confidence that the true percentage of defective items is above the larger of the two values defining the AQL class.

If the AQL value is fairly high and the number of sample items from which the process average is computed is large enough for the distribution of observed process averages to be reasonably well approximated by the normal distribution, the lower limit is equal to the smaller of the two values defining the AQL class minus 1.96 times the standard deviation of the process average; the upper limit is equal to the larger of the two values defining the AQL class plus 1.96 times the standard deviation of the process average.* This is the method used to compute most of the entries in Table 11.5. For the smaller AQL values and smaller number of observations, however, the normal approximation is not satisfactory. For these, the lower and upper limits were computed more accurately by use of the Poisson approximation to the distribution.†

4.1.3. Minimum Number of Items for Computation of Process Average.—Limits like those in Table 11.5 could be derived for process

* Suppose the AQL class is $100p_1$ to $100p_2$, n is the number of sample items from which the process average is computed, L is the lower limit and U the upper limit. Then, if the normal approximation is satisfactory,

$$\frac{L}{100} = p_1 - 1.96\sqrt{\frac{p_1(1-p_1)}{n}}$$
$$\frac{U}{100} = p_2 + 1.96\sqrt{\frac{p_2(1-p_2)}{n}}$$

† The exact method used to compute these is as follows (the symbols have the same meaning as in the preceding footnote): If d_1 is such that the probability is 0.025 of getting d_1 or fewer defects in a random sample of n items drawn from an inspection lot in which the fraction defective is p_1, then $\frac{L}{100}$ is $\frac{d_1}{n}$. Similarly, if d_2 is such that the probability is 0.025 of getting d_2 or more defects in a random sample of n items drawn from an inspection lot in which the fraction defective is p_2, then $\frac{U}{100}$ is $\frac{d_2}{n}$. Molina's *Poisson's Exponential Binomial Limit*, Table II, gives cumulated terms of the Poisson series. Let $a = np$. Then Molina's Table II gives values of $P(c,a) = \sum_{x=c}^{\infty} \frac{a^x e^{-a}}{x!}$ for

averages computed from any number of sample items and would give the same assurance that reduced or tightened inspection would not be used when they should not be. If the number of sample items were very small, however, the limits would be so far apart that there would be insufficient assurance that reduced or tightened inspection would be used when they should be. It is therefore desirable to set a lower limit on the number of sample items from which the process average should be computed.

The somewhat arbitrary criterion used in deriving the minimum number of sample items is that if the true percentage defective is more than two AQL classes above the AQL class designated for the product, there should be less than 1 chance in 40 that a computed process average would be less than the upper limit of the process average. The minimum number of items satisfying this criterion was computed for each AQL class.* The resulting values were then smoothed to obtain the values given explicitly in Part III and implicitly in Table 11.5.

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values of a from 0.001 to 100. If np_1 was a value of a for which $P(c,a)$ is given, then c_1 was determined by linear interpolation such that $P(c_1,np_1) = 0.975$. To allow for the discreteness of the series, $L/100$ was taken as $(c_1 - 1)/n$. If np_2 was a value of a for which $P(c,a)$ is given, then c_2 was determined by linear interpolation such that $P(c_2,np_2) = 0.025$. $U/100$ was taken as c_2/n. When np_1 (or np_2) was a value of a for which $P(c,a)$ is not given, use was made of the fact that when n is even the probability of exceeding any given value of χ^2 is reducible to the partial sum of a Poisson series. That is,

$$\frac{1}{\frac{\nu-2}{2}!}\int_{2a}^{\infty}\left(\frac{1}{2}\chi^2\right)^{\frac{1}{2}(\nu-2)}e^{-\frac{1}{2}\chi^2}d\left(\frac{1}{2}\chi^2\right) = \sum_{x=0}^{\frac{\nu-2}{2}}\frac{a^x e^{-a}}{x!}$$

where ν = degrees of freedom of χ^2.

Let $\chi_\nu^2(P)$ be the value of χ^2 such that the probability of exceeding this value of χ^2 for ν degrees of freedom is P. Thompson's "Table of Percentage Points of the χ^2 Distribution" gives values of $\chi_\nu^2(P)$ for various values of ν and P. To find L, ν_1 was determined by linear interpolation such that $\chi_{\nu_1}^2(0.025) = 2np_1$. $L/100$ was taken as $(\nu_1 - 2)/2n$. To find U, ν_2 was determined by linear interpolation such that $\chi_{\nu_2}^2(0.975) = 2np_2$. $U/100$ was taken as $\nu_2/2n$.

* Let the AQL class for the product be $100p_1$ to $100p_2$; the next AQL class, $100p_2$ to $100p_3$; and the next one, $100p_3$ to $100p_4$; U, the upper limit for the AQL class $100p_1$ to $100p_2$; and n, the number of sample items from which the process average is computed. Assume that the normal approximation is valid. Then

$$\frac{U}{100} = p_2 + 1.96\sqrt{\frac{p_2(1-p_2)}{n}}$$

and the condition stated in the text is that

$$p_4 - 1.96\sqrt{\frac{p_4(1-p_4)}{n}} \geq \frac{U}{100}$$

Using the equality sign and substituting for U gives an equation from which the minimum number of sample items can be computed.

CHAPTER 17

METHODS OF COMPUTATION

1. OPERATING-CHARACTERISTIC (OC) CURVES

1.1. Alternative Definitions of Operating-characteristic (OC) Curves

The pages of Table 4 give the OC curves of all sampling plans included in this volume. Each OC curve shows the percentage of submitted inspection lots that will be accepted on the basis of the sampling plan of the indicated type on that page for each percentage of defective items in submitted product. There are two alternative ways of interpreting "percentage of defective items in submitted product" and these lead to somewhat different OC curves for small inspection lots. (*a*) The percentage of defective items can be considered as applying to each inspection lot separately. The ordinate of the OC curve then shows the percentage of inspection lots accepted if a series of identical inspection lots, each containing the same number of items and of defective items, is submitted. (*b*) The percentage of defective items can be considered as applying to the process yielding a particular inspection lot, and each inspection lot can be considered as itself a sample from a hypothetical infinite population of items that might be produced if the process remained unchanged. The ordinate of the OC curve then shows the percentage of inspection lots accepted if a series of inspection lots produced by the identical process is submitted. The actual percentage of defective items in the inspection lots would vary from inspection lot to inspection lot because of chance fluctuations.

If interpretation (*a*) is adopted, only a finite number of points on the OC curve exists and both the number of points and the corresponding ordinates depend on the size of the inspection lot. For example, if the inspection lot contains 200 items, there exist only 201 points on the OC curve, since the number of defective items can be only 0, 1, 2, . . . , 200, and only 201 percentages of defective items are possible. Suppose a series of inspection lots each containing 200 items and 2 defective items are submitted for inspection and a single-sampling plan is used with a sample size of 100 and an acceptance number of 2. Every inspection lot will necessarily be accepted since none contains more than 2 defective items and hence the sample cannot contain more than 2 defective items. If the size of the inspection lot were, say, 300 but the percentage of defective items the same, some inspection lots would be rejected since each would now contain 3 defective items and occasionally all 3 would be

included in the sample of 100. The dependence of the OC curve on the size of the inspection lot is important, of course, only when the inspection lot is small compared to the sample. If the inspection lot is, say, more than about 10 times as large as the sample, the dependence on inspection-lot size can be neglected.

If interpretation (*b*) is adopted, the resultant OC curve does not depend on the size of the inspection lot. To use the example of the preceding paragraph, successive inspection lots of 200 will not all contain 2 defective items; some will contain 3 or more defective items, and some samples from these will also contain 3 or more defective items. The sample of 100 is in effect selected by first taking a sample of 200 from an infinite population, then a subsample of 100. The distribution of these subsamples by the number of defective items they contain will be exactly the same as the distribution of samples of 100 taken directly from the infinite population. Hence, the size of the inspection lot does not affect the OC curve.

This book is not entirely consistent in the interpretation placed on the OC curves. In most of the textual discussion, interpretation (*a*) is adopted. On the other hand, the OC curves are in fact computed under interpretation (*b*) in order to avoid the necessity of giving them as functions of inspection-lot size. Further, the suggestion is made that the percentage of inspection lots likely to be accepted can be estimated from the ordinate of the OC curve corresponding to the process average. This procedure is correct only for interpretation (*b*) and only if the process average for a process is in control.* The justification for these inconsistencies is that, in practice, inspection lots are likely to be large compared to the samples from them. When this is true, the two interpretations lead to the same OC curve.

The interpretation of the OC curve also affects the meaning and numerical value of AOQ and AOQL, though, again, the values yielded by the two interpretations come together as the size of the inspection lot increases relative to the sample. The AOQL classes in Table 4, as well as the AOQL values in Table 15.4, are computed under the second interpretation.

1.2. Operating-characteristic (OC) Curves for Single-sampling Plans

The probability that r defective items will be contained in a sample of n from an inspection lot with proportion defective p is given by the

* If the process is in control and the sample size times the process average expressed in proportion defective is not too small (say not less than 5), the percent defective in successive inspection lots will be distributed almost symmetrically about the process average. If in addition the OC curve is approximately a straight line in the neighborhood of the process average, the interpretation offered is very accurate. As long as the process is in control the approximation will be useful, even if the OC curve departs considerably from linearity near the process average.

$(r + 1)$st term of the binomial expansion $[(1 - p) + p]^n$, namely, $\frac{n!}{r!(n - r)!}(1 - p)^{n-r}p^r$. Consequently, the probability of acceptance (ordinate of the OC curve at p), if the acceptance number is a, is the sum of the first $(a + 1)$ terms of the binomial expansion. This direct method of computation is laborious if n is large, though various tables make it feasible for a fairly wide range of values of n. For small p and fairly large n, the binomial expansion can be approximated by the Poisson exponential, the probability of r defective items being given by $m^re^{-m}/r!$ where $m = np$ is the expected number of defective items. For n sufficiently large, the binomial can also be approximated by a normal distribution with mean np and variance $np(1 - p)$.

Only the exact method and the Poisson approximation were used in computing the OC curves in Table 4 for single-sampling plans. The exact method was used for sample-size letters A through F. For such small values of n the Poisson approximation is not adequate. Fortunately, for n less than 50 it is fairly easy to compute the exact probabilities from Pearson's *Tables of the Incomplete Beta Function.*

The OC curves for sample-size letters G through O were calculated by use of the Poisson approximation. Since the Poisson approximation becomes poor for large p, the OC curves for single- (and double-) sampling plans must be considered only rough approximations for values of p greater than, say, 0.10 for sample-size letters G through O. A few exact calculations were made to determine the size of the error for such values of p. These indicated that the error, while appreciable, is not large enough to impair seriously the usefulness of the OC curves in judging the consequences of adopting a particular sampling plan.

1.3. Operating-characteristic (OC) Curves for Double-sampling Plans

The double-sampling plans in Table 2 and Table 4 are described by three parameters: the size of the first sample, n_1, the acceptance number for the first sample, a_1, and the acceptance number for the second sample, a_2. The size of the second sample is always $2n_1$ and the rejection numbers for both samples equal $(a_2 + 1)$.

The probability of accepting an inspection lot is

$$P''(a_1;n_1) + \sum_{k=a_1+1}^{a_2} p(k;n_1)P''(a_2 - k;2n_1)$$

where $p(c;d)$ denotes the probability of c defective items in d items and $P''(c;d)$ denotes the probability of c or fewer defective items in d items.

Computation of OC curves for the double-sampling plans was greatly facilitated by use of the Aberdeen Ballistic Research Laboratory Mem-

orandum Report No. 264, *Tables of Acceptance Probabilities for Double Sampling Plans with* $n_2 = 2n_1$, by Shaw and Stein. These tables give for various combinations of a_1 and a_2, the probability of acceptance as a function of n_1p, the expected number of defective items in the first sample. Since these tables are based on the Poisson approximation to the binomial distribution, they were used only for sample-size letters G through O. The OC curves for sample-size letters A through F were computed exactly by evaluating the terms in the expression for the probability of acceptance through the use of *Tables of the Incomplete Beta Function*, as described in Sec. 2 below.

1.4. Operating-characteristic (OC) Curves for Sequential-sampling Plans

Section 2 below, which describes the method of determining the acceptance and rejection numbers for the sequential-sampling plans, also describes the method of calculating the exact probability of acceptance for a grouped and truncated sequential-sampling plan—what may be called a multiple-sampling plan. This method is, however, extremely laborious and it was not, therefore, feasible to use it in obtaining the OC curves in Table 4. Instead, the OC curve for the item-by-item sequential plan from which the multiple-sampling plan was derived was taken as an approximation to the OC curve for the multiple-sampling plan.

The multiple-sampling plan is constructed so as to have approximately the same probability of acceptance as the item-by-item sequential plan at two points—the intended AQL and LTPD. Table 15.1 shows how close it was possible to come. This table gives the exact probabilities of acceptance for the multiple plans at the intended AQL and LTPD. The probability of acceptance at p_1, the intended AQL, which is exactly 0.95 for the item-by-item sequential plan, is 0.945 to 0.955 for 95 of the 123 multiple plans. The largest discrepancy is for sample-size letter G, AQL class 0.32 to 0.65, for which the probability of acceptance for the multiple plan is 0.969. The probability of acceptance at p_2, the intended LTPD, which is exactly 0.10 for the sequential, is 0.09 to 0.11 for 109 of the 123 multiple plans. The largest discrepancy is for sample-size letter D, AQL class 2.2 to 3.2, for which the probability of acceptance for the multiple plan is 0.132.

At p_1 and p_2, therefore, the OC curve for the item-by-item sequential plan gives approximately the correct probabilities for the corresponding multiple plan. The validity of using the OC curve for the item-by-item sequential plan as an approximation to the OC curve for the multiple plan at other values of the percentage defective was tested by computing for a few plans exact probabilities at other percentages defective. The agreement was uniformly excellent.

The method used to compute the OC curves for the sequential plans is described in *Sequential Analysis of Statistical Data: Applications.** The parameters of the item-by-item sequential plans are given in Table 15.6.

2. CONSTRUCTION OF MULTIPLE-SAMPLING PLANS FROM SEQUENTIAL PRINCIPLES

2.1. The Problem

An item-by-item likelihood-ratio sequential-sampling plan for attribute inspection can be summarized in a graph like Fig. 17.1. After each item is inspected, a point is plotted on the graph, the abscissa of the point being the total number of items inspected so far; the ordinate, the number of defective items found so far. If the plotted point ever falls above the upper sloping line, that is, in the region labeled "Reject," the inspection lot is rejected; if it ever falls below the lower line, that is, in the region labeled "Accept," the inspection lot is accepted; so long as it remains between the two lines, inspection is continued. The regions are separated by two parallel straight lines called the "rejection and acceptance lines." The three parameters defining these lines—the two intercepts and the common slope—are computed from the formulas given in *Sequential Analysis of Statistical Data: Applications.*† A sequential plan of this type has the desirable property that it requires close to the minimum average amount of inspection of any plan giving the same or a smaller chance of rejecting an inspection lot of quality p_1 or of accepting an inspection lot of quality p_2.

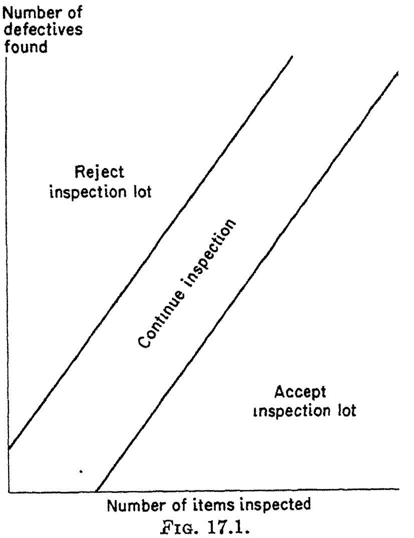


FIG. 17.1.

For reasons already given, it is frequently preferable in practice to inspect items in groups and to set a definite upper limit on the maximum number of items that may ever have to be inspected from one inspection lot. Such a grouped and truncated sequential plan has the advantages that it is not necessary to ensure that the order in which items are inspected is strictly random, it is generally simpler to administer, and the variation in amount of inspection from inspection lot to inspection

* Section 2, Second Revision, pp. 2.47 to 2.56; see also pp. 2.81 to 2.101.

† Section 2, Second Revision, formulas 2.203, 2.204, and 2.205.

lot is reduced. A grouped and truncated sequential plan (multiple-sampling plan) can attain these advantages and yet retain most of the efficiency of an item-by-item sequential plan if the groups are not too large and there are enough groups.

The problem considered in this section is how to derive from an item-by-item sequential plan a multiple-sampling plan having essentially the same OC curve and retaining maximum efficiency. A general treatment of this problem would have to consider the determination of (*a*) the size and number of samples, (*b*) the acceptance and rejection numbers. This section neglects the first problem, since for the plans in this book the size and number of samples were determined for several plans at once and hence could not be based solely on considerations of efficiency. The size and number of samples are therefore taken as given. Since, with one minor exception, the successive samples are equal in size for the plans in Table 4, the discussion is in terms of multiple-sampling plans having a succession of equal samples. However, this limitation is unimportant, since the modifications that need to be made in the procedure to allow for unequal samples sizes are fairly obvious.

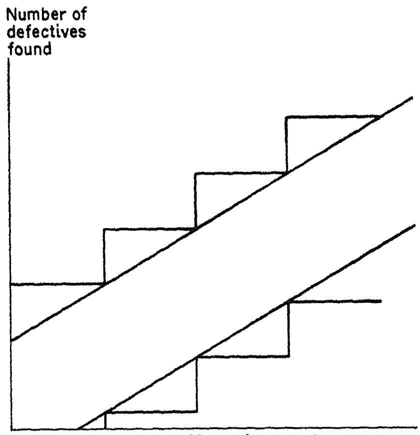


FIG. 17.2.

The effect of grouping items is essentially to substitute a step function for the straight lines of Fig. 17.1, as is shown in Fig. 17.2. In Fig. 17.2, the vertical lines are at the boundaries of the groups. The use of a single acceptance and rejection number for each such group means that the acceptance or rejection number must be represented by a horizontal line. The particular horizontal lines represent one particular method of deriving a step function, namely, by making the rejection number for the end and the acceptance number for the beginning of a group apply to the group. It is clear that this method adds to the region of indecision and subtracts from both the rejection region and acceptance region. It will therefore clearly require at least as much inspection for each inspection lot as the item-by-item sequential plan and will tend to make the OC curve steeper.

In order to retain the same OC curve, the step functions must not be entirely on one side of the acceptance and rejection lines; they must, as in Fig. 17.3, cut the acceptance and rejection lines, both adding to and subtracting from each of the three relevant regions. There seems, however, to be no simple way of determining where the step functions

should be placed so as to balance the effect on the probability of acceptance of the additions to a region and the subtractions from the region. Attempts to develop a general mathematical theory of the effects of grouping and truncation have so far met with little success.

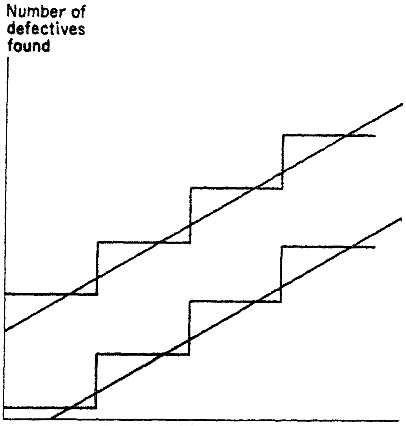


Fig. 17.3.

In the absence of such a general theory, a trial-and-error method can be used to determine the acceptance and rejection numbers. There are only a limited number of step functions that approximate the straight lines of the item-by-item sequential plan—that is, sets of acceptance and rejection numbers that are approximately linear and are in general between the highest and lowest item-by-item acceptance and rejection numbers for the corresponding sample. These can be tried to see if they yield the desired probabilities of acceptance and one of them chosen.

This process is feasible only if the exact probability of acceptance can be computed for the multiple-sampling plan in a manner that facilitates the early elimination of incorrect solutions. Section 2.2 of this chapter describes such a procedure. Section 2.3 of this chapter indicates how it is used to determine the acceptance and rejection numbers.

While the objective is to match the entire OC curve of the item-by-item sequential plan, it has been found sufficient to substitute the more limited objective of matching the OC curve at two points. The experiments made to test the validity of approximating the OC curve of a multiple-sampling plan by the OC curve of the corresponding item-by-item sequential plan indicate that, if the multiple plan gives the same probability of acceptance as the item-by-item sequential plan at the proportions defective (p_1 and p_2) for which the item-by-item sequential plan is most efficient, it will also give about the same probability of acceptance elsewhere. Accordingly, in what follows, the problem is further restricted to deriving a multiple plan that will give a probability of acceptance of $1 - \alpha$ at p_1 and of β at p_2, where p_1, p_2, α, and β are the four quantities used in constructing the item-by-item sequential plan. For the plans in this book, α equals 0.05 and β equals 0.10.

2.2. Computation of Probability of Acceptance for Multiple-sampling Plans

2.2.1. Alternative Procedures.—An algebraic expression for the probability, L_p, of accepting an inspection lot containing a fraction p of

defective items can be set up as a function of p for any specific multiple-sampling plan. This procedure has the advantage that it is general; and, once the algebraic expression has been found, it can be used to determine L_p for any value of p. This advantage is more than offset by two other considerations: (*a*) for any but the simplest plan, the algebraic expression is inordinately lengthy and complicated; (*b*) it is not convenient to get a rough idea from the algebraic expression of the final answer at an intermediate stage so as to be able to discard a tentative but incorrect solution.

It has been found more efficient to use instead a simple arithmetic procedure in which nearly all the work must be repeated for each new value of p but in which the desired probabilities are obtained step-by-step.

TABLE 17.1

EXAMPLE OF CONTROL TABLE

Sample (*i*th)	Combined samples			Proportion defective = p_1			Proportion defective = p_2		
	Size (n)	a_i	r_i	$P_A^{(i)}$	$P_R^{(i)}$	$P_{A+R}^{(i)}$	$P_A^{(i)}$	$P_R^{(i)}$	$P_{A+R}^{(i)}$
(1)	(2)	(3)	(4)	(5)	(6)	(7) = (5) + (6)	(8)	(9)	(10) = (8) + (9)
First......	50								
Second.....	100								
Third......	150								
Fourth....	200								
Fifth	250								
Sixth......	300								
Seventh....	350								
Eighth....	400								
Total....					$\Sigma = P_R(p_1)$	1.00	$\Sigma = P_A(p_2)$		1.00

2.2.2. The Control Table.—The heart of this arithmetic procedure is a control table summarizing the results of the computations at each step. Table 17.1 is an example of this control table for a multiple-sampling plan consisting of eight samples of 50 items each. To simplify exposition, it is assumed throughout this section that each sample is completely inspected (curtailing inspection within a sample obviously has no effect on the OC curve). In Table 17.1, n stands for the number of items inspected up to any point, a_i for the acceptance number associated with the ith sample, r_i for the rejection number associated with the ith sample, $P_A^{(i)}$ for the probability of accepting the inspection lot at the end of i samples and not sooner, $P_R^{(i)}$ for the probability of rejecting the inspection lot at the end of i samples and not sooner, $P_{A+R}^{(i)}$ for the probability of coming to a decision at the end of i samples and not sooner, $P_A(p)$ for the probability of accepting an inspection lot containing a fraction p of defective items, and $P_R(p)$ for the probability of rejecting an

inspection lot containing a fraction p of defective items. Table 17.1 is set up so as to contain the results of computations at two values of p, p_1 and p_2, since these are the points at which it is desired to match the OC curves; that is, it is desired to make $P_R(p_1) = \alpha$, $P_A(p_2) = \beta$.

The significance of the various entries in Table 17.1 is as follows: The entry in the fourth row and column (5) is the probability of accepting the inspection lot (with proportion defective equal to p_1) at the end of four samples and not sooner. The entry in the fourth row and column (6) is the probability of rejecting the inspection lot (with proportion defective equal to p_1) at the end of four samples and not sooner. The entry in the fourth row and column (7), which is the sum of the entries in columns (5) and (6), is the probability of reaching a decision at the end of four samples and not sooner. The sum of the first four entries in column (7) is the probability of coming to a decision at or before the end of four samples and is the complement of the probability of continuing to a fifth sample.

It is clear that the desired values $P_R(p_1)$ and $P_A(p_2)$ are the sums of columns (6) and (8), respectively. At any stage, for example at the end of four samples, an estimate of these quantities is given by the sum of the first four entries in columns (6) and (8). These partial sums, in conjunction with the corresponding sums of columns (7) and (10) (which tell what portion of the total probability has already been accounted for), give a means of deciding whether the a_i and r_i up to this stage should be changed; furthermore, they indicate whether the remaining a_i and r_i will yield an acceptable plan or, if not, how to revise them.

2.2.3. Example of Computation of Control Table.—The method of computing the entries in Table 17.1 is the same for p_1 and p_2, so it is sufficient to consider only columns (5) and (6) of the table (entries in column (7) are the sums of entries in columns (5) and (6)). For illustrative purposes, we shall use the example in Table 17.1 with $p_1 = 0.0410$ and $p_2 = 0.0920$ and a_i and r_i as given in Table 17.2.

A p_1 strip is first set up containing three columns of probabilities computed under the assumption that the true proportion defective is p_1. The first column contains the probabilities P_i, the second column the probabilities P'_i, and the third column the probabilities P''_i, where

$$\begin{aligned} P_i &= \text{probability of exactly } i \text{ defectives } \textit{in a sample of } 50 \\ P'_i &= \text{probability of } i \text{ or more defectives } \textit{in a sample of } 50 \\ P''_i &= \text{probability of } i \text{ or fewer defectives } \textit{in a sample of } 50 \end{aligned}$$

The range of i depends on the a_i and r_i. In this illustration there is needed P_i $(i = 0, \ldots, 8)$, P'_i $(i = 4, \ldots, 9)$, and P''_i $(i = 0, \ldots, 4)$. These probabilities can be obtained from various sources, among which are Molina's tables and Pearson's *Tables of the Incomplete Beta*

*Function.** The required p_1 strip for this example, based on Molina's tables, is given below.

Number of defectives in sample of 50 i	P_i		P'_i		P''_i	
0	P_0	0.128896			P''_0	0.128896
1	P_1	0.263915			P''_1	0.392811
2	P_2	0.270344			P''_2	0.663155
3	P_3	0.184730			P''_3	0.847885
4	P_4	0.094728	P'_4	0.152117	P''_4	0.942613
5	P_5	0.038883	P'_5	0.057390		
6	P_6	0.013308	P'_6	0.018506		
7	P_7	0.003906	P'_7	0.005198		
8	P_8	0.001004	P'_8	0.001292		
9			P'_9	0.000287		

TABLE 17.2

FINAL CONTROL TABLE FOR p_1

Sample (ith) (1)	n (2)	a_i (3)	r_i (4)	$P_A^{(i)}$ (5)	$P_R^{(i)}$ (6)	$P_{A+R}^{(i)}$ (7) = (5) + (6)
First	50	1	6	0.392811	0.018506	0.411317
Second	100	3	9	0.130005	0.016175	0.146180
Third	150	7	13	0.244573	0.003286	0.247859
Fourth	200	10	16	0.080462	0.002967	0.083429
Fifth	250	13	19	0.042902	0.002040	0.044942
Sixth	300	16	22	0.024738	0.001309	0.026047
Seventh	350	19	25	0.014801	0.000818	0.015619
Eighth	400	24	25	0.018497	0.006114	0.024611
Total				0.948789	0.051215	1.000004

The method of obtaining the entries in Table 17.2, which is the final control table for p_1, is as follows:

First Row of Table 17.2.—The entries in the first row are obtained directly from the p_1 strip, since

$$P_A^{(1)} = P''_1 \qquad \text{and} \qquad P_R^{(1)} = P'_6$$

Second Row of Table 17.2.—The entries in the second row are obtained from the following formulas:

$$P_A^{(2)} = P_2 \cdot P''_1 + P_3 \cdot P''_0 \tag{1}$$
$$P_R^{(2)} = P_2 \cdot P'_7 + P_3 \cdot P'_6 + P_4 \cdot P'_5 + P_5 \cdot P'_4 \tag{2}$$

Computing Scheme 1 is convenient for the computation of $P_A^{(2)}$ and $P_R^{(2)}$.

* See Sec. 2.4 below for the method of obtaining P_i, P'_i, and P''_i.

COMPUTING SCHEME 1

Number of defectives in sample of 50	Number of defectives at end of 1 sample: 2	3	4	5
	Probability of the above			
	0.270344	0.184730	0.094728	0.038883
0	Accept		4	5
1		4	5	6
2	4	5	6	7
3	5	6 Continue	7 inspection	8
4	6	7	8	
5	7	8		
6	8		Reject	
7				
8				

Acceptance line

Rejection line

The probabilities on top of the columns, the only computations that are copied onto the scheme, are taken directly from the strip—they are P_2, P_3, P_4, and P_5.

To compute $P_A^{(2)}$, place the strip next to the scheme so that the rows correspond, and cumulate products of entries in the third column of the strip by entries at the heads of the columns of the scheme. The acceptance line is used to determine which row entries are to be multiplied by which column entries. In each column use the box directly above the acceptance line, and take the product of the corresponding row entry (from the third column of the strip) and column entry (from the top of the column of the scheme). Cumulate all such products to obtain $P_A^{(2)}$.

In this example, two products are cumulated, and it can easily be verified that the result is $P_A^{(2)}$ as given by equation (1).

To compute $P_R^{(2)}$, cumulate products of entries in the second column of the strip and entries at the heads of the columns of the scheme. The rejection line is used as follows: use the box directly below the rejection line and take the product of its row entry (from the second column of the strip) and its column entry. The cumulative sum of all such products is $P_R^{(2)}$. It can again be verified that for this example the cumulative sum of the four indicated products is $P_R^{(2)}$ as given by equation (2).

Third Row of Table 17.2.—The items in the third row are obtained from the following formulas.

$$P_A^{(3)} = P_4^{(2)} \cdot P_3'' + P_5^{(2)} \cdot P_2'' + P_6^{(2)} \cdot P_1'' + P_7^{(2)} \cdot P_0'' \tag{3}$$
$$P_R^{(3)} = P_4^{(2)} \cdot P_9' + P_5^{(2)} \cdot P_8' + P_6^{(2)} \cdot P_7' + P_7^{(2)} \cdot P_6' + P_8^{(2)} \cdot P_5' \tag{4}$$

where $P_k^{(i)}$ = probability of exactly k defectives in i samples of 50, the k defectives being obtained in such a way that no decision (acceptance or rejection) is possible before the ith sample.

(*Note:* $P_k^{(1)} = P_k$.)

Before computing $P_A^{(3)}$ and $P_R^{(3)}$, formulas are needed for $P_4^{(2)}$, $P_5^{(2)}$, $P_6^{(2)}$, $P_7^{(2)}$, and $P_8^{(2)}$. They are

$$P_4^{(2)} = P_2 \cdot P_2 + P_3 \cdot P_1 + P_4 \cdot P_0 \tag{5}$$
$$P_5^{(2)} = P_2 \cdot P_3 + P_3 \cdot P_2 + P_4 \cdot P_1 + P_5 \cdot P_0 \tag{6}$$
$$P_6^{(2)} = P_2 \cdot P_4 + P_3 \cdot P_3 + P_4 \cdot P_2 + P_5 \cdot P_1 \tag{7}$$
$$P_7^{(2)} = P_2 \cdot P_5 + P_3 \cdot P_4 + P_4 \cdot P_3 + P_5 \cdot P_2 \tag{8}$$
$$P_8^{(2)} = P_2 \cdot P_6 + P_3 \cdot P_5 + P_4 \cdot P_4 + P_5 \cdot P_3 \tag{9}$$

Use can again be made of a computing scheme to facilitate the computations, for example Computing Scheme 2.

The probabilities at the tops of the columns are $P_4^{(2)}$, $P_5^{(2)}$, $P_6^{(2)}$, $P_7^{(2)}$, and $P_8^{(2)}$. These are obtained from Computing Scheme 1. In Computing Scheme 1, numbers have been listed in the upper left-hand corners of some boxes. They represent the combined number of defectives at the end of two samples obtained by getting a number of defectives in the first sample equal to the column heading and a number of defectives in the second sample equal to the row number. Numbers have been listed only when a decision is not possible.

To obtain $P_4^{(2)}$, use the boxes containing a 4 and take the product of the row entry (from the first column of the strip) and column entry. The cumulative sum of all such products is $P_4^{(2)}$. Similarly to compute $P_5^{(2)}$, use the boxes containing a 5, and so on. It is easy to verify that the results are exactly those given by equations (5) to (9).

Having filled in the column entries in Computing Scheme 2, compute $P_A^{(3)}$ and $P_R^{(3)}$ in exactly the same way as $P_A^{(2)}$ and $P_R^{(2)}$.

COMPUTING SCHEME 2

Number of defectives in sample of 50	Number of defectives at end of 2 samples: 4	5	6	7	8
	Probability of the above: 0.134049	0.129893	0.095605	0.056022	0.026937
0					8
1				8	9
2			8	9	10
3		8	9	10	11
4	8	9	10	11	12
5	9	10	11	12	
6	10	11	12		
7	11	12			
8	12				
9					
10					

Fourth, Fifth, Sixth, Seventh, and Eighth Rows of Table 17.2.—The items in the remaining rows of the table are computed in exactly the same way as the items in the third row with the help of five more computing schemes—3 to 7. The complete computing scheme for this illustration with column entries for the p_1 computations is given in Table 17.3. The same computing scheme is used for the p_2 computations,

TABLE 17.3
EXAMPLE OF COMPUTING SCHEME

Number of defectives in sample of 50	Computing Scheme 1				Computing Scheme 2					Computing Scheme 3					Computing Scheme 4				
	Number of defectives at the end of 1 sample				Number of defectives at the end of 2 samples					Number of defectives at the end of 3 samples					Number of defectives at the end of 4 samples				
	2	3	4	5	4	5	6	7	8	8	9	10	11	12	11	12	13	14	15
	0.270344	0.184730	0.094728	0.038883	0.134049	0 129893	0 095605	0.056022	0.026937	0 080797	0 057432	0 033522	0.016253	0 006644	0 041394	0 032471	0 020922	0 011282	0.005150
0			4	5					8				11	12				14	15
1		4	5	6				8	9			11	12	13			14	15	16
2	4	5	6	7			8	9	10		11	12	13	14		14	15	16	17
3	5	6	7	8		8	9	10	11	11	12	13	14	15	14	15	16	17	18
4	6	7	8		8	9	10	11	12	12	13	14	15		15	16	17	18	
5	7	8			9	10	11	12		13	14	15			16	17	18		
6	8				10	11	12			14	15				17	18			
7					11	12				15					18				
8					12														
9																			

Number of defectives in sample of 50	Computing Scheme 5					Computing Scheme 6					Computing Scheme 7				
	Number of defectives at the end of 5 samples					Number of defectives at the end of 6 samples					Number of defectives at the end of 7 samples				
	14	15	16	17	18	17	18	19	20	21	20	21	22	23	24
	0.023401	0.019217	0.012960	0 007272	0.003427	0 013876	0.011631	0.007995	0.004556	0.002173	0 008405	0 007107	0.004923	0.002923	0.001353
0				17	18				20	21					
1			17	18	19			20	21	22					
2		17	18	19	20		20	21	22	23					
3	17	18	19	20	21	20	21	22	23	24					
4	18	19	20	21		21	22	23	24						
5	19	20	21			22	23	24							
6	20	21				23	24								
7	21					29									
8															
9															

but a p_2 strip—one containing probabilities calculated under the assumption that the true proportion defective is p_2—is used instead of a p_1 strip.

There is a continuous check on the computations. If four lines in Table 17.2 have been completed, then the check is as follows: The sum of the four items in column (7) of Table 17.2 plus the sum of the column entries of Computing Scheme 4 should equal one. The first sum is the probability of coming to a decision by the end of four samples, and the second sum is the probability of continuing to a fifth sample. The sum should obviously equal one.

2.2.4. Construction of Computing Scheme.—Consider a multiple-sampling plan consisting of s samples of t observations each and associated acceptance numbers $a_1, a_2, \ldots, a_s$ and rejection numbers $r_1, r_2, \ldots, r_s$. In tabular form,

TABLE 17.4
TABULAR REPRESENTATION OF A MULTIPLE-SAMPLING PLAN

Sample	Combined samples		
	Size	a_i	r_i
First	t	a_1	r_1
Second	$2t$	a_2	r_2
ith	it	a_i	r_i
sth	st	a_s	r_s

$$a_s = r_s - 1$$

Set up the computing schemes in the following way (there will be $s - 1$ schemes in all):

1. For any scheme, set up a column for each number of defectives that will result in continued inspection; that is, for Computing Scheme i, set up columns for all integers greater than a_i and less than r_i.

2. Fill in the numbers in the boxes by summing the corresponding column and row headings (rows are always numbered 0, 1, 2, . . .). For Computing Scheme i, list only those sums that are equal to the numbers listed at the top of the columns of Computing Scheme $i + 1$.

3. The acceptance and rejection lines can then be drawn as the lines that separate the boxes with numbers from the boxes without numbers—the upper line is the acceptance line, the lower is the rejection line.

4. In Computing Scheme $s - 1$, the acceptance and rejection lines coincide. This one line is the line that separates the boxes with sums less than or equal to a_s from the boxes with sums greater than or equal to r_s.

2.3. Choice of Acceptance and Rejection Numbers

The first step in choosing suitable a_i and r_i is to draw the acceptance and rejection lines corresponding to the sequential plan defined by α, β, p_1, and p_2. For the illustration above, the lines are shown in Fig. 17.4.

Figure 17.4 is used as a guide in choosing the a_i and r_i. To determine the a_i for any value of n, read off the ordinate of the acceptance line corresponding to that value of n and choose an integer near this value—usually a higher integer. To determine the r_i, read off the corresponding ordinate of the rejection line and choose an integer near this value—usually a lower integer. This rule obviously does not define the a_i and

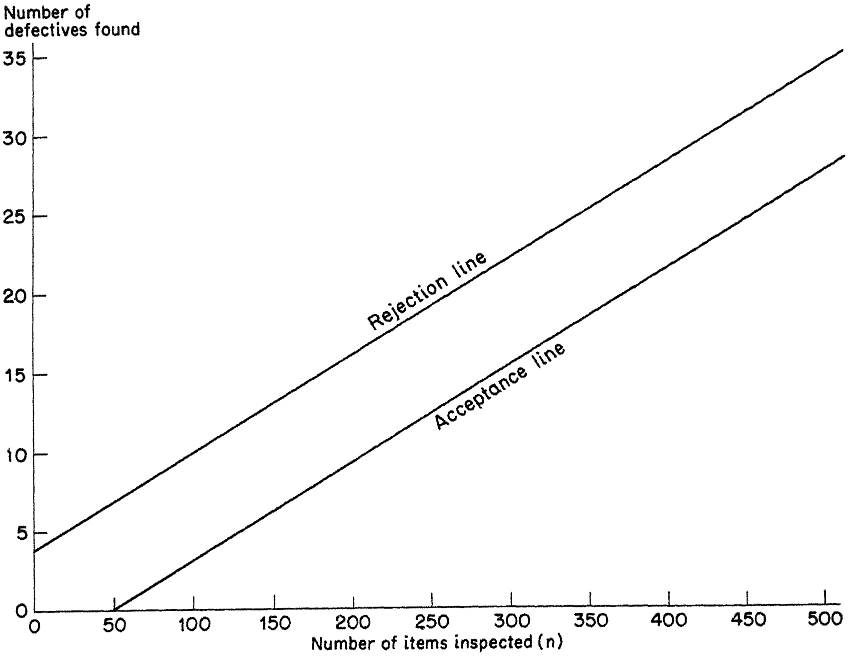


FIG. 17.4.

r_i uniquely and is used merely as a guide. (As a matter of fact, a_1 and r_1 can usually be obtained without reference to the graph—it is necessary only to fill in the first row to find out whether the choice is a proper one.) At each stage the choices can be tested by taking partial sums of columns (6), (7), (8), and (10) of Table 17.1.

If, on the basis of results up to a certain stage, it is necessary to make it easy to reject at p_1, use rejection numbers that are smaller than those indicated on the rejection line; if it is necessary to make it easy to accept at p_2, use acceptance numbers that are greater than those indicated on the acceptance line. Thus all a_i and r_i can be determined; the only restriction is that the last acceptance number must be one less than the last rejection number in order to ensure a decision, at the latest, after the final sample has been inspected.

This procedure seems to leave much leeway at each stage. In prac-

tice, however, this is seldom so. The acceptance and rejection numbers for the first sample are usually unique, since there is only one set that makes it possible to get the desired values of $P_R(p_1)$ and $P_A(p_2)$. The choice, thereafter, is also fairly limited. Further, $P_R(p_1)$ is much more sensitive to changes in the r_i than to changes in the a_i; and $P_A(p_2)$ to changes in the a_i than to changes in the r_i. Therefore, the r_i can be used mainly to control $P_R(p_1)$ and the a_i to control $P_A(p_2)$.

Up to this point, only one criterion has been considered by which to judge whether a plan is acceptable, namely, the closeness of $P_R(p_1)$ and $P_A(p_2)$ to α and β respectively. This is usually accomplished if the proposed a_i and r_i bear some resemblance to those obtained from the acceptance and rejection lines, which means that the a_i and r_i increase linearly; that is, the differences between two consecutive a_i (or r_i) are approximately constant.

If several plans satisfy the first criterion,* $P_R(p_1) \doteq 0.05$ and $P_A(p_2) \doteq 0.10$, a second criterion is needed by which to choose one of them. Such a criterion is to choose that plan that requires the least inspection. An approximation to the average amount of inspection is given by $\Sigma n P^{(i)}_{A+R}$, that is, by assuming that a decision cannot be reached before the end of a sample. When all the subsample sizes are equal to t, $n = ti$, so in the example of Table 17.1 the value of n in the fourth line of column (2) is $4 \times 50 = 200$. Thus, among plans judged equally good on the basis of the first criterion, it seems reasonable to choose that one for which $\Sigma n P^{(i)}_{A+R}$ is a minimum. The summation indicated can be carried out very easily; for example, in Table 17.1 one merely adds up the eight products obtained by multiplying the numbers in column (2) by the numbers on the same line in column (7). (See Sec. 3.2.1 of this chapter for further discussion.)

2.4. Calculation of P_i, P'_i, and P''_i

Consider a sample of size n and assume that the true proportion defective is p; then

$$
\begin{aligned}
P_i &= \text{probability of exactly } i \text{ defectives in a sample of } n \\
&= {}_nC_i p^i (1-p)^{n-i} \\
P'_i &= \text{probability of } i \text{ or more defectives in a sample of } n \\
&= \sum_{x=i}^{n} {}_nC_x p^x (1-p)^{n-x} \\
P''_i &= \text{probability of } i \text{ or fewer defectives in a sample of } n \\
&= \sum_{x=0}^{i} {}_nC_x p^x (1-p)^{n-x}
\end{aligned}
$$

* The symbol $\doteq$ denotes "approximately equal to."

where ${}_nC_x$ stands for the number of combinations of n things taken x at a time.

P_i, P'_i, and P''_i can be obtained directly from the formulas above. It is not necessary, however, to calculate all of them, for the following relations exist:

$$P_i = P'_i - P'_{i+1} \tag{10}$$
$$P_i = P''_i - P''_{i-1} \tag{11}$$
$$P'_i = 1 - P''_{i-1} \tag{12}$$
$$P''_i = 1 - P'_{i+1} \tag{13}$$
$$P''_i = P_0 + P_1 + \cdots + P_i \tag{14}$$

In order to avoid the laborious computations necessitated by the exact formulas, use can be made of Molina's table of *Poisson's Exponential Binomial Limit,* which tabulates the individual (Table I) and cumulated (Table II) terms of the Poisson exponential expansion.

The individual term $\frac{a^x e^{-a}}{x!}$ (where $a = np$) which is the $(x + 1)$st term of the Poisson expansion is an approximation to the $(x + 1)$st term of the expansion of the binomial $[(1 - p) + p]^n$, that is,

$$P_x = {}_nC_x p^x (1 - p)^{n-x}$$

The cumulated term

$$P(c,a) = \sum_{x=c}^{\infty} \frac{a^x e^{-a}}{x!}$$

is an approximation to the corresponding binomial sum

$$P'_c = \sum_{x=c}^{n} {}_nC_x p^x (1 - p)^{n-x}$$

Therefore, Molina's Table I can be used to obtain P_i and his Table II to obtain P'_i. P''_i can then be derived from these probabilities by (13) or (14).

This approximation is satisfactory when p is less than 0.10 and was used in the computations for the plans in this book in all cases for which n was 20 or greater and p was 0.10 or less.

Pearson's *Tables of the Incomplete Beta Function* provides another means of obtaining P_i, P'_i, and P''_i. This volume tabulates the function*

$$I_x(p,q) = \frac{\int_0^x x^{p-1}(1 - x)^{q-1}\,dx}{\int_0^1 x^{p-1}(1 - x)^{q-1}\,dx}$$

* The p and q which Pearson uses as the arguments of the Beta function should not be confused with the p and $1 - p$ used in this volume to represent fraction defective.

It can be shown that this function represents the sum of the last q terms in the expansion of the binomial $[(1 - x) + x]^{p+q-1}$, that is,

$$I_x(p,q) = \sum_{y=p}^{p+q-1} {}_{p+q-1}C_y \, x^y(1 - x)^{p+q-1-y}$$

Using this relation, P_i' can be evaluated as follows:

$$\begin{aligned} P_i' &= \sum_{x=i}^{n} {}_nC_x p^x(1 - p)^{n-x} \\ &= \text{sum of the last } n + 1 - i \text{ terms of } [(1 - p) + p]^n \\ &= I_p(i,n + 1 - i) \end{aligned}$$

Therefore, to obtain P_i', enter the table of $I_x(p,q)$ with $x = p$, $p = i$, and $q = n + 1 - i$. [The function $I_x(p,q)$ is tabulated only for $p \geq q$. If $p < q$, use the following relation:

$$I_x(p,q) = 1 - I_{1-x}(q,p)]$$

P_i and P_i'' can then be derived from P_i' by using (10) and (13).

3. AVERAGE AMOUNT OF INSPECTION

3.1. Introduction

Generally, the amount of inspection is not the same for every inspection lot. If inspection is curtailed as soon as a decision can be reached, the amount of inspection will vary from inspection lot to inspection lot even for single sampling. In double and multiple (grouped and truncated sequential) sampling, there are two possible sources of variation in the amount of inspection: the number of samples required to reach a decision will vary; and inspection may be curtailed within samples in the same manner as in single sampling. A complete description of the amount of inspection then consists of the probability that each possible amount of inspection will occur. It is obvious that these probabilities will depend on the proportion, p, of defective items in the inspection lot (and, if the inspection lot is small, on its size), so a complete description would consist of the probability distribution of the amount of inspection as a function of p.

For many purposes, the most important feature of the probability distribution of the amount of inspection is its average (arithmetic mean). This is the amount of inspection per inspection lot to be expected if inspection is done on many inspection lots and is an appropriate basis for comparing the efficiency of different sampling plans.

Figure 9.2 shows, for several selected plans, the average amount of inspection required by single, double, and sequential plans having almost the same OC curve. Similar data would be desirable for all plans in this book; it was not, however, feasible to provide such data because of the

amount of computational labor required. Table 15.7 gives some supplementary data on the average amount of inspection required by the multiple-sampling plans at p_1 and p_2.

The present section describes a method of computing the average amount of inspection for single-, double-, and multiple-sampling plans. Since double- and single-sampling plans are special cases of multiple sampling, multiple sampling is considered first, double sampling next, and single sampling last.

It is specified in the standard procedure in this book that the entire first sample be inspected in double or multiple sampling in order to facilitate computation of the process average, even though it is possible to reach a decision before completing inspection of the first sample. Inspection of later samples is, however, to be curtailed as soon as a decision can be reached. The computational methods described below for double and multiple sampling therefore assume that the entire first sample is inspected. To simplify the computations, they make the additional assumption that inspection is continued to the end of the sample being inspected, even though a decision to accept can be reached earlier, and is curtailed only when a decision to reject can be reached, that is, when the number of defectives found is equal to the rejection number. This assumption is made because, for plans of the kind in this book, that is, plans with relatively low values of AQL, curtailing for rejection has a much larger effect on the average amount of inspection than curtailing for acceptance. The results obtained by taking account solely of curtailing for rejection ("semicurtailing") are therefore a good approximation to the correct results when inspection is curtailed for acceptance as well.

For single sampling, if conducted as specified in the standard procedure, the amount of inspection is simply the sample size. However, it may sometimes be desired to curtail in single sampling, and Fig. 3.6 gives an example of how curtailing in single sampling affects the average amount of inspection. Accordingly, Sec. 3.4 of this chapter describes how to compute the average amount of inspection for curtailed single sampling. To provide comparability with the preceding sections, it gives a method when curtailing is solely for rejection; it also gives a method when curtailing is for both acceptance and rejection.

3.2. Multiple Sampling

3.2.1. Formula for Average Amount of Inspection.—Consider a multiple-sampling plan of s samples with t observations in each sample. Denote the acceptance number associated with the ith sample ($i = 1, \ldots, s$) by a_i and the rejection number associated with the ith sample by r_i.

By definition, the average amount of inspection per inspection lot

(which may be called the "average sample number" and abbreviated to ASN) is

$$\text{ASN} = \sum_{n=1}^{st} nP(n) \tag{15}$$

where $P(n)$ = probability of making a decision at the nth observation and not before. Since the entire first sample is assumed inspected, $P(n)$ is zero for n less than t; hence (15) becomes

$$\text{ASN} = \sum_{n=t}^{st} nP(n) \tag{16}$$

In evaluating (16), it is necessary to consider separately three sets of values of n.

1. $n = t$. This is the end of the first sample and is the earliest point at which a decision can be reached. The decision can be either to accept or to reject.

2. $n = ti$ $(i = 2, \ldots, s)$. These values are at the end of the second and later samples. A decision can be made at these values of n either to accept or to reject. For rejection to occur at any of these values of n, however, the last item inspected must be defective, since otherwise the rejection number would have been reached earlier and inspection would have been discontinued.

3. $n = ti + j$ $\begin{Bmatrix} i = 1, \ldots, s-1 \\ j = 1, \ldots, t-1 \end{Bmatrix}$. These values are intermediate values in the second and later samples. It is assumed that only a decision to reject is reached at these values. Even if a decision to accept is possible, inspection is assumed continued to the end of the sample. For rejection to occur at any of these values, the last item inspected must be defective, since otherwise the rejection number would have been reached earlier and inspection would have been discontinued.

A formal expression for $P(n)$ for values of n in each of these sets is given in the following table:

n	$P(n)$
t	$P_A^{(1)} + P_R^{(1)}$
$ti(i = 2, \ldots, s)$	$P_A^{(i)} + \sum\limits_{k=a_{i-1}+1}^{r_{i-1}-1} P_k^{(i-1)} P \left\{ \begin{matrix} [t-1] \\ [r_i - k - 1] \end{matrix} \text{ and 1 on the } (ti)\text{th} \right\}$
$ti + j \begin{pmatrix} i = 1, \ldots, s-1 \\ j = 1, \ldots, t-1 \end{pmatrix}$	$\sum\limits_{k=a_i+1}^{r_i-1} P_k^{(i)} P \left\{ \begin{matrix} [j-1] \\ [r_{i+1} - k - 1] \end{matrix} \text{ and 1 on the } (ti+j)\text{th} \right\}$

where $P_A^{(i)}$ = probability of acceptance on the basis of the ith sample and not sooner

$P_R^{(i)}$ = probability of rejection on the basis of the ith sample and not sooner

$P_k^{(i)}$ = probability of exactly k defectives in i samples, the k defectives being obtained in such a way that no decision (acceptance or rejection) is possible before the ith sample

$P\begin{Bmatrix}[c] \\ [d]\end{Bmatrix}$ and 1 on the fth$\Big\}$ = probability of exactly d defectives in c observations and one defective on the fth observation

If each value of $P(n)$ is multiplied by the corresponding n and the resulting products summed for all values of n in the same set, the contributions of each set of values of n to (16) are given below:

n	Contribution of each set of values of n to $\sum_{n=t}^{st} nP(n)$
t	$t(P_A^{(1)} + P_R^{(1)})$
$ti(i = 2, \ldots, s)$	$t\left(\sum_{i=2}^{s} iP_A^{(i)} + \sum_{i=2}^{s} iF(i)\right)$ where $F(i) = \sum_{k=a_{i-1}+1}^{r_{i-1}-1} P_k^{(i-1)} P \left\{\begin{matrix}[t-1] \\ [r_i - k - 1]\end{matrix}\right.$ and 1 on the (ti)th$\Big\}$
$ti + j\begin{pmatrix} i = 1, \ldots, s-1 \\ j = 1, \ldots, t-1\end{pmatrix}$	$t\sum_{i=1}^{s-1} i \sum_{k=a_i+1}^{r_i-1} P_k^{(i)} G(i,k) + \sum_{i=1}^{s-1} \sum_{k=a_i+1}^{r_i-1} P_k^{(i)} H(i,k)$ where $G(i,k) = \sum_{j=r_{i+1}-k}^{t-1} P \left\{\begin{matrix}[j-1] \\ [r_{i+1} - k - 1]\end{matrix}\right.$ and 1 on the $(ti+j)$th$\Big\}$ $H(i,k) = \sum_{j=r_{i+1}-k}^{t-1} jP \left\{\begin{matrix}[j-1] \\ [r_{i+1} - k - 1]\end{matrix}\right.$ and 1 on the $(ti+j)$th$\Big\}$ the lower limit for j assuming the assigned value, since for smaller values $r_{i+1} - k - 1$ is greater than $j - 1$, and $P \left\{\begin{matrix}[j-1] \\ [r_{i+1} - k - 1]\end{matrix}\right.$ and 1 on the $(ti+j)$th$\Big\}$ is zero.

Let $P_{n:x}$ = probability of getting exactly x defectives in n trials
$P'_{n:x}$ = probability of getting x or more defectives in n trials
$P''_{n:x}$ = probability of getting x or fewer defectives in n trials
${}_nC_x$ = number of combinations of x things out of n

Now, by definition,

$$G(i,k) = \sum_{j=r_{i+1}-k}^{t-1} P\begin{Bmatrix} [j-1] & \\ [r_{i+1}-k-1] & \text{and 1 on the } (ti+j)\text{th} \end{Bmatrix} \tag{17}$$

But this is equal to the probability of getting $r_{i+1} - k$ or more defectives out of $t - 1$ trials, since a set of mutually exclusive and exhaustive ways of getting $r_{i+1} - k$ or more defectives out of $t - 1$ trials is to get (1) $r_{i+1} - k$ defectives in the first $r_{i+1} - k$ trials; (2) $r_{i+1} - k - 1$ defectives in the first $r_{i+1} - k$ trials and one defective on the $(r_{i+1} - k + 1)$th trial; (3) $r_{i+1} - k - 1$ defectives in the first $r_{i+1} - k + 1$ trials and one defective on the $(r_{i+1} - k + 2)$th trial; . . . ; $(t - r_{i+1} + k)$ $r_{i+1} - k - 1$ defectives in the first $t - 2$ trials and one defective on the $(t - 1)$th trial. Since the successive terms in equation (17) are the probabilities of these events, their sum is the probability of getting $r_{i+1} - k$ or more defectives out of $t - 1$ trials; that is,

$$G(i,k) = P'_{t-1:r_{i+1}-k} \tag{18}$$

By definition,

$$H(i,k) = \sum_{j=r_{i+1}-k}^{t-1} jP\begin{Bmatrix} [j-1] & \\ [r_{i+1}-k-1] & \text{and 1 on the } (ti+j)\text{th} \end{Bmatrix} \tag{19}$$

Now

$$\begin{aligned} P\begin{Bmatrix} [j-1] & \\ [r_{i+1}-k-1] & \text{and 1 on the } (ti+j)\text{th} \end{Bmatrix} &= {}_{j-1}C_{r_{i+1}-k-1}\,p^{r_{i+1}-k}q^{j-r_{i+1}+k} \\ &= \frac{r_{i+1}-k}{jp}\,{}_jC_{r_{i+1}-k}\,p^{r_{i+1}-k+1}q^{j-r_{i+1}+k} \\ &= \frac{r_{i+1}-k}{jp}\,P\begin{Bmatrix} [j] & \\ [r_{i+1}-k] & \\ & \text{and 1 on the } (ti+j+1)\text{th} \end{Bmatrix} \end{aligned} \tag{20}$$

Substituting in (19),

$$H(i,k) = \frac{r_{i+1}-k}{p} \sum_{j=r_{i+1}-k}^{t-1} P\begin{Bmatrix} [j] & \\ [r_{i+1}-k] & \text{and 1 on the } (ti+j+1)\text{th} \end{Bmatrix} \tag{21}$$

But, by reasoning like that given above, the summation in the last expres-

sion is the probability of getting $r_{i+1} - k + 1$ or more defectives in t trials. Hence,

$$H(i,k) = \frac{r_{i+1} - k}{p} P'_{t:r_{i+1}-k+1} \tag{22}$$

Adding the contributions of each set of values of n to $\sum_{n=t}^{st} nP(n)$, replacing $G(i,k)$ by (18), and $H(i,k)$ by (22), gives

$$\text{ASN} = tP_R^{(1)} + t\sum_{i=1}^{s} iP_A^{(i)} + t\sum_{i=2}^{s} iF(i) + t\sum_{i=1}^{s-1} i \sum_{k=a_i+1}^{r_i-1} P_k^{(i)} P'_{t-1:r_{i+1}-k} \tag{23}$$
$$+ \frac{1}{p}\sum_{i=1}^{s-1} \sum_{k=a_i+1}^{r_i-1} P_k^{(i)} (r_{i+1} - k) P'_{t:r_{i+1}-k+1}$$

Let

$$A_i = \sum_{k=a_i+1}^{r_i-1} P_k^{(i)} P'_{t-1:r_{i+1}-k} \tag{24}$$

and

$$B_i = \sum_{k=a_i+1}^{r_i-1} P_k^{(i)} (r_{i+1} - k) P'_{t:r_{i+1}-k+1} \tag{25}$$

Making these substitutions, and simplifying, gives

$$\text{ASN} = t\left\{ P_R^{(i)} + \sum_{i=1}^{s} i\left[P_A^{(i)} + F(i) + A_i \right]\right\} + \frac{1}{p}\sum_{i=1}^{s-1} B_i \tag{26}$$

where $F(i)$ is defined only for $i = 2, \ldots, s$, and A_i is defined only for $i = 1, \ldots, s - 1$.

3.2.2. Computation of Average Amount of Inspection.—To evaluate (26), A_i and B_i are computed for $i = 1, \ldots, s - 1$; $P_R^{(i)}$ and $P_A^{(i)}$ are obtained from the computations for the OC curve (see Sec. 2.2); and the $F(i)$ are computed from the relation*

$$F(i) + A_{i-1} = P_R^{(i)} \tag{27}$$

Replacing $F(i)$ in (26) by its equivalent from (27) gives

$$\text{ASN} = t\sum_{i=1}^{s} i\left[P_A^{(i)} + P_R^{(i)} + A_i - A_{i-1} \right] + \frac{1}{p}\sum_{i=1}^{s-1} B_i \tag{28}$$

where A_i is defined only for $i = 1, \ldots, s - 1$.

* $F(i)$ is the probability of rejection on the (ti)th observation and not sooner. A_{i-1} is the probability of rejection on one of the preceding $(t - 1)$ observations of the ith sample and not sooner. $F(i) + A_{i-1}$ is therefore the probability of rejection on the basis of the ith sample and not sooner; and this, by definition, is $P_R^{(i)}$.

The computing scheme used in the OC curve computations is used in the calculation of A_i and B_i. This computing scheme gives, for the first $s - 1$ samples, a column for each number of defectives that would result in continued inspection. The rows of this scheme represent the number of defectives (x), and within each scheme there are drawn an acceptance line and a rejection line. For a given value of p, the probabilities that inspection will continue are listed at the top of the columns for each scheme. For this given value of p, a strip on which is listed $P'_{i-1:x}$ can be lined up with the rows of the scheme. Then, A_i is the cumulative sum of products of entries on the strip by entries at the top of the columns for Computing Scheme i. For each column entry, the strip entry used is that corresponding to the row below the rejection line for that column. B_i is the cumulative sum of products of entries on a strip $[(x - 1)P'_{i:x}]$ by entries at the top of the columns for Computing Scheme i. For each column entry, the strip entry used is that corresponding to two rows below the rejection line for that column.

To illustrate the method of computing the ASN for a given p, consider a multiple-sampling plan of 8 samples, each of 50 observations. $P_A^{(i)}$ and $P_R^{(i)}$ are obtained in the computations for the OC curve. A_i and B_i are computed by the method described above. The remaining calculations can be set up as shown below.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
i	$P_A^{(i)}$	$P_R^{(i)}$	B_i	A_i	$\Delta_i = A_i - A_{i-1}$	(2) + (3) + (6)
1					A_1	
2						
3						
4						
5						
6						
7						
8					$-A_7$	

The ASN is then computed as follows:

$$\text{ASN} = 50\left[\sum (1)(7)\right] + \frac{1}{p}\sum (4) \tag{29}$$

where the first summation is over the product of elements on the same line in columns (1) and (7) and the second summation is the sum of the elements in column (4).

3.3. Double Sampling

3.3.1. Formula for Average Amount of Inspection.—Consider the following double-sampling plan:

Sample	Combined samples		
	Size	Acceptance number	Rejection number
First..................................	n_1	a_1	r_1
Second..................................	n_2	a_2	$r_2(= a_2 + 1)$

The ASN for a given p is derived in the same manner as for multiple sampling by considering the contribution of each n from n_1 to n_2 to the quantity $\sum\limits_{n=n_1}^{n_2} nP(n)$.

The table on page 210 gives $P(n)$ and the contribution to $\Sigma nP(n)$ for each set of values of n.

By the kind of reasoning used in Sec. 3.2.1 above, it can be shown that

$$F(k) = P'_{n_2-n_1-1:r_2-k} \tag{30}$$

$$G(k) = \frac{r_2 - k}{p} P'_{n_2-n_1:r_2-k+1} \tag{31}$$

and

$$P'_{n_2-n_1-1:r_2-k} + pP_{n_2-n_1-1:r_2-k-1} = P'_{n_2-n_1:r_2-k} \tag{32}$$

Making these substitutions, replacing a_2 by $r_2 - 1$, and simplifying,

$$\text{ASN} = n_1 (P''_{n_1:a_1} + P'_{n_1:r_1})$$

$$+ \sum_{k=a_1+1}^{r_1-1} P_{n_1:k} \left[n_2 - (n_2 - n_1)P'_{n_2-n_1-1:r_2-k} + \frac{r_2 - k}{p} P'_{n_2-n_1:r_2-k+1} \right] \tag{33}$$

3.3.2. Computation of Average Amount of Inspection.—For a given double-sampling plan (given n_1, n_2, a_1, a_2, r_1, r_2) the ASN for any p can be computed as shown in the table at the top of page 211.

The ASN is computed as follows:

$$\text{ASN} = n_1(P''_{n_1:a_1} + P'_{n_1:r_1}) + \Sigma(4)(8) \tag{34}$$

where the indicated summation is over the product of elements appearing on the same lines in columns (4) and (8).

n	$P(n)$	Contribution of each set of values of n to $\sum_{n=n_1}^{n_2} nP(n)$
n_1	$P''_{n_1:a_1} + P'_{n_1:r_1}$	$n_1(P''_{n_1:a_1} + P'_{n_1:r_1})$
$n_1 + i$ $(i = 1, \ldots, n_2 - n_1 - 1)$	$\sum_{k=a_1+1}^{r_1-1} P_{n_1:k} P \left\{ \begin{matrix} [i-1] \\ [r_2 - k - 1] \end{matrix} \text{ and 1 on the } (n_1+i)\text{th} \right\}$	$n_1 \sum_{k=a_1+1}^{r_1-1} P_{n_1:k} F(k) + \sum_{k=a_1+1}^{r_1-1} P_{n_1:k} G(k)$ where $F(k) = \sum_{i=r_2-k}^{n_2-n_1-1} P \left\{ \begin{matrix} [i-1] \\ [r_2 - k - 1] \end{matrix} \text{ and 1 on the } (n_1+i)\text{th} \right\}$ $G(k) = \sum_{i=r_2-k}^{n_2-n_1-1} iP \left\{ \begin{matrix} [i-1] \\ [r_2 - k - 1] \end{matrix} \text{ and 1 on the } (n_1+i)\text{th} \right\}$
n_2	$\sum_{k=a_1+1}^{r_1-1} P_{n_1:k} \left[P''_{n_2-n_1:a_2-k} + P \left\{ \begin{matrix} [n_2 - n_1 - 1] \\ [r_2 - k - 1] \end{matrix} \text{ and 1 on the } (n_2)\text{th} \right\} \right]$	$n_2 \sum_{k=a_1+1}^{r_1-1} P_{n_1:k}(P''_{n_2-n_1:a_2-k} + pP_{n_2-n_1-1:r_2-k-1})$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
k	$r_2 - (1)$	$(2) + 1$	$P_{n_1:k}$	$P'_{n_2-n_1-1:(2)}$	$P'_{n_2-n_1:(3)}$	$\frac{(2)}{p}$	$n_2 - (n_2 - n_1)(5) + (6)(7)$
$a_1 + 1$							
.							
.							
.							
$r_1 - 1$							

The ASN curves for two special cases are given below:

Case 1:

$$a_1 = r_2 - 2$$
$$a_2 = r_2 - 1$$
$$r_1 = r_2$$

$$\text{ASN} = n_1 + \frac{1}{p} P_{n_1:r_2-1} \cdot P'_{n_2-n_1:1} \tag{35}$$

Case 2:

$$a_1 = r_2 - 3$$
$$a_2 = r_2 - 1$$
$$r_1 = r_2$$

$$\text{ASN} = n_1 + \frac{1}{p} [P_{n_1:r_2-2} (2P'_{n_2-n_1:1} - P_{n_2-n_1:1}) + P_{n_1:r_2-1} (P'_{n_2-n_1:1})] \tag{36}$$

3.4. Single Sampling

3.4.1. Semicurtailed Single Sampling.—Consider the following single-sampling plan:

$$n_1 = \text{sample size}$$
$$a = \text{acceptance number}$$
$$r = \text{rejection number}$$

The ASN for a given p is derived in the same manner as for multiple sampling by considering the contribution of each n from 1 to n_1 to the quantity $\sum_{n=1}^{n_1} nP(n)$.

The following table gives $P(n)$ and the contribution to $\Sigma nP(n)$ for each set of values of n.

Inspection is curtailed as soon as a decision of rejection can be reached.

n	$P(n)$	Contribution of each set of values of n to $\sum_{n=1}^{n_1} nP(n)$
$1, \ldots, n_1 - 1$	$P\left\{\begin{matrix}[n-1] \\ [r-1]\end{matrix} \text{ and 1 on the } n\text{th}\right\}$	$\sum_{n=1}^{n_1-1} {}_{n-1}C_{r-1}p^rq^{n-r} = \frac{r}{p}P'_{n_1:r+1}$
n_1	$P''_{n_1:a} + P\left\{\begin{matrix}[n_1-1] \text{ and 1 on the} \\ [r-1] \qquad n_1\text{th}\end{matrix}\right\}$	$n_1(P''_{n_1:a} + pP_{n_1-1:r-1})$

Adding the contributions of each set of values of n to $\sum_{n=1}^{n_1} nP(n)$, and using the relation,

$$n_1p\, P_{n_1-1:r-1} + \frac{r}{p}P'_{n_1:r+1} = \frac{r}{p}P'_{n_1+1:r+1} \tag{37}$$

gives

$$\text{ASN} = n_1P''_{n_1:a} + \frac{r}{p}P'_{n_1+1:r+1} \tag{38}$$

3.4.2. Curtailed Single Sampling.—Consider the following single-sampling plan:

n_1 = sample size
a = acceptance number
r = rejection number

The ASN for a given p is derived in the same manner as for multiple sampling by considering the contribution of each n from 1 to n_1 to the quantity $\sum_{n=1}^{n_1} nP(n)$.

The table on page 213 gives $P(n)$ and the contribution to $\Sigma nP(n)$ for each set of values of n.

Inspection is curtailed as soon as a decision of acceptance or rejection can be reached.

Adding the contributions of each set of values of n to $\sum_{n=1}^{n_1} nP(n)$ gives

$$\begin{aligned}\text{ASN} = \sum_{n=1}^{n_1-a-1} n\, {}_{n-1}C_{r-1}p^rq^{n-r} \\ + \sum_{i=0}^{a} (n_1 - a + i)\, {}_{n_1-a+i-1}C_{r-1}\, p^rq^{n_1-a+i-r} \\ + \sum_{i=0}^{a} (n_1 - a + i)\, {}_{n_1-a+i-1}C_i\, p^iq^{n_1-a}\end{aligned} \tag{39}$$

n	$P(n)$	Contribution of each set of values of n to $\sum_{n=1}^{n_1} nP(n)$
$1, 2, \ldots, n_1 - a - 1$	$P\left\{\begin{matrix}[n-1] \\ [r-1]\end{matrix} \text{ and 1 on the } n\text{th}\right\}$	$\sum_{n=1}^{n_1-a-1} n\,{}_{n-1}C_{r-1}p^r q^{n-r}$
$n_1 - a + i$ $(i = 0, \ldots, a)$	$P\left\{\begin{matrix}[n_1 - a + i - 1] \\ [r-1]\end{matrix} \text{ and 1 on the } (n_1 - a + i)\text{th}\right\}$ $+ P\left\{\begin{matrix}[n_1 - a + i - 1] \\ [i]\end{matrix} \text{ and none on the } (n_1 - a + i)\text{th}\right\}$	$\sum_{i=0}^{a} (n_1 - a + i)\left({}_{n_1-a+i-1}C_{r-1}p^r q^{n_1-a+i-r} + {}_{n_1-a+i-1}C_i p^i q^{n_1-a}\right)$

The sum of the first two summations in (39) can be written as

$$\sum_{n=1}^{n_1} n_{n-1}C_{r-1}\, p^r q^{n-r}$$

and, by the kind of reasoning used in Sec. 3.2.1 above, it can be shown that

$$\sum_{n=1}^{n_1} n_{n-1}C_{r-1}\, p^r q^{n-r} = \frac{r}{p} P'_{n_1+1:r+1} \tag{40}$$

Now

$$\begin{aligned}
\sum_{i=0}^{a} (n_1 - a + i)_{n_1-a+i-1}C_i\, p^i q^{n_1-a} \\
&= \sum_{i=0}^{a} (n_1 - a + i)\frac{(n_1 - a + i - 1)!}{i!(n_1 - a - 1)!} p^i q^{n_1-a} \\
&= \sum_{i=0}^{a} \frac{(n_1 - a + i)!}{i!(n_1 - a - 1)!} p^i q^{n_1-a} \\
&= (n_1 - a)\sum_{i=0}^{a} {}_{n_1-a+i}C_i\, p^i q^{n_1-a} \\
&= \frac{n_1 - a}{q}\sum_{i=0}^{a} {}_{n_1-a+i}C_i\, p^i q^{n_1-a+1} \qquad (41)
\end{aligned}$$

By the kind of reasoning used in Sec. 3.2.1 above, it can be shown that the summation in the last expression is the probability of getting a or fewer defectives in $n_1 + 1$ trials. Hence,

$$\sum_{i=0}^{a} (n_1 - a + i)_{n_1-a+i-1}C_i\, p^i q^{n_1-a} = \frac{n_1 - a}{q} P''_{n_1+1:a} \tag{42}$$

Replacing the summations in (39) by (40) and (42) gives

$$\text{ASN} = \frac{r}{p} P'_{n_1+1:r+1} + \frac{n_1 - a}{q} P''_{n_1+1:a} \tag{43}$$

Part V

TABLES

TABLE 1

SAMPLE-SIZE LETTER, BY INSPECTION LEVEL AND SIZE OF INSPECTION LOT

Size of inspection lot	Sample-size letter for inspection level				
	I	II	III	IV	V
Under 25	A	A	B	C	D
25– 50	A	B	C	D	E
50– 100	B	C	D	E	F
100– 200	C	D	E	F	G
200– 300	D	E	F	G	H
300– 500	E	F	G	H	I
500– 800	F	G	H	I	J
800– 1,300	G	H	I	J	K
1,300– 3,200	H	I	J	K	L
3,200– 8,000	I	J	K	L	M
8,000– 22,000	J	K	L	M	N
22,000–110,000	K	L	M	N	N
110,000–550,000	L	M	N	N	O
550,000 and over	N	N	O	O	O
Approximate relative number of items	1	1½	2	3	4

CONDITIONS UNDER WHICH TABLE 1 IS USED

1. If sampling plan is specified by acceptable-quality level.

PROCEDURE

Normal Inspection

2. For inspection level specified for product, find sample-size letter for size of inspection lot to be used in inspecting product.

3. Use this sample-size letter, together with specified acceptable-quality level, to find plans in Table 2 or Table 4.

Reduced Inspection

4. If inspection level III, IV, or V is used for normal inspection, use a level two numbers lower for reduced inspection. If inspection level II is used for normal inspection, use I for reduced inspection. If inspection level I is used for normal inspection, reduced inspection is not available.

Tightened Inspection

5. Use same inspection level and sample-size letter as for normal inspection.

NOTES

6. Inspection level is a term used to designate the relative amount of inspection to be given a product. The exact amount of inspection depends also on the size of the inspection lot and the type of sampling (single, double, sequential). For a given size of inspection lot and type of sampling, the relative number of items inspected is approximately as indicated on the last line of the table.

7. Inspection level III will be used for most products for normal inspection; the other levels will be used for products that require unusually high or low relative amounts of inspection.

TABLE 3

Sample-size Letters for Average Outgoing-quality Limit (AOQL) Classes and Acceptable-quality Level (AQL) Classes

AQL class (in % defective)	AOQL class (in % defective): 0.05–0.075	0.075–0.10	0.10–0.15	0.15–0.22	0.22–0.30	0.30–0.50	0.50–0.90	0.90–1.5	1.5–2.5	2.5–3.5	3.5–5.0	5.0–7.0	7.0–11.0	11.0–16.0
0.024– 0.035	O	N	M		J									
0.035– 0.06		O	MN	L		H								
0.06 – 0.12			O	LMN	K		G							
0.12 – 0.17				O	LMN	JK		F						
0.17 – 0.22					NO	JKLM	I	E						
0.22 – 0.32						LMNO	HIJK		D					
0.32 – 0.65							LMNO	GHIJK	C		B			
0.65 – 1.2								KLMNO	FGHIJ	E		A		
1.2 – 2.2									JKLMN	FGHI	DE			
2.2 – 3.2										HIJKLM	DEFG	C		
3.2 – 4.4											FGHIJKLM	CDE	B	
4.4 – 5.3												DEFGHIJKL	BC	
5.3 – 6.4												GHIJKL	CDEF	AB
6.4 – 8.5													DEFGHIJK	ABC
8.5 –11.0													JK	

If a sampling plan is selected by specifying the AQL and the sample-size letter, the AOQL of the sampling plan can be found in Table 3.

If a sampling plan is to be selected on the basis of the AOQL, it can be obtained from Table 3 as follows:

Step 1. Select the AOQL class that includes the desired AOQL.

Step 2. For that AOQL class find the sample-size letter A or the sample-size letter nearest A (the smallest average amount of inspection per inspection lot that assures the desired AOQL).

Step 3. Find the AQL class corresponding to the selected AOQL class and sample-size letter. If this AQL class is suitable, use the single-, double-, or sequential-sampling plan in Table 2 having this AQL class and the selected sample-size letter.

Remarks

a. If in the desired AOQL class the sample-size letter A or the sample-size letter nearest A occurs more than once, choose the one associated with the most suitable AQL class.

b. The suitability of the AQL class found in Step 3 may be determined by comparing the AQL class with the process average. If the process average is within the AQL class, approximately 95 percent of all inspection lots of process-average quality will be accepted. If the process average is above the AQL class, less than 95 percent of submitted inspection lots of process-average quality will be accepted.

c. Further information on the suitability of the selected sampling plan is contained in its operating-characteristic (OC) curve, shown in Table 4.

SUPPLEMENT TO TABLE 3

Average Outgoing-quality Limits (AOQL's) of Selected Single-sampling Plans for Small Inspection Lots

Single-sampling plan: Size	Acceptance number	Rejection number	Inspection-lot sizes for which average outgoing-quality limit (ADQL) is (% defective): Less than 0.15	0.15–0.22	0.22–0.3	0.3–0.5	0.5–0.9	0.9–1.5	1.5–2.5	2.5–3.5	3.5–5.0	5.0–7.0	7.0–11.0	AOQL for very large inspection lots
5	0	1						6	7	8 to 11	11 to 20	20 & over		6.698
6**	0	1					7	8	9 to 11	11 to 16	16 to 52	52 & over		5.665
7**	0	1					8	9 to 11	11 to 15	15 to 25	25 & over			4 909
8**	0	1				9	10	11 to 13	13 to 19	19 to 42	42 & over			4 330
9**	0	1				10	11	12 to 15	15 to 26	26 to 94	94 & over			3 874
10	0	1				11	12 to 14	14 to 18	18 to 35	35 to 7,096	7,096 & over			3 505
10	1	2					11	12	13 to 15	15 to 18	18 to 26	26 to 71	71 & over	8 165
15	0	1	16		17	18 to 20	20 to 25	25 to 41	41 & over					2.374
15	1	2				16	17	18 to 21	21 to 28	28 to 42	42 to 168	168 & over		5 492
15	2	3					16	17	18 to 21	21 to 25	25 to 34	34 to 66	66 & over	9 079
20	0	1	21	22	23 to 25	25 to 28	28 to 41	41 to 122	122 & over					1.795
20	1	2		21		22	23 to 26	26 to 32	32 to 51	51 to 130	130 & over			4.138
20	2	3				21	22 to 24	24 to 26	26 to 32	32 to 42	42 to 76	76 & over		6 818
20	3	4				21	22	23	24 to 27	27 to 32	32 to 42	42 to 71	71 & over	9 746
30	0	1	31 to 35	35 to 37	37 to 40	40 to 52	52 to 119	119 & over						1 206
30	1	2	31	32	33	34 to 37	37 to 45	45 to 66	66 to 306	306 & over				2 772
30	2	3	31		32	33	34 to 38	38 to 45	45 to 67	67 to 130	130 & over			4 552
30	3	4		31		32	33 to 35	35 to 40	40 to 49	49 to 66	66 to 131	131 & over		6.487
30	4	5			31		32 to 34	34 to 37	37 to 43	43 to 51	51 to 73	73 to 167	167 & over	8.542
40	0	1	41 to 48	48 to 53	53 to 60	60 to 89	89 to 4,343	4,343 & over						0 908
40	1	2	41 to 44	44	45 to 47	47 to 53	53 to 71	71 to 143	143 & over					2 084
40	2	3	41	42	43	44 to 47	47 to 55	55 to 72	72 to 150	150 & over				3.417
40	3	4	41		42	43 to 45	45 to 50	50 to 58	58 to 83	83 to 143	143 & over			4 862
40	4	5		41		42 to 44	44 to 47	47 to 53	53 to 66	66 to 89	89 to 184	184 & over		6.393
40	5	6		41		42	43 to 46	46 to 50	50 to 59	59 to 72	72 to 107	107 to 322	322 & over	7.994
55	0	1	56 to 72	72 to 83	83 to 101	101 to 224	224 & over							0.663
55	1	2	56 to 62	62 to 65	65 to 69	69 to 82	82 to 135	135 to 4,448	4,448 & over					1 519
55	2	3	56 to 59	59 to 61	61 to 63	63 to 69	69 to 87	87 to 139	139 & over					2.487
55	3	4	56 to 58	58	59 to 61	61 to 65	65 to 74	74 to 96	96 to 188	188 to 5,665	5,665 & over			3 534
55	4	5	56	57	58	59 to 62	62 to 69	69 to 82	82 to 120	120 to 224	224 & over			4 642
55	5	6	56	57	58	59 to 61	61 to 66	66 to 75	75 to 97	97 to 139	139 to 400	400 & over		5.798
55	8	9		56		57 to 59	59 to 61	61 to 66	66 to 75	75 to 88	88 to 117	117 to 210	210 & over	9 493

** These plans are not included in Table 2*a*, Table 3, or Table 4. They are included here to permit the attainment of a wider range of AOQL values with small inspection lots and small samples.

Notes: For small inspection lots, the AOQL of a sampling plan depends on the size of the inspection lot. This supplement to Table 3 shows, for selected single-sampling plans, the inspection-lot sizes for which the AOQL is in each AOQL class. The size ranges are inclusive of the lower limits but exclusive of the upper limits shown in the table.

TABLE 4

Sampling Plans and Operating-characteristic (OC) Curves, Classified by Sample-size Letter, and by Acceptable-quality Level (AQL) and Average Outgoing-quality Limit (AOQL)

Notes

1. Each complete page of Table 4 contains single-, double-, and sequential-sampling plans having approximately the same operating-characteristic (OC) curves.

2. Instructions on pages of Table 4 directing the use of other sample-size letters or the use of other types of sampling apply only when the sampling plan is specified by acceptable-quality level (AQL).

3. The plans in Table 4 are summarized in Table 2 and Table 3.

4. The notes to Table 1, to the sections of Table 2, and to Table 3 give details of procedure.

5. The accuracy of the four-color charts, as actually printed for the first edition, is discussed on page x.

TABLE 4 (*continued*)

A	Sample-size letter
0.024–0.035	AQL classes (percent defective)
0.035–0.06	
0.06 –0.12	

Smallest sample size giving an AQL in any of these AQL classes will exceed inspection-lot size. 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

Sample-size letter	**A**
AQL classes (percent defective)	**0.12–0.17**
	0.17–0.22
	0.22–0.32
	0.32–0.65

For AQL class	Use sample-size letter
0.12–0.17	F
0.17–0.22	E
0.22–0.32	D
0.32–0.65	B

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

A Sample-size letter
0.65–1.2 AQL class (percent defective)
5.0 –7.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	5	5	0	1
Double	Use single sampling				
Sequential	Use single sampling				

b. Operating-characteristic Curves

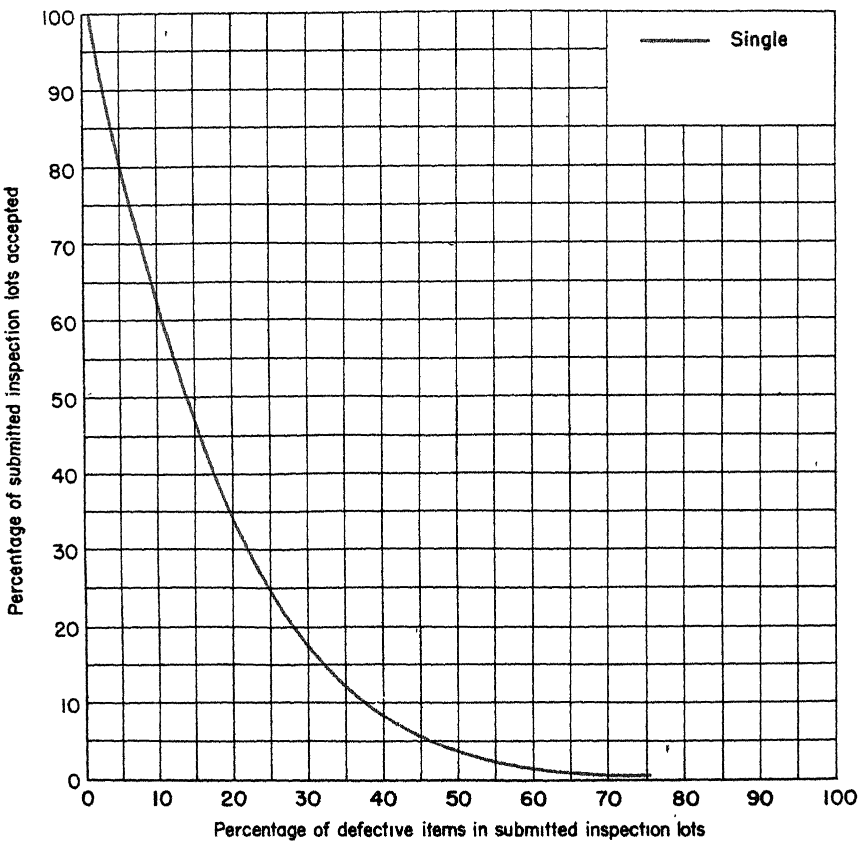


TABLE 4 (*continued*)

Sample-size letter	**A**
AQL classes (percent defective)	**1.2–2.2**
	2.2–3.2
	3.2–4.4
	4.4–5.3

For AQL class	Use sample-size letter
1.2–2.2	D
2.2–3.2	C
3.2–4.4	B
4.4–5.3	B

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

A Sample-size letter
5.3– 6.4 AQL class (percent defective)
11.0–16.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	5	5	1	2
Double	First	3	3	0	2
	Second	6	9	1	2
Sequential	First	3	3	0	2
	Second	2	5	0	2
	Third	2	7	1	2

b. Operating-characteristic Curves

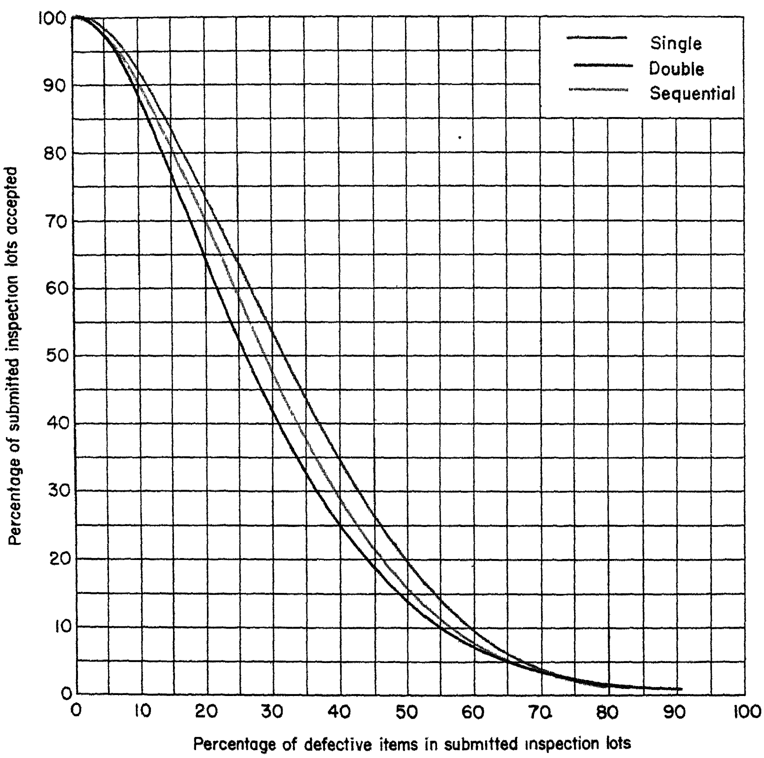


TABLE 4 (*continued*)

Sample-size letter **A**
AQL class (percent defective) **6.4– 8.5**
AOQL class (percent defective) **11.0–16.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	5	5	1	2
Double	First	3	3	0	2
	Second	6	9	1	2
Sequential	First	3	3	0	2
	Second	2	5	0	2
	Third	2	7	2	3

b. Operating-characteristic Curves

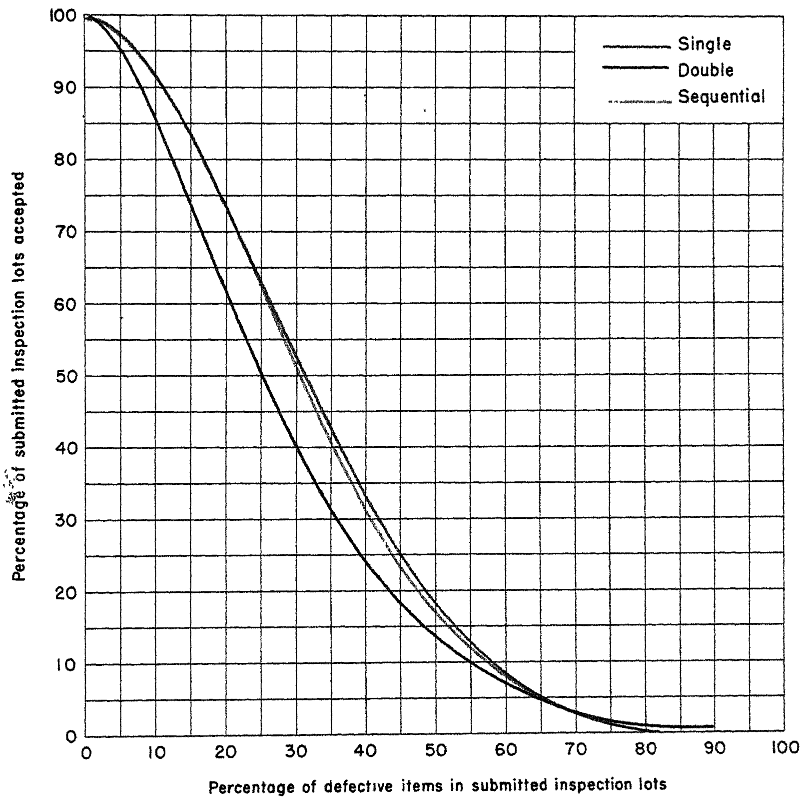


TABLE 4 (*continued*)

B Sample-size letter
0.024–0.035 AQL class (percent defective)

Smallest sample size giving this AQL will exceed inspection-lot size. 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

Sample-size letter	**B**
AQL classes (percent defective)	**0.035–0.06**
	0.06 –0.12
	0.12 –0.17
	0.17 –0.22
	0.22 –0.32

For AQL class	Use sample-size letter
0.035–0.06	H
0.06 –0.12	G
0.12 –0.17	F
0.17 –0.22	E
0.22 –0.32	D

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

B Sample-size letter TABLE 4 (*continued*)

0.32–0.65 AQL class (percent defective)

3.5 –5.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	10	10	0	1
Double	Use single sampling				
Sequential	Use single sampling				

b. Operating-characteristic Curves

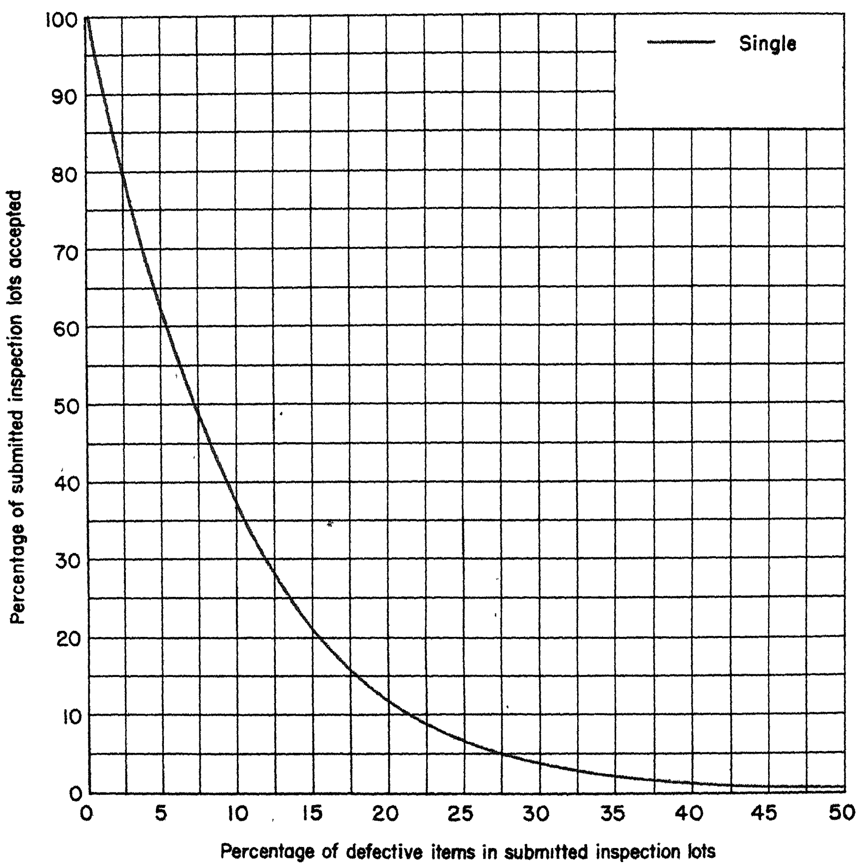


TABLE 4 (*continued*)

Sample-size letter	**B**
AQL classes (percent defective)	**0.65–1.2**
	1.2 –2.2
	2.2 –3.2

For AQL class	Use sample-size letter
0.65–1.2	E
1.2 –2.2	D
2.2 –3.2	C

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

B Sample-size letter TABLE 4 (*continued*)
3.2– 4.4 AQL class (percent defective)
7.0–11.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	10	10	1	2
Double	First....................	7	7	0	3
	Second..................	14	21	2	3
Sequential	First....................	4	4	*	2
	Second..................	4	8	0	2
	Third....................	4	12	1	3
	Fourth..................	4	16	1	3
	Fifth....................	4	20	3	4

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

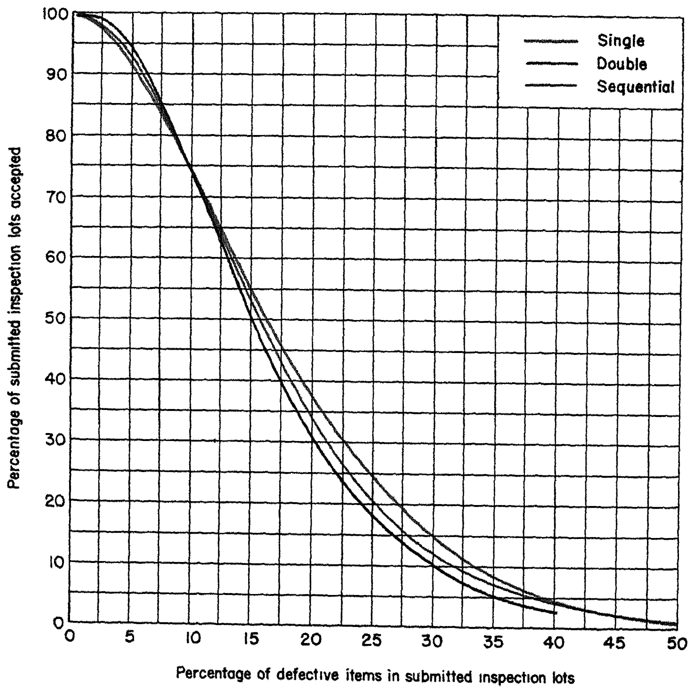


TABLE 4 (*continued*)

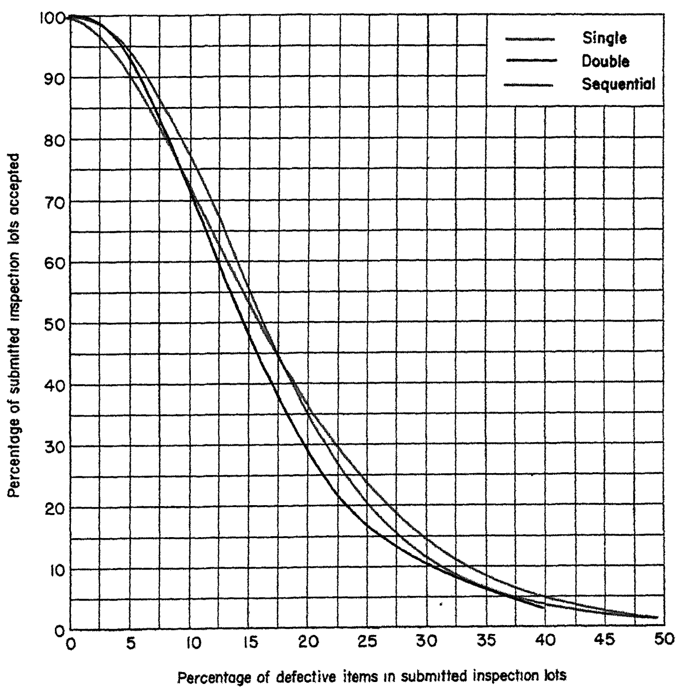
Sample-size letter **B**
AQL class (percent defective) **4.4– 5.3**
AOQL class (percent defective) **7.0–11.0**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	10	10	1	2
Double	First....................	7	7	0	3
	Second..................	14	21	2	3
Sequential	First....................	4	4	*	2
	Second..................	4	8	0	3
	Third...................	4	12	1	3
	Fourth..................	4	16	1	3
	Fifth....................	4	20	3	4

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES



B Sample-size letter
5.3– 6.4 AQL class (percent defective)
11.0–16.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	10	10	2	3
Double	First	7	7	1	3
	Second	14	21	2	3
Sequential	First	4	4	0	2
	Second	4	8	0	3
	Third	4	12	1	4
	Fourth	4	16	2	4
	Fifth	4	20	4	5

b. Operating-characteristic Curves

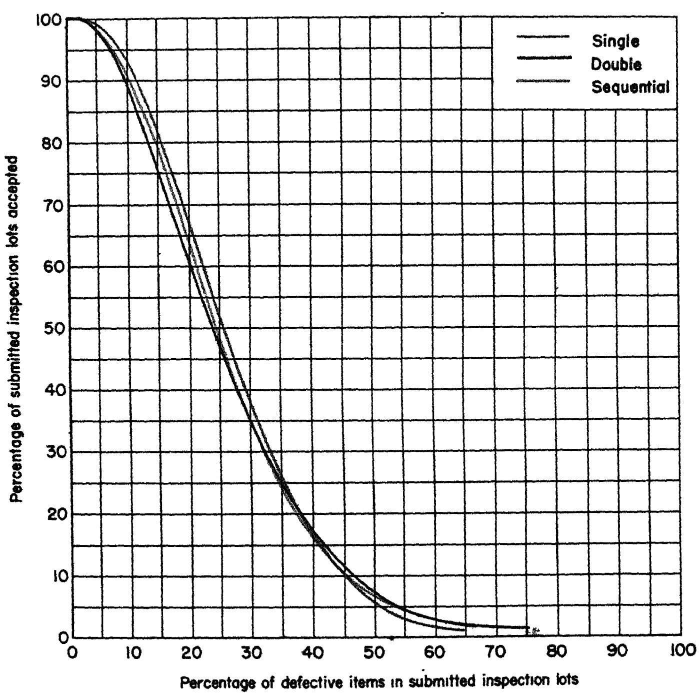


TABLE 4 (*continued*)

Sample-size letter	**B**
AQL class (percent defective)	**6.4– 8.5**
AOQL class (percent defective)	**11.0–16.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	10	10	2	3
Double	First	7	7	1	4
	Second	14	21	3	4
Sequential	First	4	4	0	2
	Second	4	8	0	3
	Third	4	12	1	4
	Fourth	4	16	2	5
	Fifth	4	20	4	5

b. Operating-characteristic Curves

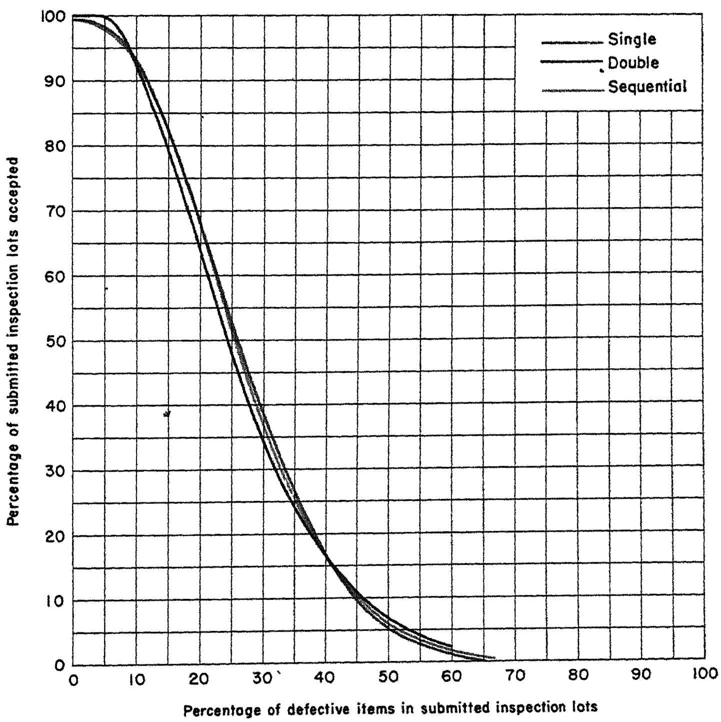


TABLE 4 (*continued*)

For AQL class	Use sample-size letter
0.024–0.035	J
0.035–0.06	H
0.06 –0.12	G
0.12 –0.17	F
0.17 –0.22	E
0.22 –0.32	D

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

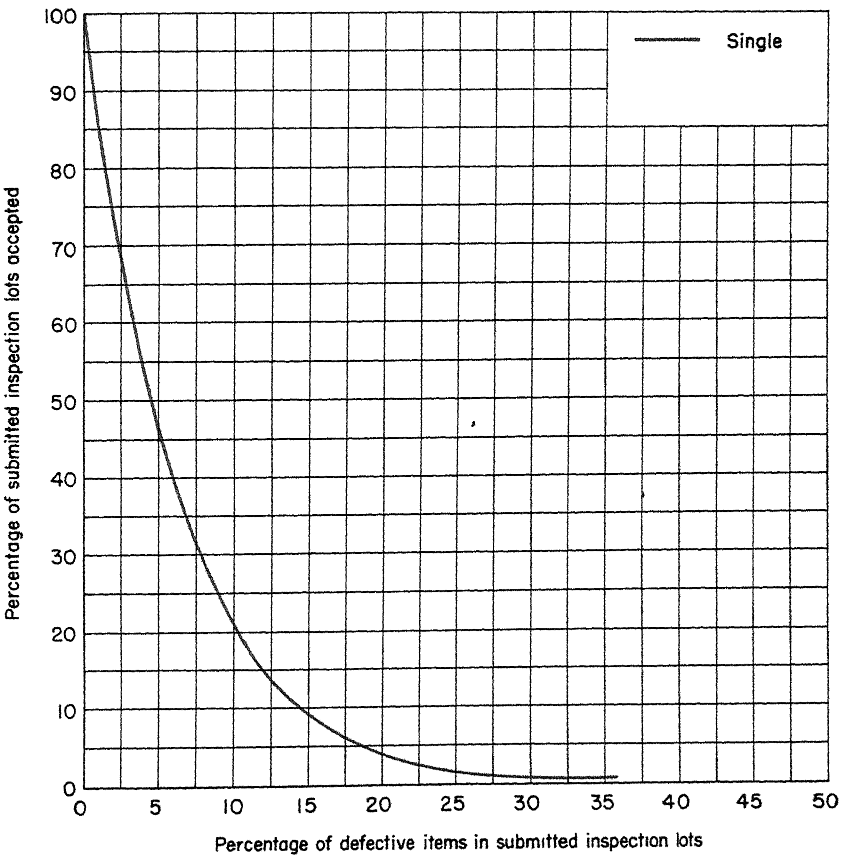
TABLE 4 (*continued*)

Sample-size letter **C**
AQL class (percent defective) **0.32–0.65**
AOQL class (percent defective) **1.5 –2.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	15	15	0	1
Double	Use single sampling				
Sequential	Use single sampling				

b. Operating-characteristic Curves



C Sample-size letter TABLE 4 (*continued*)
0.65–1.2 AQL classes (percent defective)
1.2 –2.2

For AQL class	Use sample-size letter
0.65–1.2	E
1.2 –2.2	D

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

Sample-size letter **C**
AQL class (percent defective) **2.2–3.2**
AOQL class (percent defective) **5.0–7.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First	15	15	1	2
Double	First	10	10	0	3
	Second	20	30	2	3
Sequential	First	5	5	*	2
	Second	5	10	0	2
	Third	5	15	0	3
	Fourth	5	20	1	3
	Fifth	5	25	2	3

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

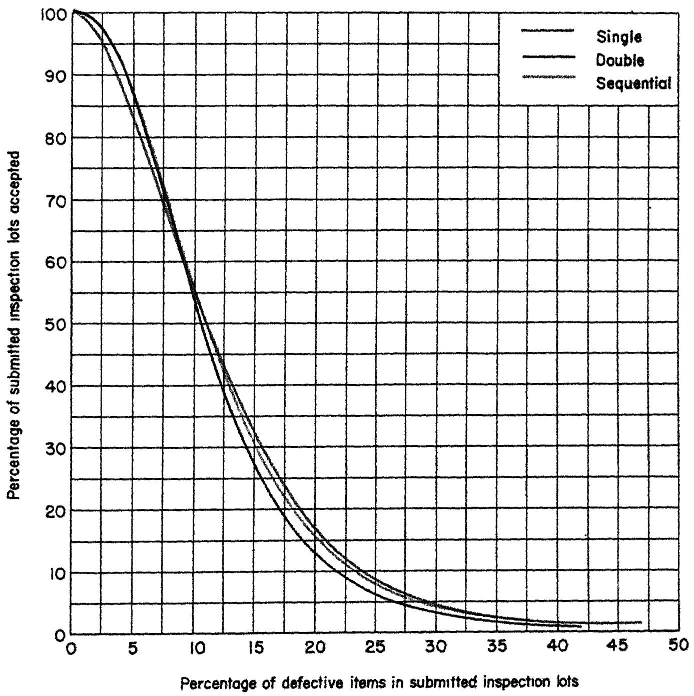


TABLE 4 (*continued*)

C Sample-size letter
3.2–4.4 AQL class (percent defective)
5.0–7.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	15	15	1	2
Double	First	10	10	0	3
	Second	20	30	2	3
Sequential	First	5	5	*	2
	Second	5	10	0	2
	Third	5	15	0	3
	Fourth	5	20	1	3
	Fifth	5	25	3	4

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

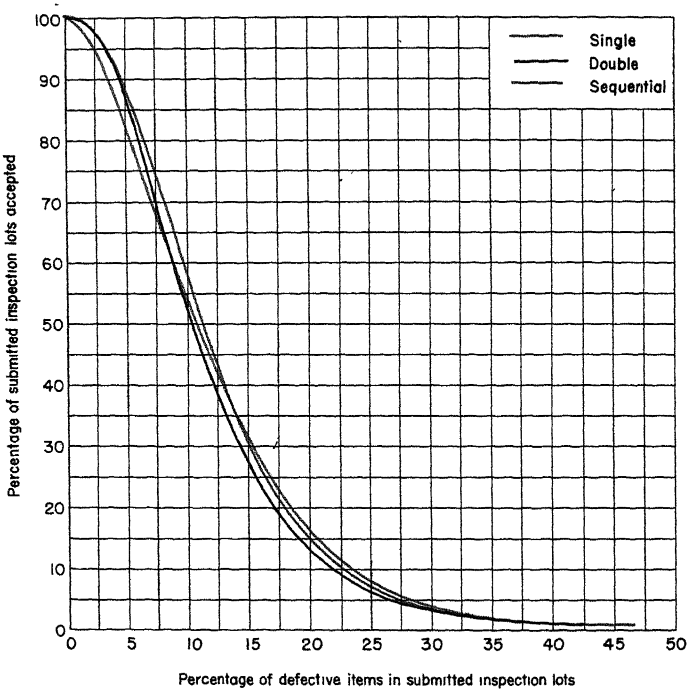


TABLE 4 (*continued*)

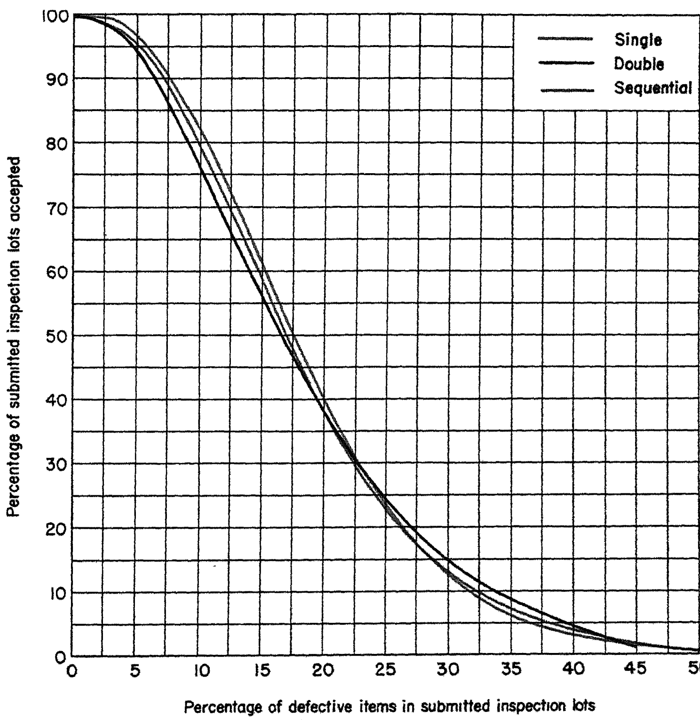
Sample-size letter **C**
AQL class (percent defective) **4.4– 5.3**
AOQL class (percent defective) **7.0–11.0**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First....................	15	15	2	3
Double	First....................	10	10	1	3
	Second..................	20	30	2	3
Sequential	First....................	5	5	*	2
	Second..................	5	10	1	3
	Third....................	5	15	1	3
	Fourth..................	5	20	1	3
	Fifth....................	5	25	3	4

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES



C Sample-size letter
5.3– 6.4 AQL class (percent defective)
7.0–11.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	15	15	2	3
Double	First	10	10	1	4
	Second	20	30	3	4
Sequential	First	5	5	*	2
	Second	5	10	1	3
	Third	5	15	1	4
	Fourth	5	20	1	4
	Fifth	5	25	3	4

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES

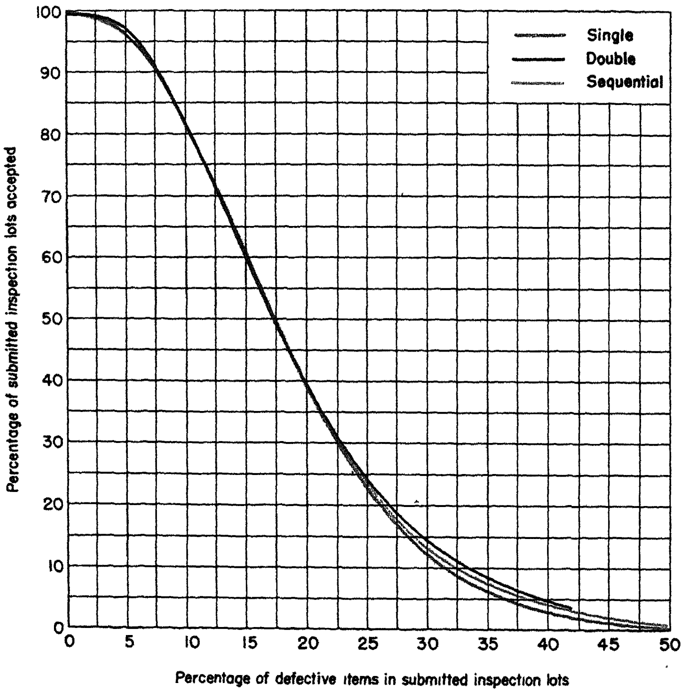


TABLE 4 (*continued*)

Sample-size letter	**C**
AQL class (percent defective)	**6.4– 8.5**
AOQL class (percent defective)	**11.0–16.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First....................	15	15	3	4
Double	First....................	10	10	1	6
	Second..................	20	30	5	6
Sequential	First....................	5	5	0	3
	Second..................	5	10	0	3
	Third....................	5	15	2	4
	Fourth..................	5	20	3	6
	Fifth....................	5	25	5	6

b. Operating-characteristic Curves

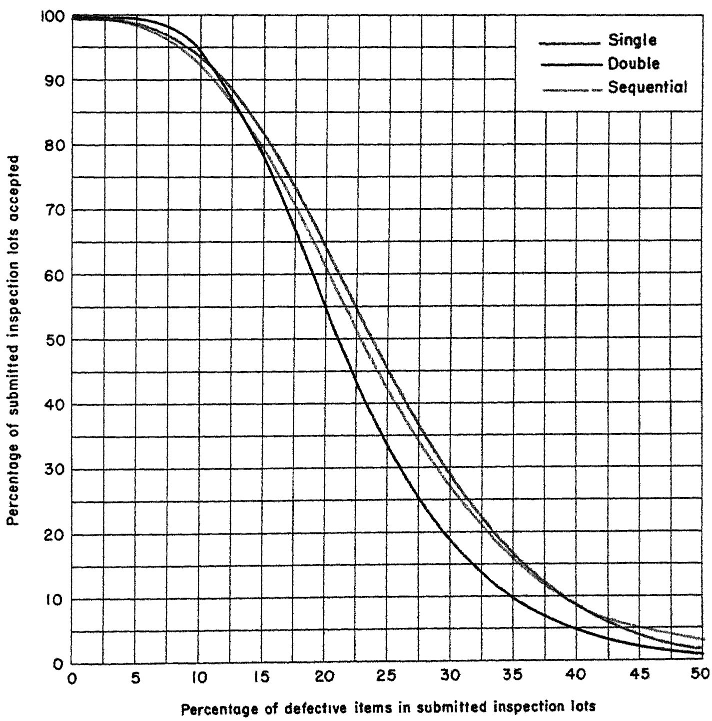


TABLE 4 (*continued*)

D	Sample-size letter
0.024–0.035	AQL classes (percent defective)
0.035–0.06	
0.06 –0.12	
0.12 –0.17	
0.17 –0.22	

For AQL class	Use sample-size letter
0.024–0.035	J
0.035–0.06	H
0.06 –0.12	G
0.12 –0.17	F
0.17 –0.22	E

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

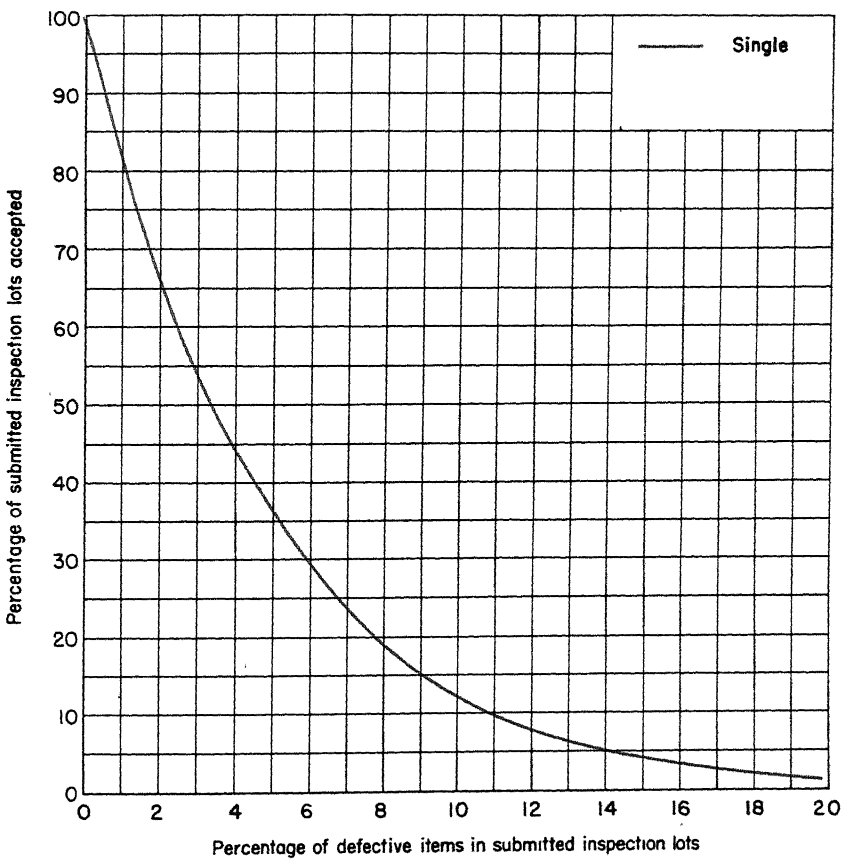
TABLE 4 (*continued*)

Sample-size letter **D**
AQL class (percent defective) **0.22–0.32**
AOQL class (percent defective) **1.5 –2.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	20	20	0	1
Double	Use single sampling				
Sequential	Use single sampling				

b. Operating-characteristic Curves



D Sample-size letter
0.32–0.65 AQL classes (percent defective)
0.65–1.2

TABLE 4 (*continued*)

For AQL class	Use sample-size letter
0.32–0.65	G
0.65–1.2	E

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

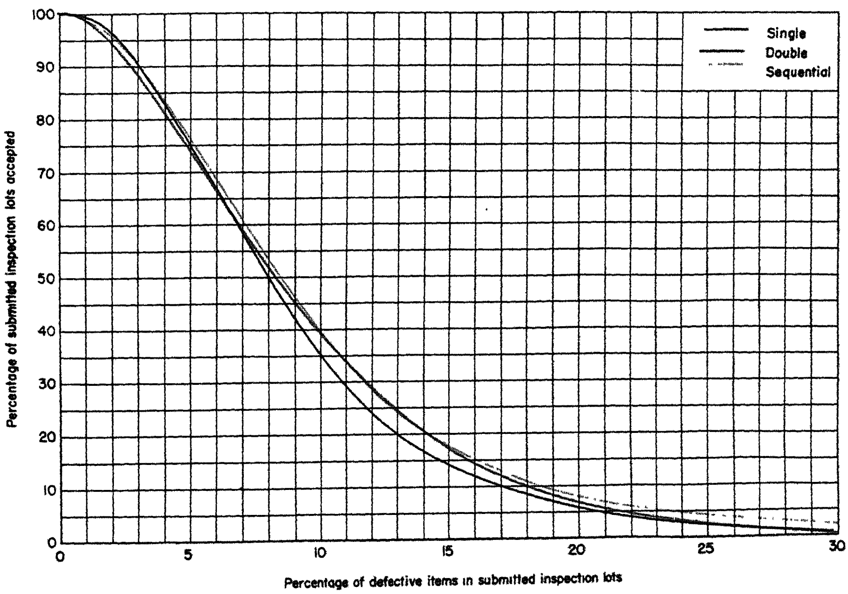
Sample-size letter **D**
AQL class (percent defective) **1.2–2.2**
AOQL class (percent defective) **3.5–5.0**

SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	20	20	1	2
Double	First....................	13	13	0	3
	Second..................	26	39	2	3
Sequential	First....................	6	6	*	2
	Second..................	6	12	0	2
	Third....................	6	18	0	2
	Fourth..................	6	24	0	3
	Fifth....................	6	30	1	3
	Sixth....................	6	36	2	3

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES



D Sample-size letter
2.2–3.2 AQL class (percent defective)
3.5–5.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	20	20	1	2
Double	First	13	13	0	3
	Second	26	39	2	3
Sequential	First	6	6	*	2
	Second	6	12	0	2
	Third	6	18	0	3
	Fourth	6	24	0	3
	Fifth	6	30	1	4
	Sixth	6	36	3	4

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

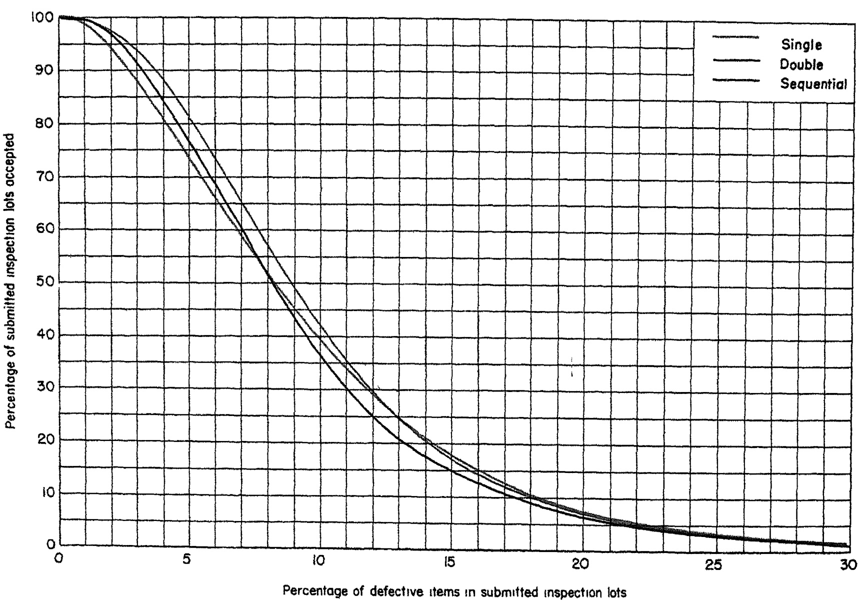


TABLE 4 (*continued*)

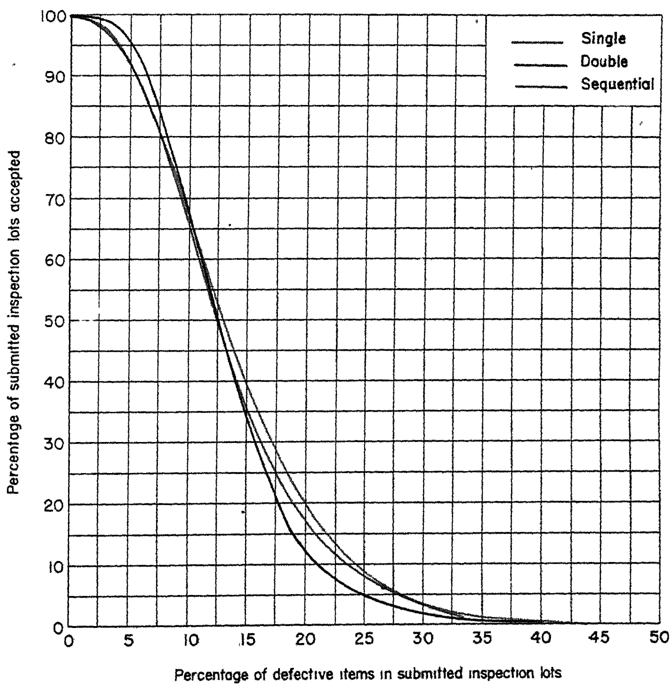
Sample-size letter **D**
AQL class (percent defective) **3.2–4.4**
AOQL class (percent defective) **5.0–7.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	20	20	2	3
Double	First	13	13	0	5
	Second	26	39	4	5
Sequential	First	6	6	*	2
	Second	6	12	0	3
	Third	6	18	1	3
	Fourth	6	24	2	4
	Fifth	6	30	2	4
	Sixth	6	36	3	4

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



D Sample-size letter
4.4–5.3 AQL class (percent defective)
5.0–7.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	20	20	2	3
Double	First	13	13	0	5
	Second	26	39	4	5
Sequential	First	6	6	*	2
	Second	6	12	0	3
	Third	6	18	1	4
	Fourth	6	24	2	4
	Fifth	6	30	3	5
	Sixth	6	36	4	5

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

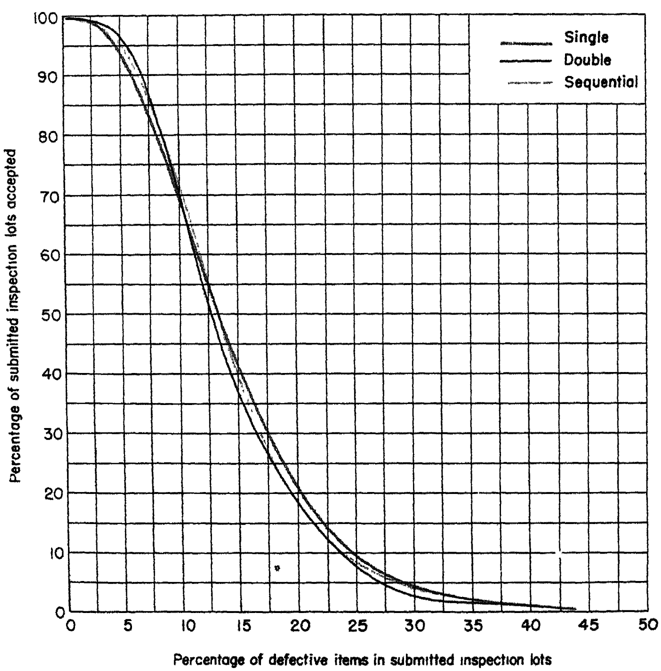


TABLE 4 (*continued*)

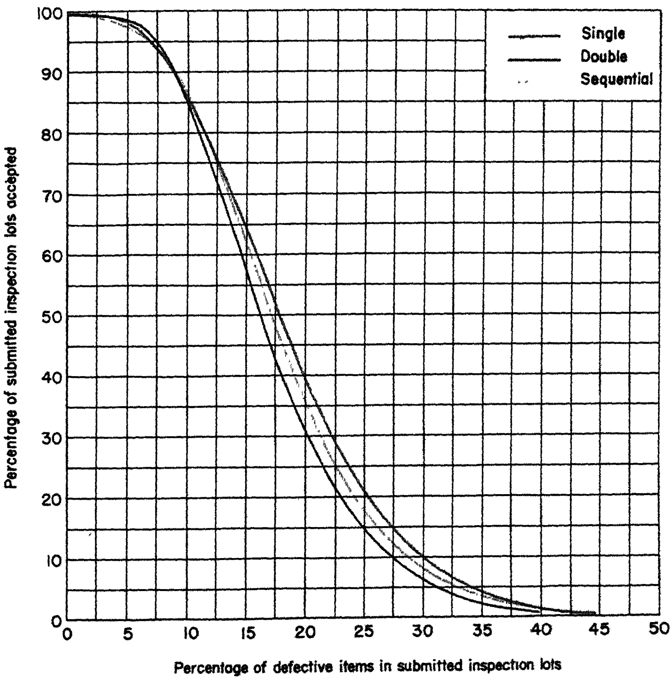
Sample-size letter	**D**
AQL class (percent defective)	**5.3– 6.4**
AOQL class (percent defective)	**7.0–11.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	20	20	3	4
Double	First....................	13	13	1	6
	Second..................	26	39	5	6
Sequential	First....................	6	6	*	3
	Second..................	6	12	1	3
	Third......	6	18	1	5
	Fourth..................	6	24	2	6
	Fifth....................	6	30	3	6
	Sixth....................	6	36	5	6

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



D Sample-size letter TABLE 4 (*continued*)
6.4– 8.5 AQL class (percent defective)
7.0–11.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	20	20	3	4
Double	First	13	13	1	6
	Second	26	39	5	6
Sequential	First	6	6	*	3
	Second	6	12	1	4
	Third	6	18	2	4
	Fourth	6	24	2	5
	Fifth	6	30	3	6
	Sixth	6	36	5	6

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

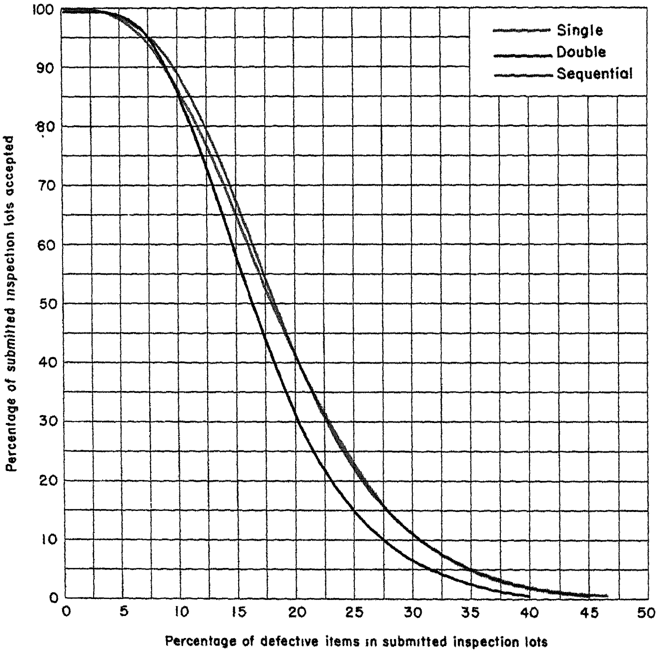


TABLE 4 *(continued)*

Sample-size letter	E
AQL classes (percent defective)	**0.024–0.035**
	0.035–0.06
	0.06 –0.12
	0.12 –0.17

For AQL class	Use sample-size letter
0.024–0.035	J
0.035–0.06	H
0.06 –0.12	G
0.12 –0.17	F

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

E Sample-size letter

0.17–0.22 AQL class (percent defective)

0.90–1.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	30	30	0	1
Double	Use single sampling				
Sequential	Use single sampling				

b. Operating-characteristic Curves

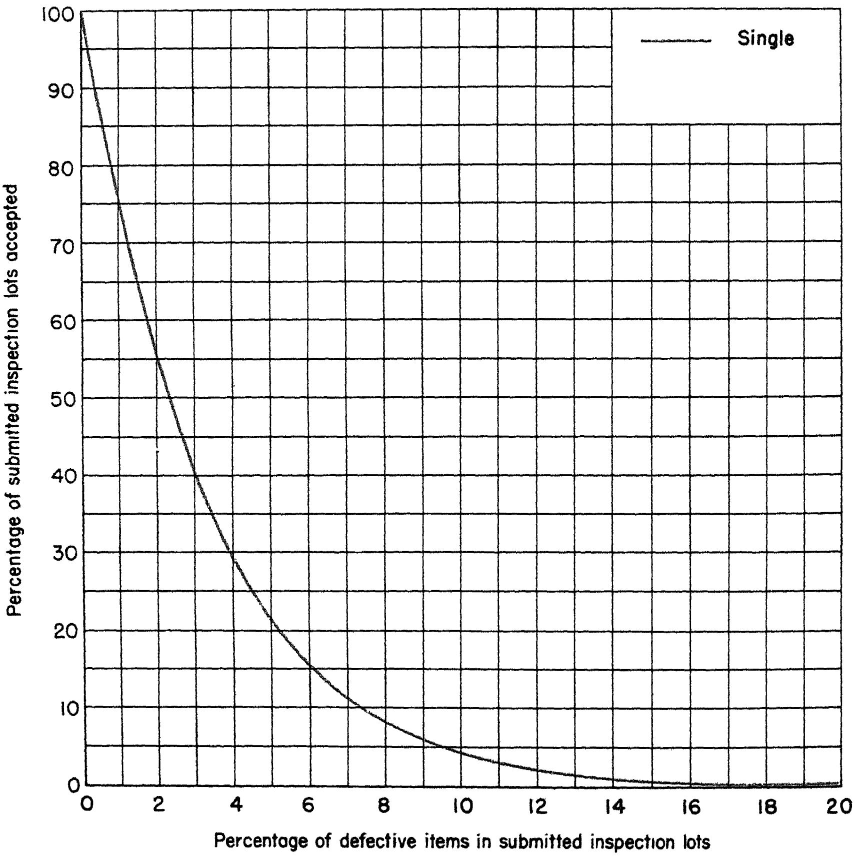


TABLE 4 (*continued*)

Sample-size letter **E**
AQL classes (percent defective) **0.22–0.32**
0.32–0.65

For AQL class	Use sample-size letter
0.22–0.32	H
0.32–0.65	G

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

E Sample-size letter
0.65–1.2 AQL class (percent defective)
2.5 –3.5 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	30	30	1	2
Double	First	20	20	0	2
	Second	40	60	1	2
Sequential	First	8	8	*	2
	Second	8	16	*	2
	Third	8	24	0	2
	Fourth	8	32	1	3
	Fifth	8	40	1	3
	Sixth	8	48	1	3
	Seventh	8	56	2	3

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

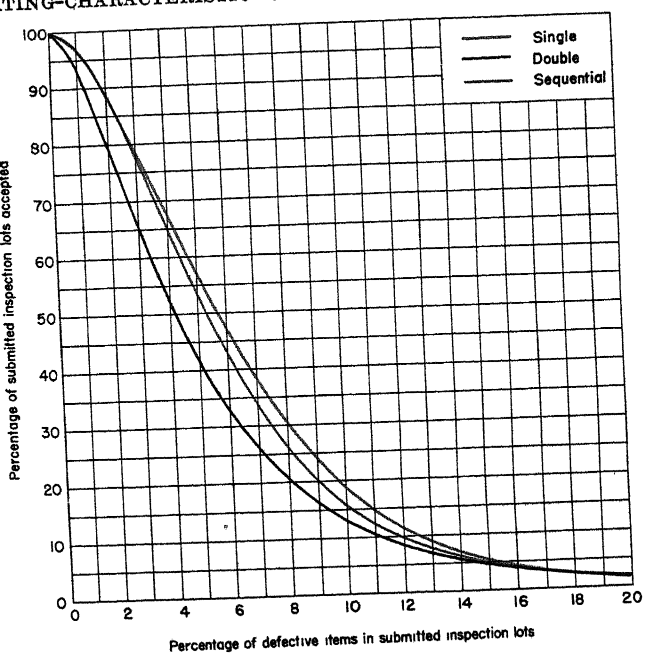


TABLE 4 (*continued*)

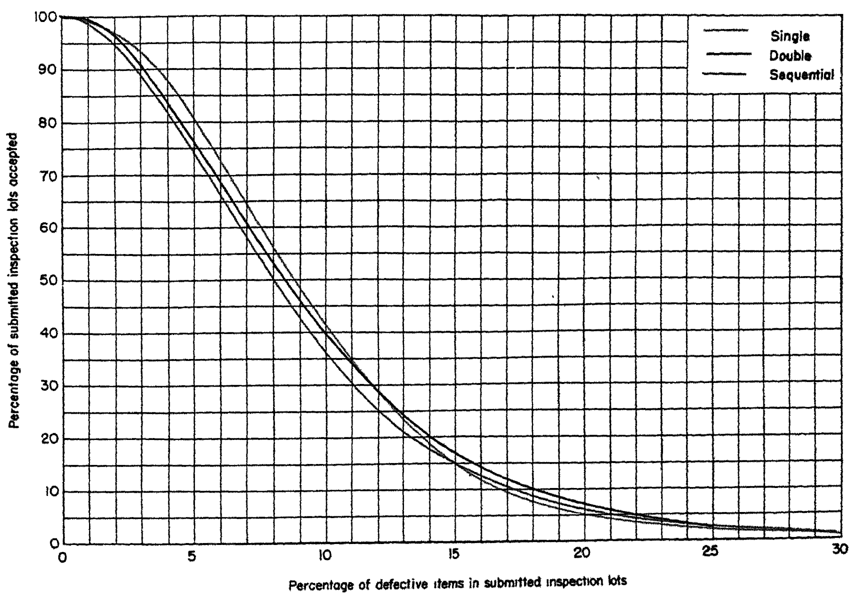
Sample-size letter **E**
AQL class (percent defective) **1.2–2.2**
AOQL class (percent defective) **3.5–5.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	30	30	2	3
Double	First....................	20	20	1	3
	Second.................	40	60	2	3
Sequential	First....................	8	8	*	2
	Second.................	8	16	0	2
	Third..................	8	24	1	3
	Fourth.................	8	32	2	4
	Fifth..................	8	40	2	4
	Sixth..................	8	48	2	4
	Seventh................	8	56	3	4

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



E Sample-size letter
2.2–3.2 AQL class (percent defective)
3.5–5.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	30	30	2	3
Double	First	20	20	1	4
	Second	40	60	3	4
Sequential	First	8	8	*	2
	Second	8	16	0	3
	Third	8	24	1	3
	Fourth	8	32	2	4
	Fifth	8	40	2	4
	Sixth	8	48	2	4
	Seventh	8	56	3	4

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

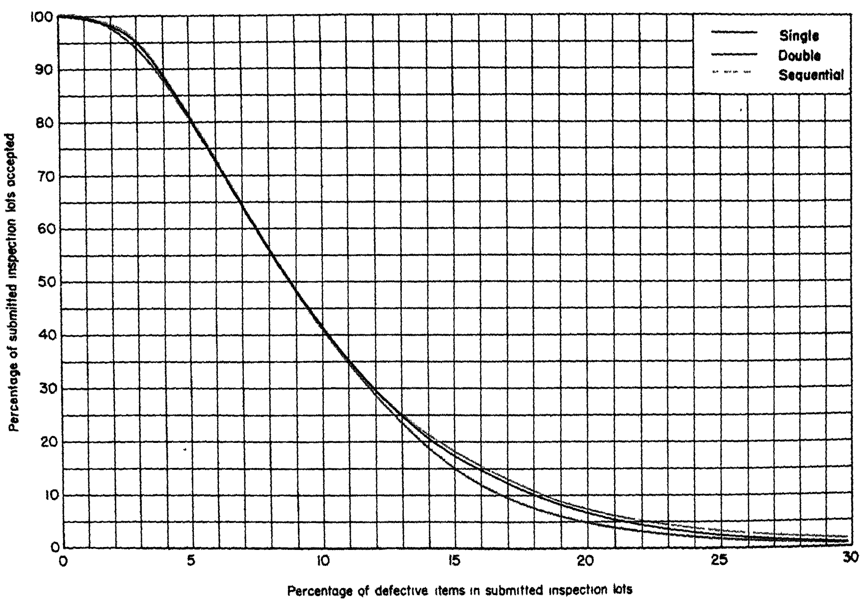


TABLE 4 (*continued*)

Sample-size letter **E**
AQL class (percent defective) **3.2–4.4**
AOQL class (percent defective) **5.0–7.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	30	30	3	4
Double	First	20	20	1	5
	Second	40	60	4	5
Sequential	First	8	8	*	2
	Second	8	16	0	4
	Third	8	24	1	4
	Fourth	8	32	2	5
	Fifth	8	40	3	6
	Sixth	8	48	5	7
	Seventh	8	56	6	7

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

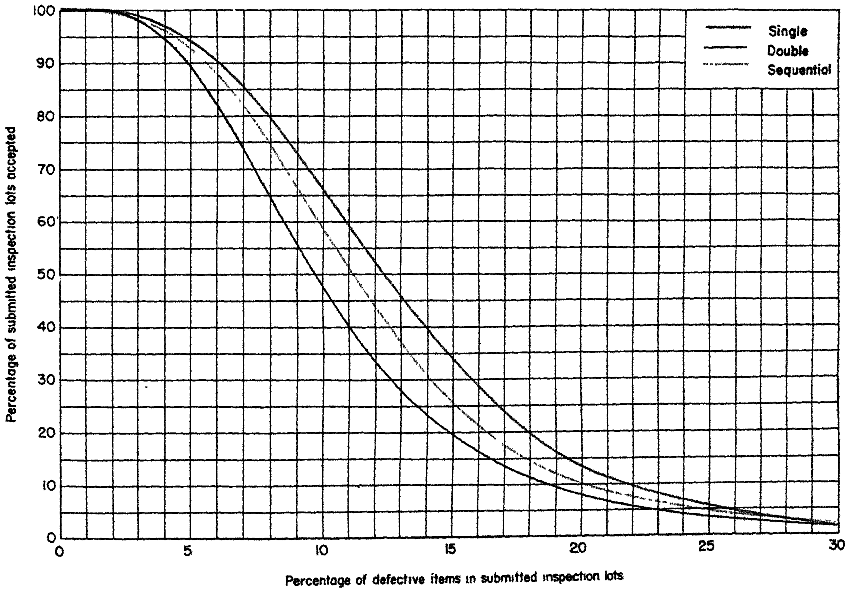


TABLE 4 (*continued*)

E Sample-size letter
4.4–5.3 AQL class (percent defective)
5.0–7.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	30	30	3	4
Double	First	20	20	1	6
	Second	40	60	5	6
Sequential	First	8	8	*	3
	Second	8	16	0	3
	Third	8	24	1	4
	Fourth	8	32	2	5
	Fifth	8	40	4	6
	Sixth	8	48	5	7
	Seventh	8	56	6	7

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

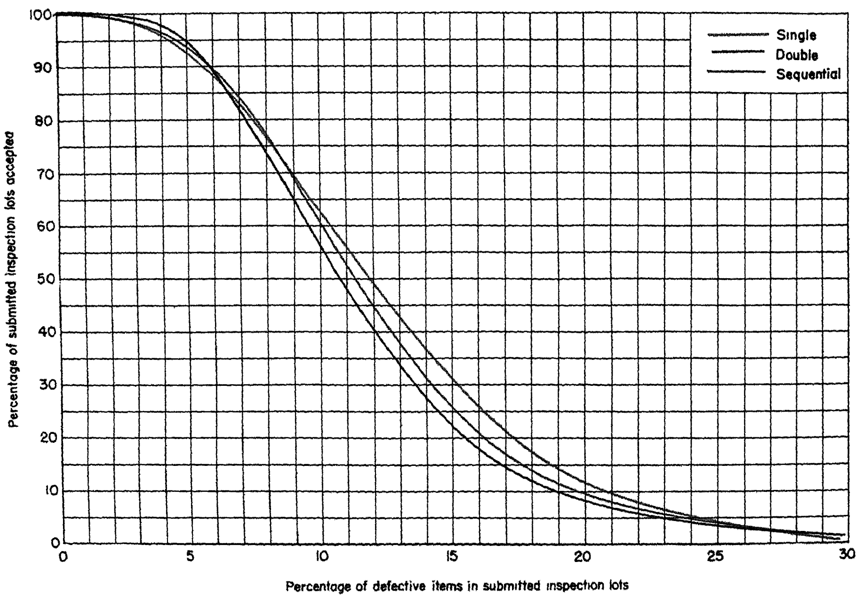


TABLE 4 (*continued*)

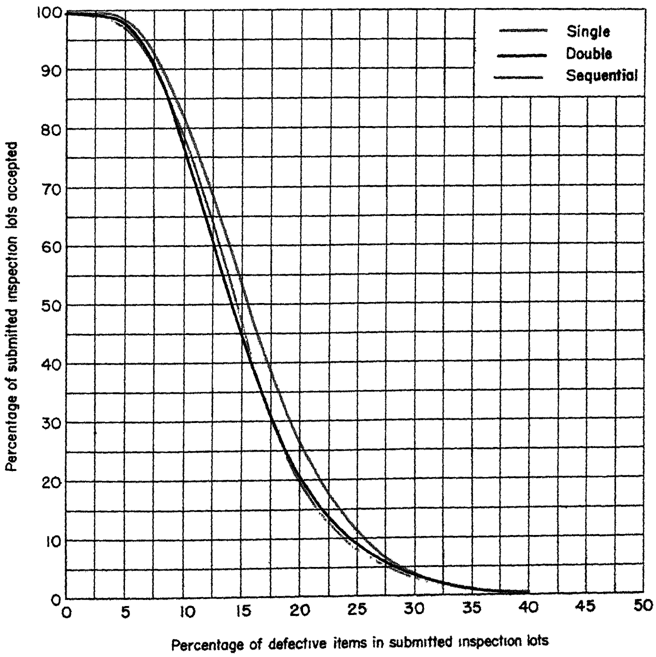
Sample-size letter **E**
AQL class (percent defective) **5.3– 6.4**
AOQL class (percent defective) **7.0–11.0**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	30	30	4	5
Double	First	20	20	2	7
	Second	40	60	6	7
Sequential	First	8	8	*	3
	Second	8	16	1	4
	Third	8	24	2	5
	Fourth	8	32	3	6
	Fifth	8	40	4	6
	Sixth	8	48	6	8
	Seventh	8	56	7	8

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES



E Sample-size letter TABLE 4 (*continued*)
6.4– 8.5 AQL class (percent defective)
7.0–11.0 AOQL class (percent defective)

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	30	30	5	6
Double	First	20	20	3	8
	Second	40	60	7	8
Sequential	First	8	8	0	3
	Second	8	16	1	5
	Third	8	24	2	6
	Fourth	8	32	3	7
	Fifth	8	40	5	8
	Sixth	8	48	6	8
	Seventh	8	56	8	9

b. OPERATING-CHARACTERISTIC CURVES

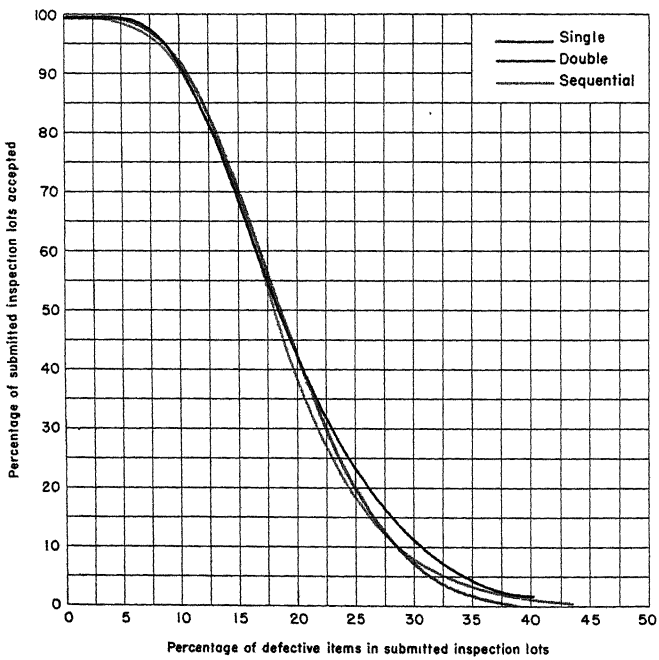


TABLE 4 (*continued*)

Sample-size letter	**F**
AQL classes (percent defective)	**0.024–0.035**
	0.035–0.06
	0.06 –0.12

For AQL class	Use sample-size letter
0.024–0.035	J
0.035–0.06	H
0.06 –0.12	G

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

F Sample-size letter
0.12–0.17 AQL class (percent defective)
0.90–1.5 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First....................	40	40	0	1
Double	Use single sampling				
Sequential	Use single sampling				

b. Operating-characteristic Curves

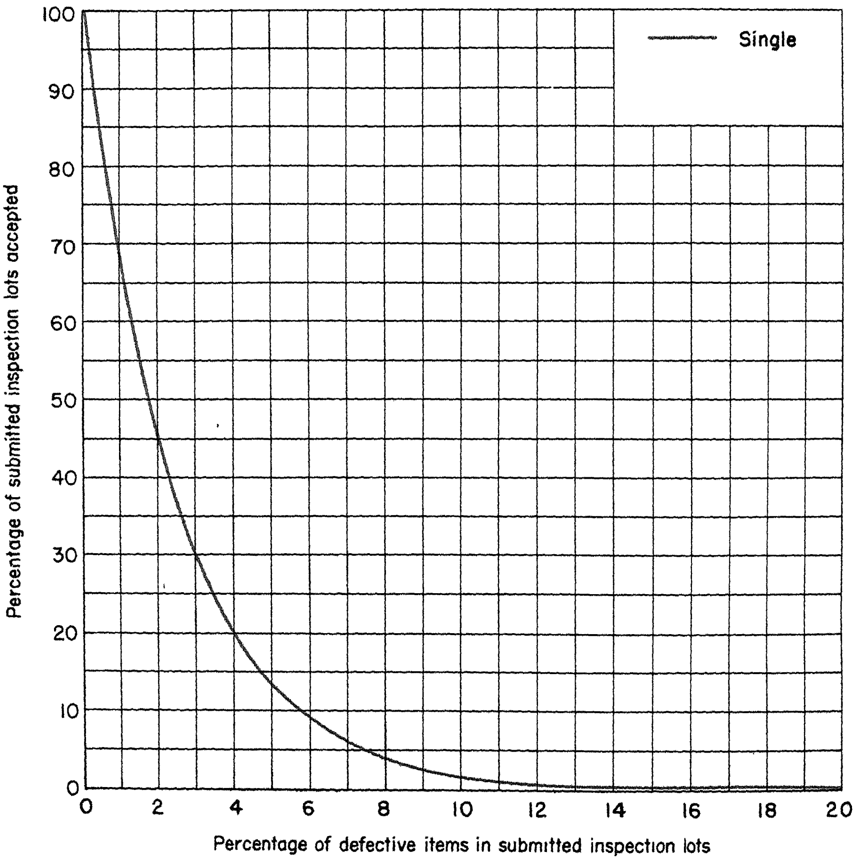


TABLE 4 (*continued*)

Sample-size letter	**F**
AQL classes (percent defective)	**0.17–0.22**
	0.22–0.32
	0.32–0.65

For AQL class	Use sample-size letter
0.17–0.22	I
0.22–0.32	H
0.32–0.65	G

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

F Sample-size letter
0.65–1.2 AQL class (percent defective)
1.5 –2.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	40	40	1	2
Double	First	25	25	0	3
	Second	50	75	2	3
Sequential	First	10	10	*	2
	Second	10	20	*	2
	Third	10	30	0	2
	Fourth	10	40	0	3
	Fifth	10	50	1	3
	Sixth	10	60	2	4
	Seventh	10	70	3	4

* Acceptance not permitted until three samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES

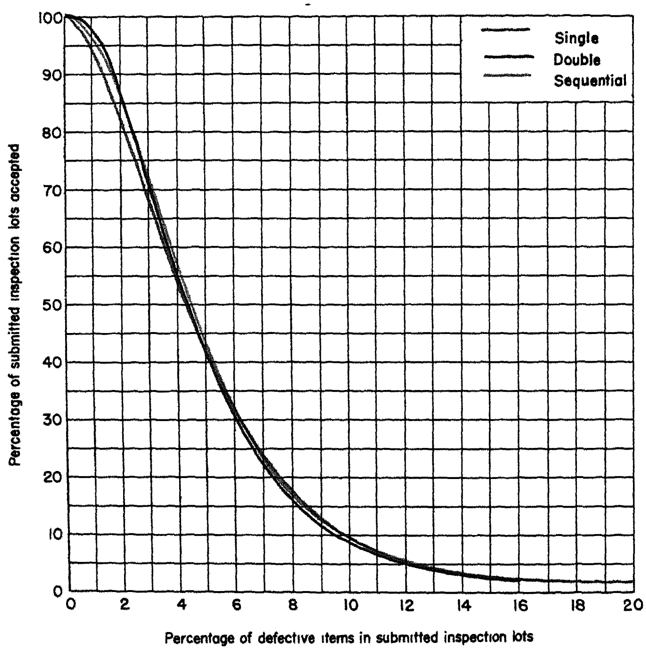


TABLE 4 (*continued*)

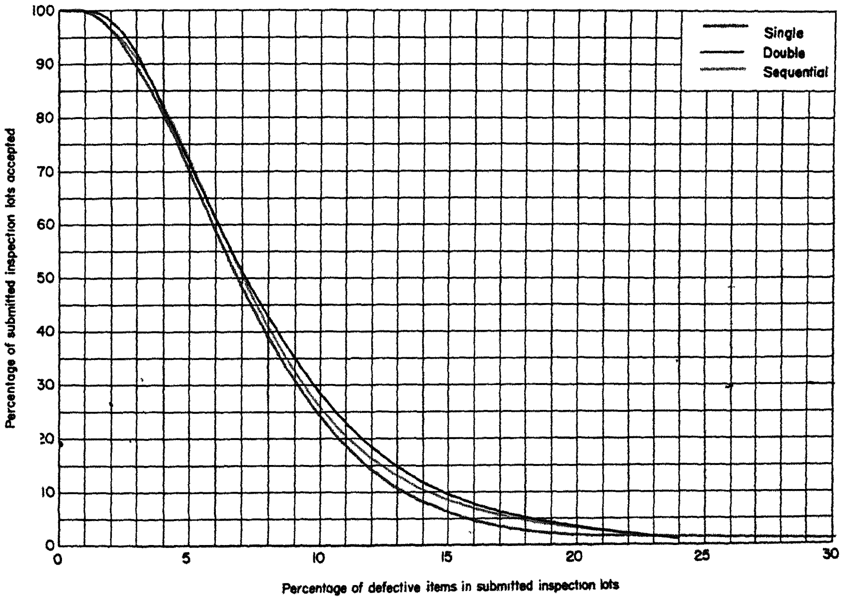
Sample-size letter **F**
AQL class (percent defective) **1.2–2.2**
AOQL class (percent defective) **2.5–3.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First....................	40	40	2	3
Double	First....................	25	25	1	4
	Second..................	50	75	3	4
Sequential	First....................	10	10	*	2
	Second..................	10	20	0	3
	Third...................	10	30	1	3
	Fourth..................	10	40	1	4
	Fifth...................	10	50	2	4
	Sixth...................	10	60	2	4
	Seventh.................	10	70	4	5

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



F Sample-size letter
2.2–3.2 AQL class (percent defective)
3.5–5.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....	40	40	3	4
Double	First....................	25	25	1	5
	Second..................	50	75	4	5
Sequential	First.......	10	10	*	2
	Second................	10	20	0	3
	Third..................	10	30	1	4
	Fourth.................	10	40	2	5
	Fifth..................	10	50	4	6
	Sixth.....	10	60	4	6
	Seventh................	10	70	5	6

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

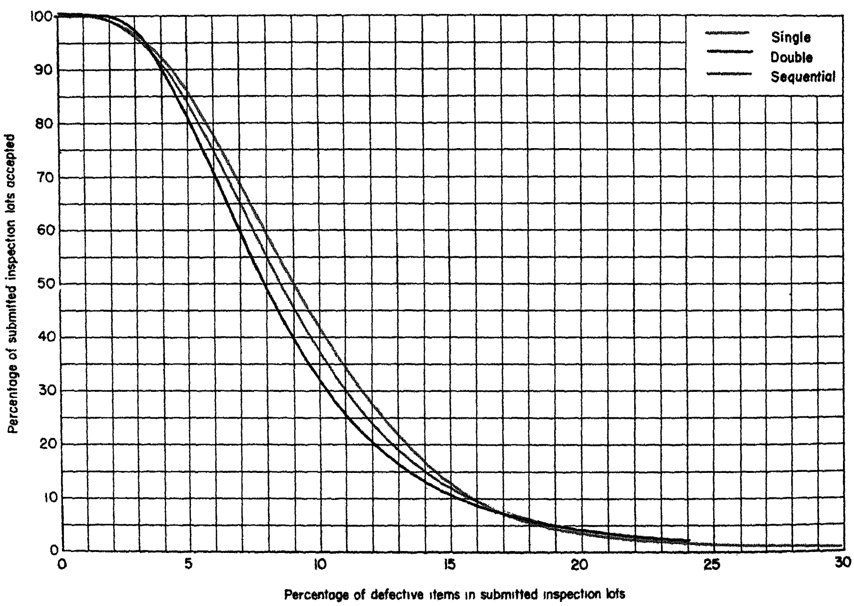


TABLE 4 (*continued*)

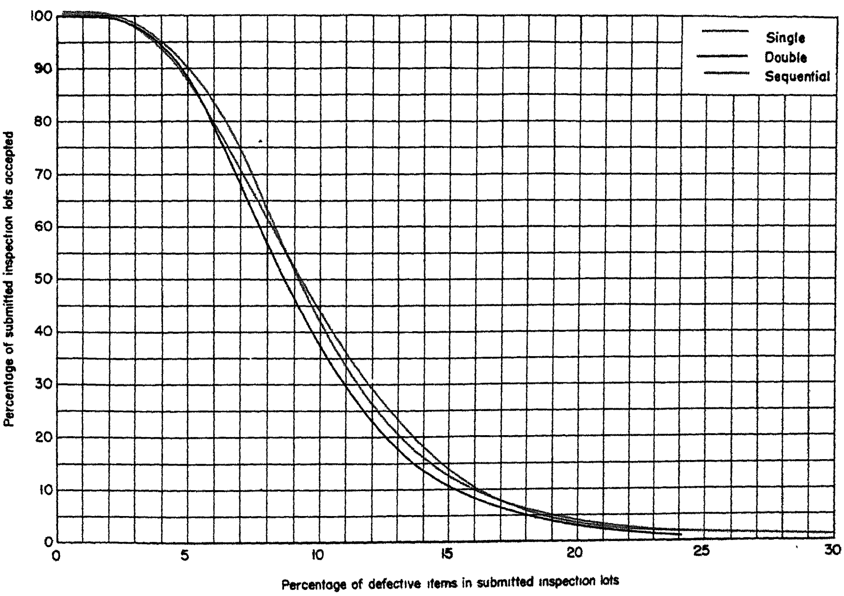
Sample-size letter **F**
AQL class (percent defective) **3.2–4.4**
AOQL class (percent defective) **3.5–5.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	40	40	3	4
Double	First	25	25	1	6
	Second	50	75	5	6
Sequential	First	10	10	*	3
	Second	10	20	0	3
	Third	10	30	1	4
	Fourth	10	40	2	6
	Fifth	10	50	3	7
	Sixth	10	60	4	7
	Seventh	10	70	6	7

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



F Sample-size letter
4.4–5.3 AQL class (percent defective)
5.0–7.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First...................	40	40	4	5
Double	First...................	25	25	2	6
	Second...................	50	75	5	6
Sequential	First...................	10	10	*	3
	Second...................	10	20	1	4
	Third...................	10	30	2	4
	Fourth...................	10	40	3	6
	Fifth...................	10	50	4	7
	Sixth...................	10	60	5	7
	Seventh...................	10	70	6	7

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

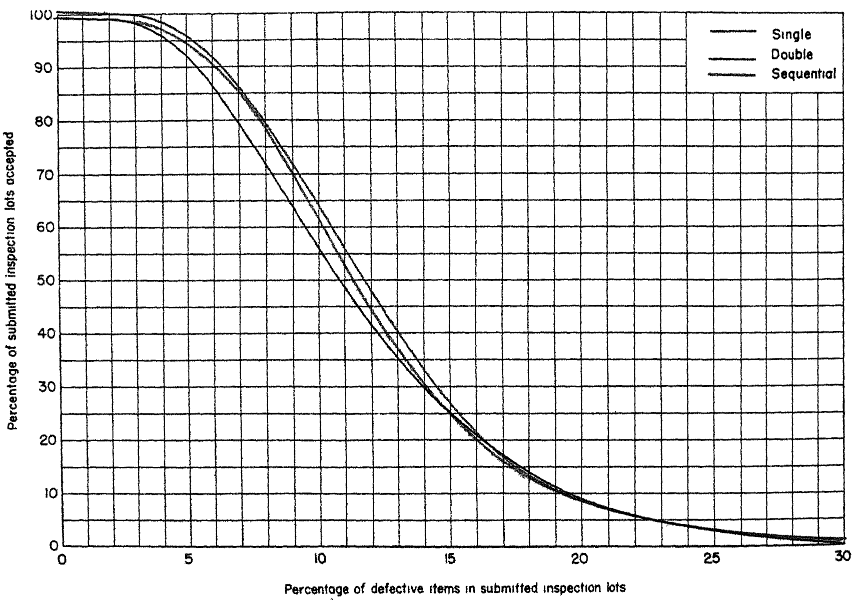


TABLE 4 (*continued*)

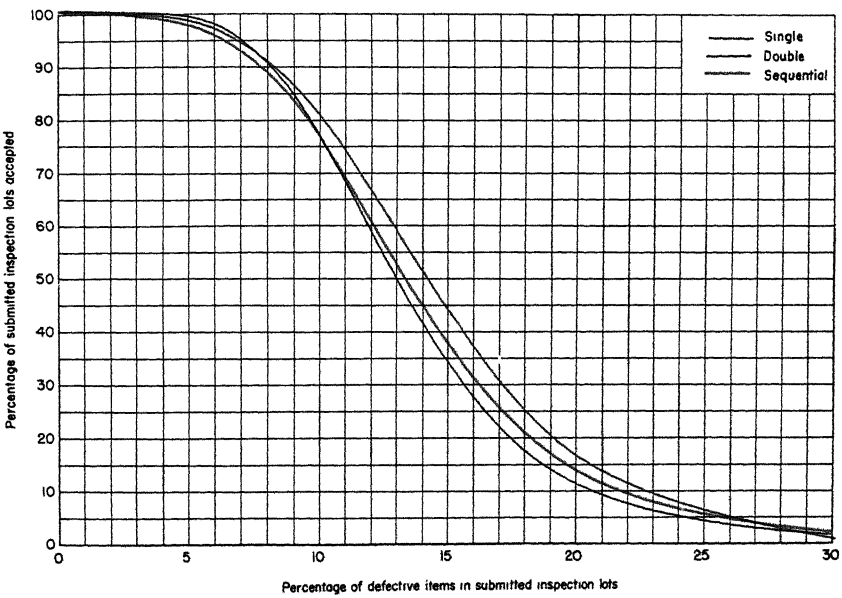
Sample-size letter **F**
AQL class (percent defective) **5.3– 6.4**
AOQL class (percent defective) **7.0–11.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	40	40	5	6
Double	First	25	25	2	9
	Second	50	75	8	9
Sequential	First	10	10	*	3
	Second	10	20	1	4
	Third	10	30	2	6
	Fourth	10	40	4	7
	Fifth	10	50	5	8
	Sixth	10	60	7	9
	Seventh	10	70	8	9

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



F Sample-size letter

6.4– 8.5 AQL class (percent defective)

7.0–11.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	40	40	6	7
Double	First	25	25	2	11
	Second	50	75	10	11
Sequential	First	10	10	0	4
	Second	10	20	1	5
	Third	10	30	3	7
	Fourth	10	40	4	8
	Fifth	10	50	5	9
	Sixth	10	60	7	10
	Seventh	10	70	10	11

b. Operating-characteristic Curves

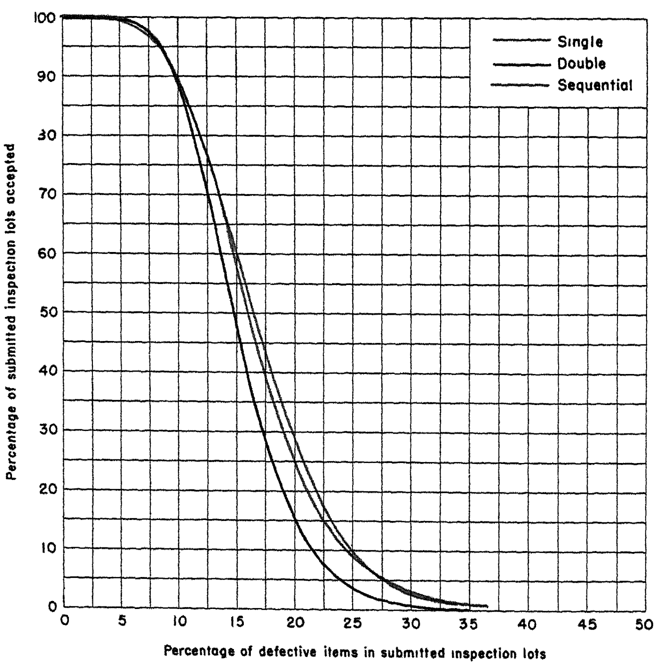


TABLE 4 (*continued*) Sample-size letter **G**
AQL classes (percent defective) **0.024–0.035**
0.035–0.06

For AQL class	Use sample-size letter
0.024–0.035	J
0.035–0.06	H

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

G Sample-size letter
0.06–0.12 AQL class (percent defective)
0.50–0.90 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	55	55	0	1
Double	Use single sampling				
Sequential	Use single sampling				

b. Operating-characteristic Curves

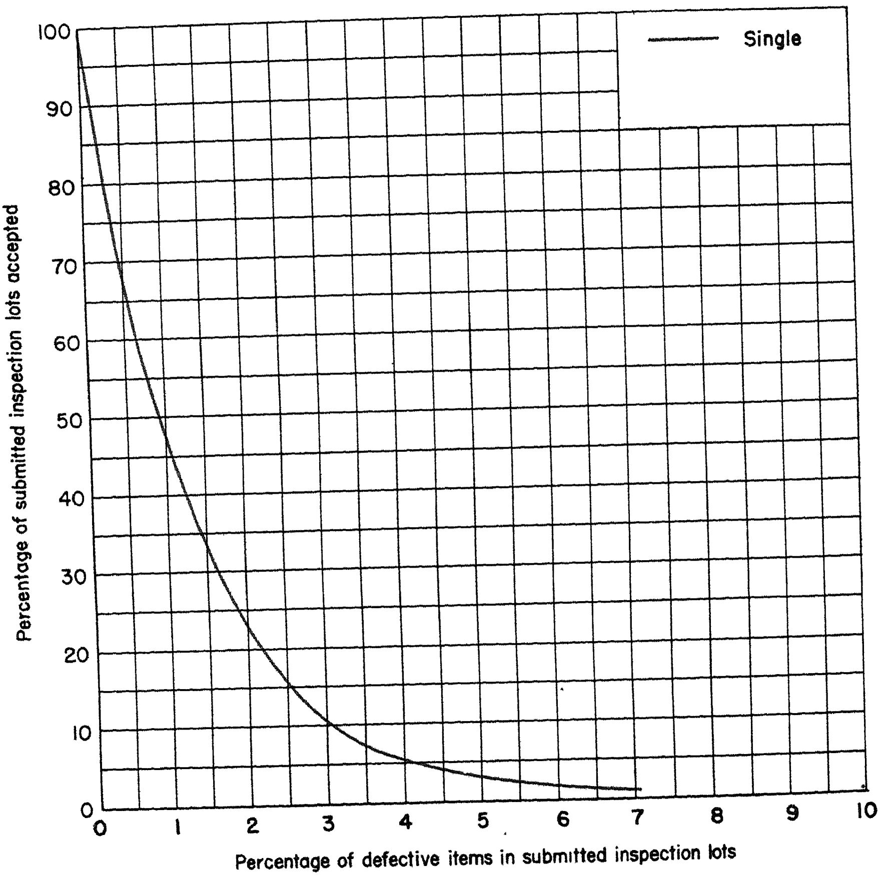


TABLE 4 (*continued*)

Sample-size letter	**G**
AQL classes (percent defective)	**0.12–0.17**
	0.17–0.22
	0.22–0.32

For AQL class	Use sample-size letter
0.12–0.17	J
0.17–0.22	I
0.22–0.32	H

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

G Sample-size letter
0.32–0.65 AQL class (percent defective)
0.90–1.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	55	55	1	2
Double	First	35	35	0	2
	Second	70	105	1	2
Sequential	First	14	14	*	2
	Second	14	28	*	2
	Third	14	42	0	2
	Fourth	14	56	0	2
	Fifth	14	70	1	3
	Sixth	14	84	2	4
	Seventh	14	98	3	4

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

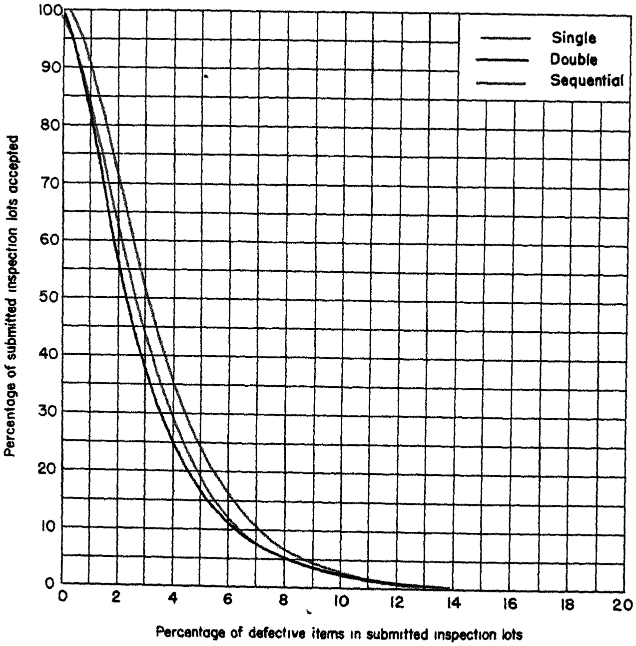


TABLE 4 (*continued*)

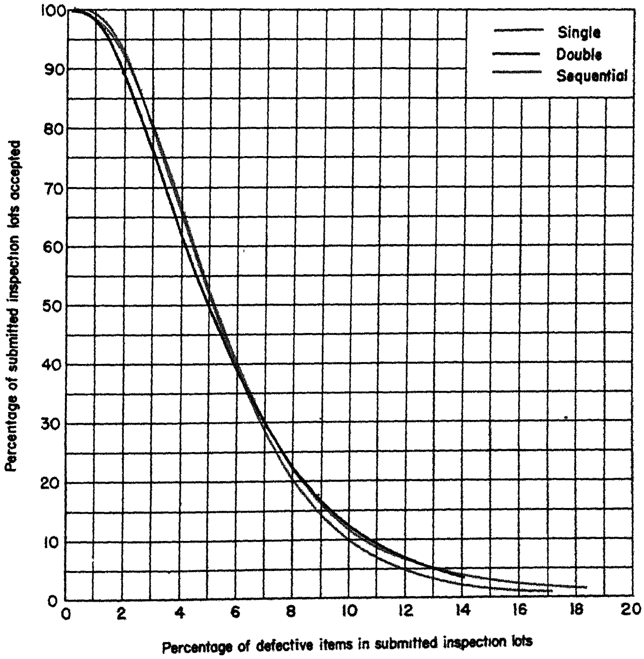
Sample-size letter **G**
AQL class (percent defective) **0.65–1.2**
AOQL class (percent defective) **1.5 –2.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First..........	55	55	2	3
Double	First.........	35	35	1	3
	Second.................	70	105	2	3
Sequential	First.............	14	14	*	2
	Second.................	14	28	0	3
	Third.............	14	42	1	3
	Fourth.................	14	56	1	3
	Fifth..................	14	70	2	4
	Sixth..................	14	84	3	5
	Seventh...........	14	98	4	5

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



G Sample-size letter

1.2–2.2 AQL class (percent defective)

2.5–3.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	55	55	3	4
Double	First	35	35	1	5
	Second	70	105	4	5
Sequential	First	14	14	*	2
	Second	14	28	0	3
	Third	14	42	1	4
	Fourth	14	56	3	5
	Fifth	14	70	3	5
	Sixth	14	84	3	5
	Seventh	14	98	4	5

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

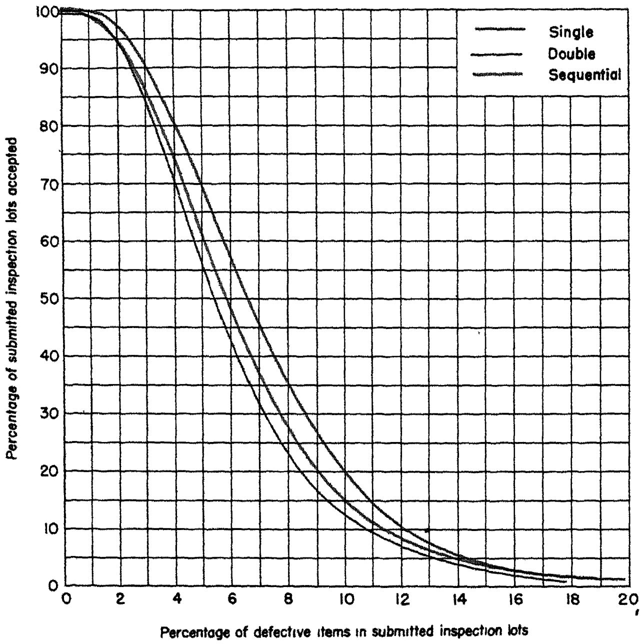


TABLE 4 (*continued*)

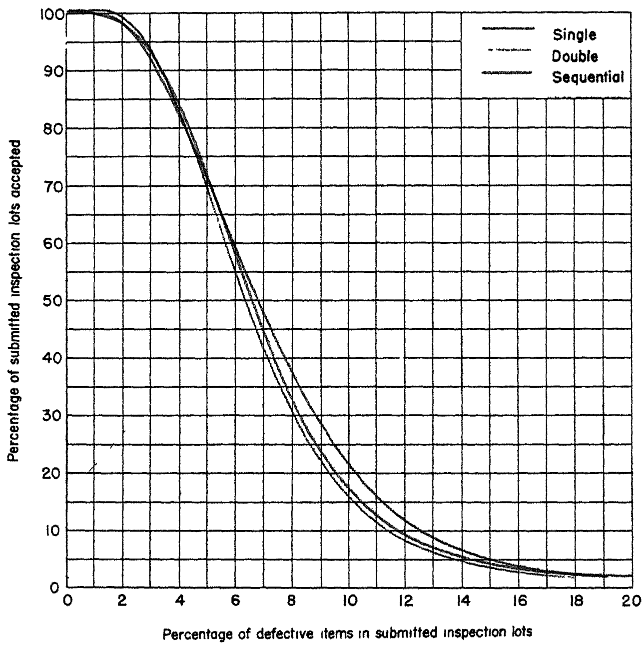
Sample-size letter **G**
AQL class (percent defective) **2.2–3.2**
AOQL class (percent defective) **3.5–5.0**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	55	55	3	4
Double	First	35	35	1	6
	Second	70	105	5	6
Sequential	First	14	14	*	3
	Second	14	28	0	3
	Third	14	42	1	4
	Fourth	14	56	2	5
	Fifth	14	70	4	6
	Sixth	14	84	4	6
	Seventh	14	98	5	6

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES



G Sample-size letter TABLE 4 (*continued*)
3.2–4.4 AQL class (percent defective)
3.5–5.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	55	55	4	5
Double	First	35	35	2	7
	Second	70	105	6	7
Sequential	First	14	14	*	3
	Second	14	28	1	4
	Third	14	42	2	4
	Fourth	14	56	3	6
	Fifth	14	70	4	6
	Sixth	14	84	5	7
	Seventh	14	98	6	7

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

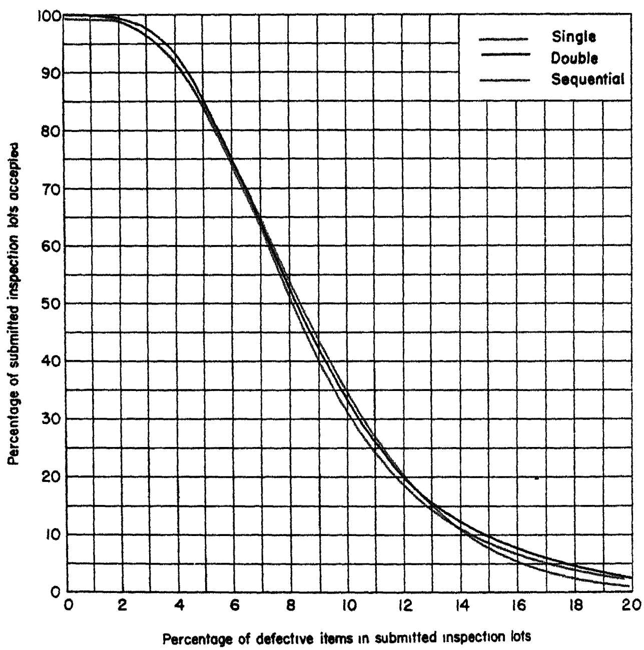


TABLE 4 (*continued*)

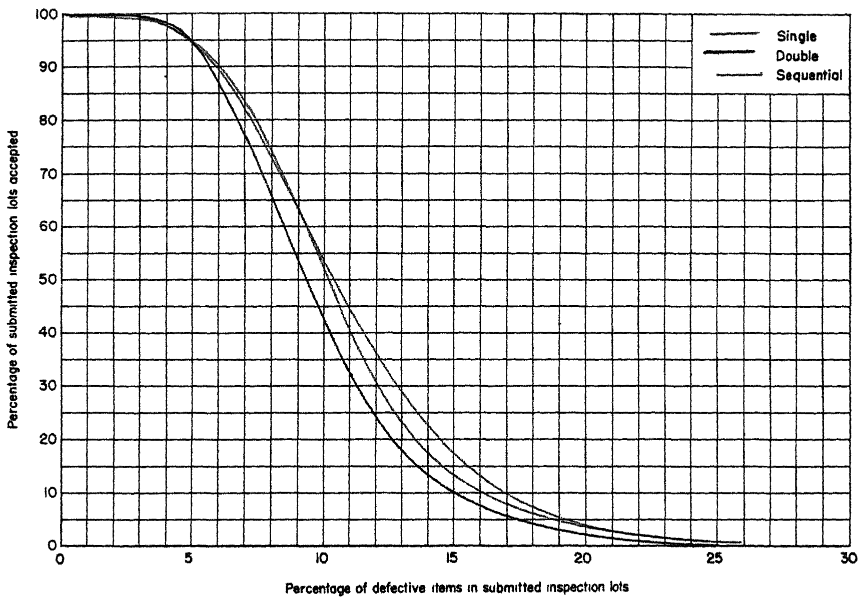
Sample-size letter **G**
AQL class (percent defective) **4.4–5.3**
AOQL class (percent defective) **5.0–7.0**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	55	55	5	6
Double	First	35	35	2	9
	Second	70	105	8	9
Sequential	First	14	14	*	3
	Second	14	28	1	5
	Third	14	42	2	6
	Fourth	14	56	4	7
	Fifth	14	70	6	9
	Sixth	14	84	7	10
	Seventh	14	98	10	11

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES



G Sample-size letter
5.3–6.4 AQL class (percent defective)
5.0–7.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	55	55	6	7
Double	First	35	35	3	10
	Second	70	105	9	10
Sequential	First	14	14	0	4
	Second	14	28	1	5
	Third	14	42	3	6
	Fourth	14	56	5	8
	Fifth	14	70	6	9
	Sixth	14	84	8	10
	Seventh	14	98	9	10

b. Operating-characteristic Curves

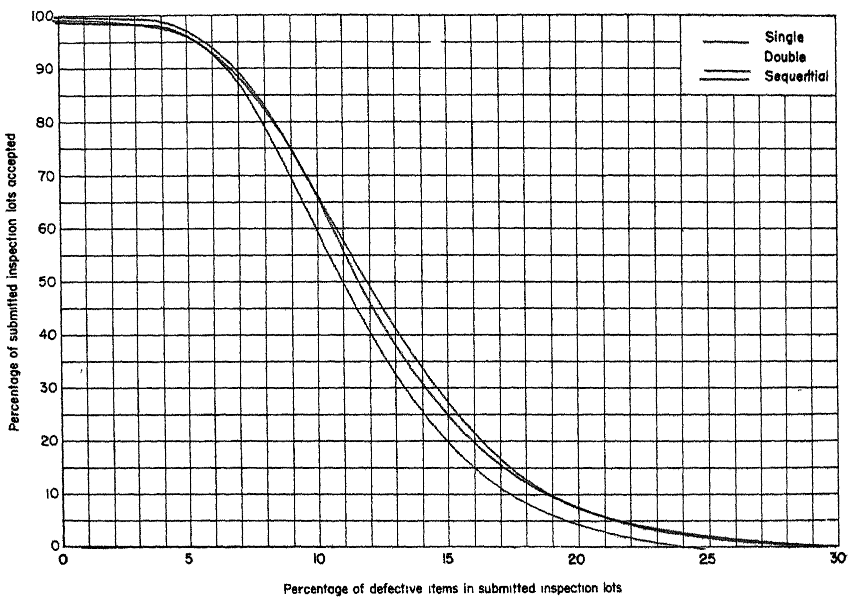


TABLE 4 (*continued*)

Sample-size letter **G**
AQL class (percent defective) **6.4– 8.5**
AOQL class (percent defective) **7.0–11.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	55	55	8	9
Double	First	35	35	4	13
	Second	70	105	12	13
Sequential	First	14	14	0	4
	Second	14	28	3	6
	Third	14	42	4	8
	Fourth	14	56	6	10
	Fifth	14	70	8	12
	Sixth	14	84	10	14
	Seventh	14	98	13	14

b. Operating-characteristic Curves

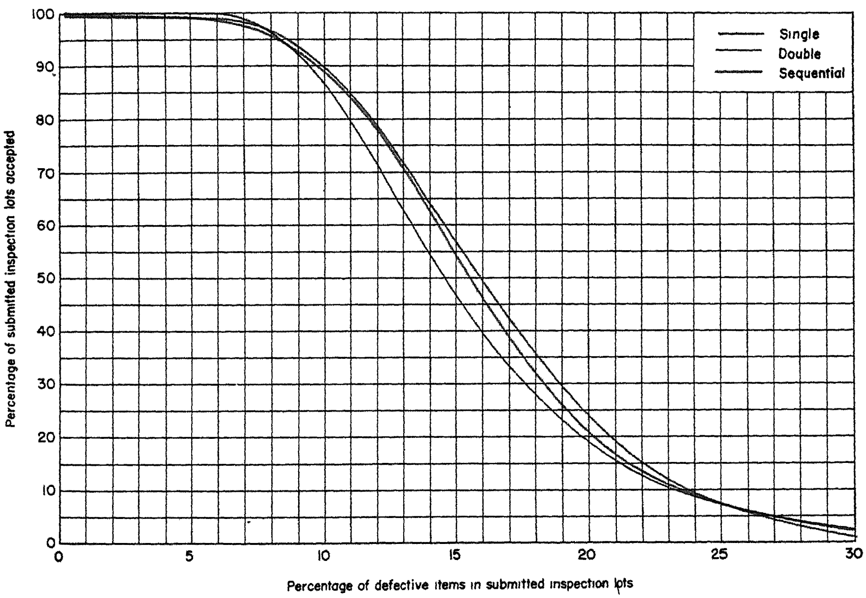


TABLE 4 (*continued*)

H Sample-size letter
0.024–0.035 AQL class (percent defective)

For AQL class	Use sample-size letter
0.024–0.035	J

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*) Sample-size letter **H**
AQL class (percent defective) **0.035–0.06**
AOQL class (percent defective) **0.30 –0.50**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First.	75	75	0	1
Double	Use single sampling				
Sequential	Use single sampling				

b. Operating-characteristic Curve

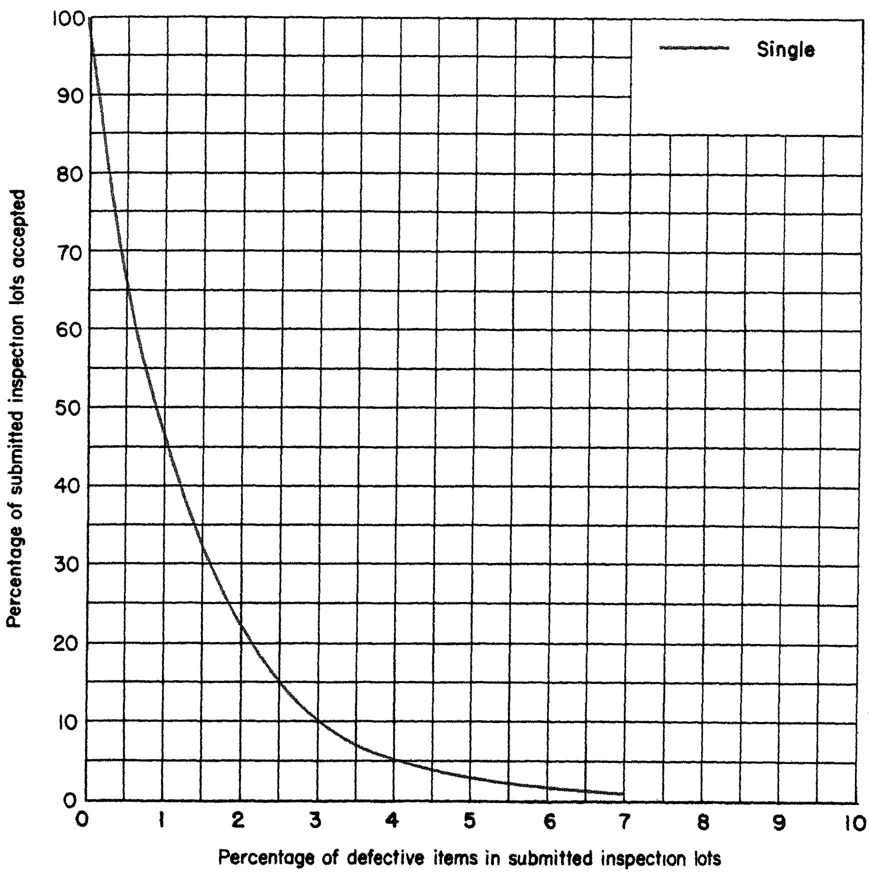


TABLE 4 (*continued*)

H Sample-size letter
0.06–0.12 AQL classes (percent defective)
0.12–0.17
0.17–0.22

For AQL class	Use sample-size letter
0.06–0.12	K
0.12–0.17	J
0.17–0.22	I

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

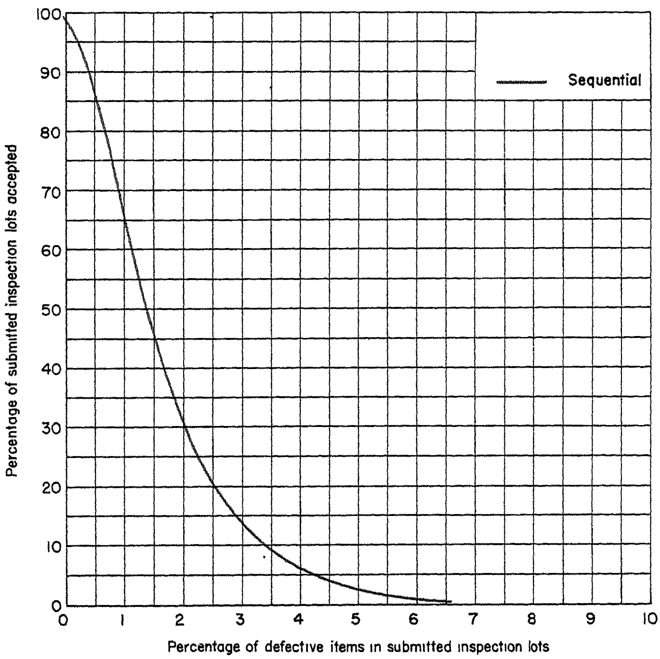
Sample-size letter **H**
AQL class (percent defective) **0.22–0.32**
AOQL class (percent defective) **0.50–0.90**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	For single sampling, use sample-size letter J			
Double	First Second	For double sampling, use sample-size letter I			
Sequential	First	20	20	*	2
	Second	20	40	*	2
	Third	20	60	*	2
	Fourth	20	80	0	2
	Fifth	20	100	0	2
	Sixth	20	120	0	2
	Seventh	20	140	2	3

* Acceptance not permitted until four samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVE



H Sample-size letter TABLE 4 (*continued*)
0.32–0.65 AQL class (percent defective)
0.90–1.5 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	75	75	1	2
Double	First	50	50	0	3
	Second	100	150	2	3
Sequential	First	20	20	*	2
	Second	20	40	*	2
	Third	20	60	0	2
	Fourth	20	80	1	3
	Fifth	20	100	1	3
	Sixth	20	120	1	3
	Seventh	20	140	2	3

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

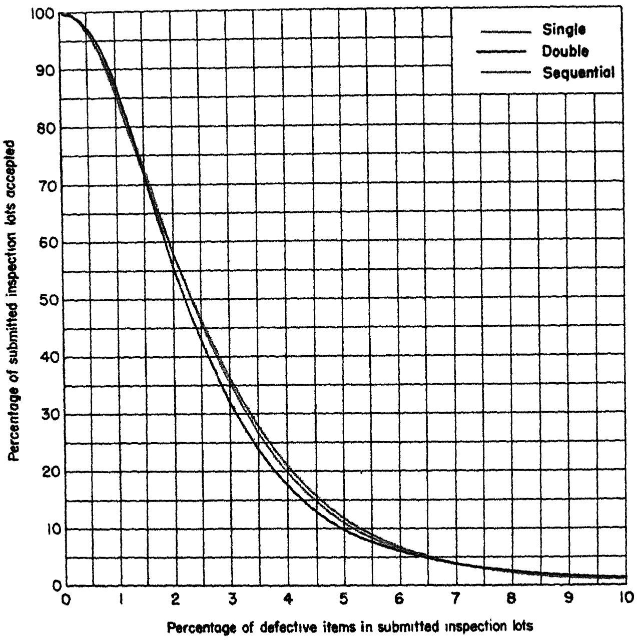


TABLE 4 (*continued*)

Sample-size letter	**H**
AQL class (percent defective)	**0.65–1.2**
AOQL class (percent defective)	**1.5 –2.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	75	75	2	3
Double	First....................	50	50	1	4
	Second..................	100	150	3	4
Sequential	First....................	20	20	*	2
	Second..................	20	40	0	3
	Third....................	20	60	1	3
	Fourth..................	20	80	2	4
	Fifth....................	20	100	2	4
	Sixth....................	20	120	2	4
	Seventh................	20	140	3	4

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

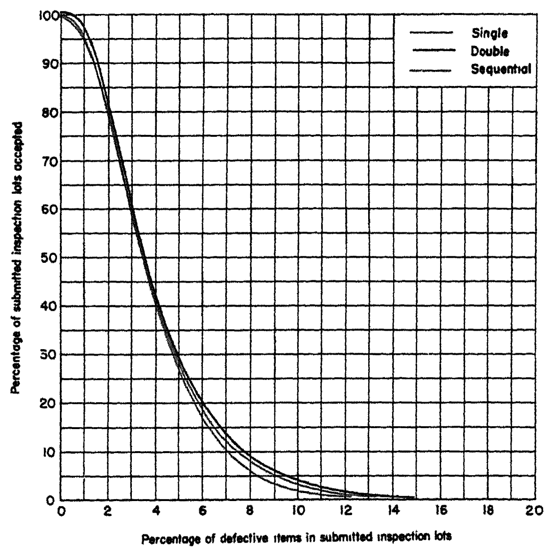


TABLE 4 (*continued*)

H Sample-size letter
1.2–2.2 AQL class (percent defective)
2.5–3.5 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	75	75	3	4
Double	First	50	50	1	6
	Second	100	150	5	6
Sequential	First	20	20	*	3
	Second	20	40	0	4
	Third	20	60	1	4
	Fourth	20	80	2	5
	Fifth	20	100	4	6
	Sixth	20	120	4	6
	Seventh	20	140	5	6

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

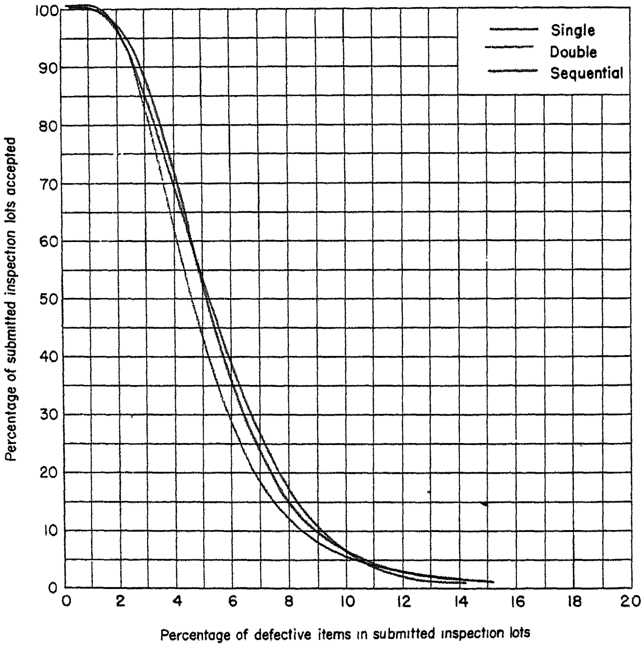


TABLE 4 (*continued*)

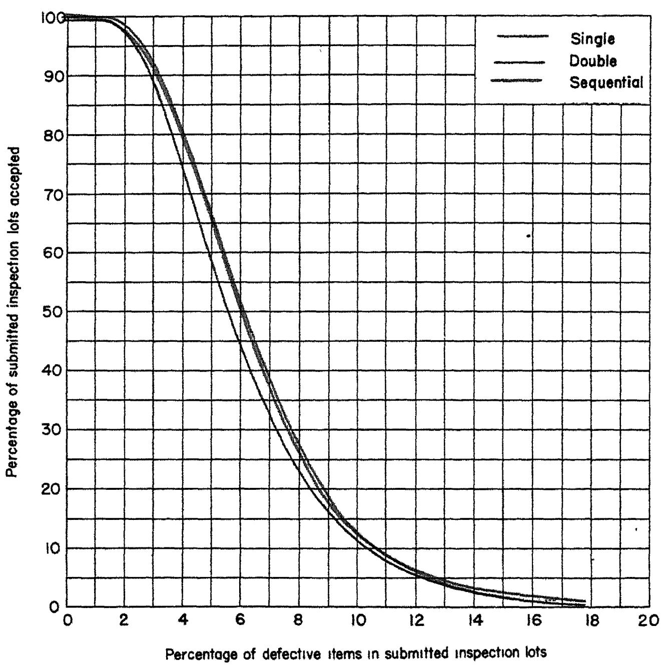
Sample-size letter **H**
AQL class (percent defective) **2.2–3.2**
AOQL class (percent defective) **2.5–3.5**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	75	75	4	5
Double	First................ ...	50	50	2	7
	Second..................	100	150	6	7
Sequential	First....................	20	20	*	3
	Second..................	20	40	1	4
	Third...................	20	60	2	5
	Fourth..................	20	80	3	6
	Fifth....................	20	100	5	7
	Sixth....................	20	120	6	8
	Seventh..	20	140	7	8

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES



H Sample-size letter TABLE 4 (*continued*)
3.2–4.4 AQL class (percent defective)
3.5–5.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	75	75	6	7
Double	First	50	50	3	10
	Second	100	150	9	10
Sequential	First	20	20	0	4
	Second	20	40	1	5
	Third	20	60	3	6
	Fourth	20	80	5	8
	Fifth	20	100	8	10
	Sixth	20	120	9	11
	Seventh	20	140	10	11

b. Operating-characteristic Curves

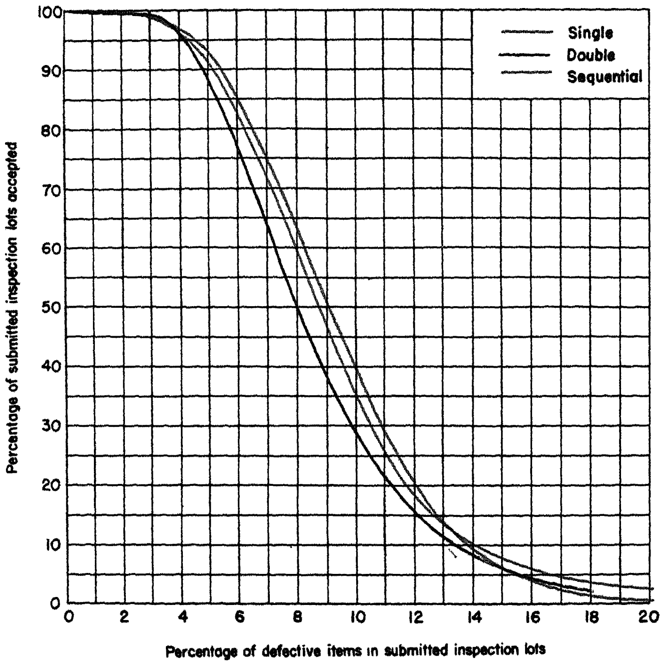


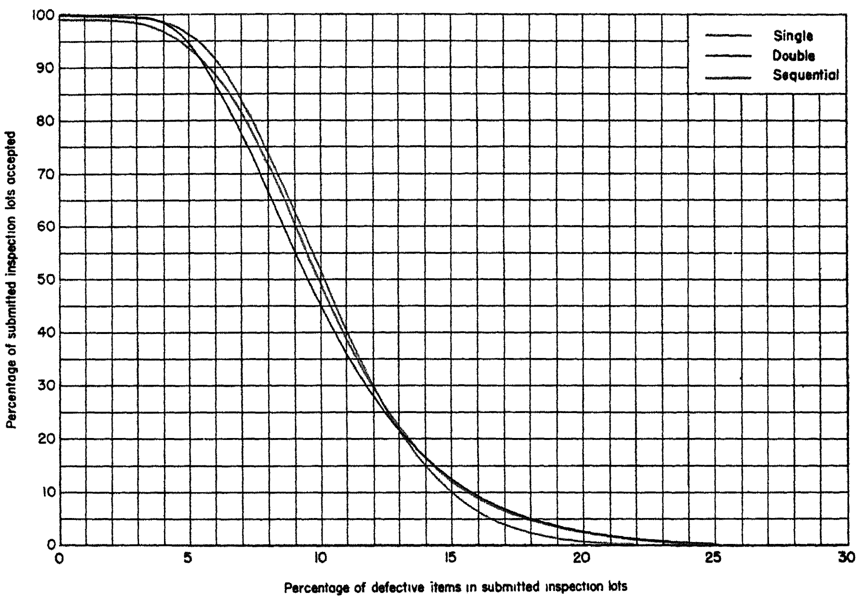
TABLE 4 (*continued*)

Sample-size letter **H**
AQL class (percent defective) **4.4–5.3**
AOQL class (percent defective) **5.0–7.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	75	75	7	8
Double	First....................	50	50	4	11
	Second..................	100	150	10	11
Sequential	First....................	20	20	0	4
	Second..................	20	40	2	6
	Third....................	20	60	5	7
	Fourth..................	20	80	6	9
	Fifth....................	20	100	8	12
	Sixth....................	20	120	11	14
	Seventh.................	20	140	13	14

b. Operating-characteristic Curves



H Sample-size letter TABLE 4 (*continued*)

5.3–6.4 AQL class (percent defective)

5.0–7.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First	75	75	8	9
Double	First	50	50	5	13
	Second	100	150	12	13
Sequential	First	20	20	0	4
	Second	20	40	3	7
	Third	20	60	6	9
	Fourth	20	80	8	11
	Fifth	20	100	10	12
	Sixth	20	120	12	14
	Seventh	20	140	14	15

b. Operating-characteristic Curves

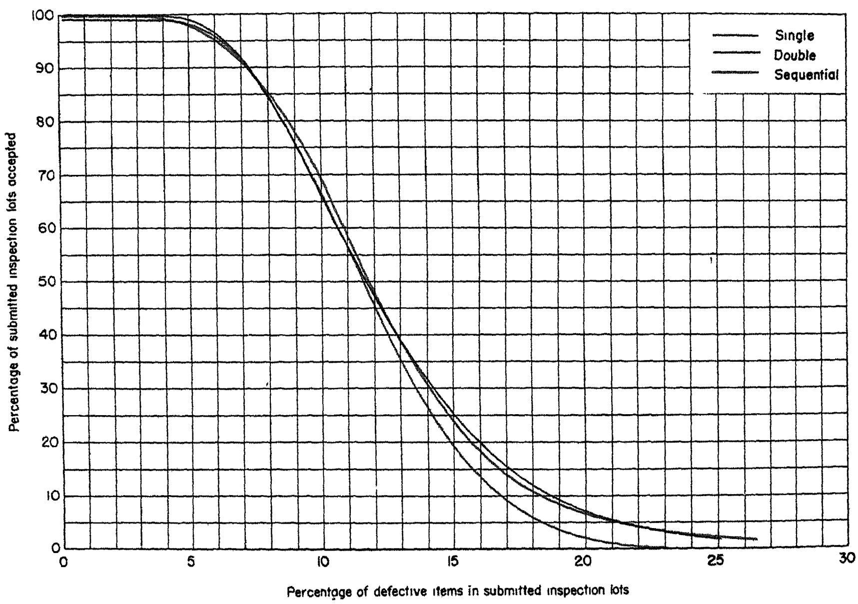


TABLE 4 (*continued*)

Sample-size letter **H**
AQL class (percent defective) **6.4– 8.5**
AOQL class (percent defective) **7.0–11.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	75	75	10	11
Double	First	50	50	6	17
	Second	100	150	16	17
Sequential	First	20	20	1	5
	Second	20	40	3	8
	Third	20	60	6	11
	Fourth	20	80	9	13
	Fifth	20	100	13	16
	Sixth	20	120	16	18
	Seventh	20	140	18	19

b. Operating-characteristic Curves

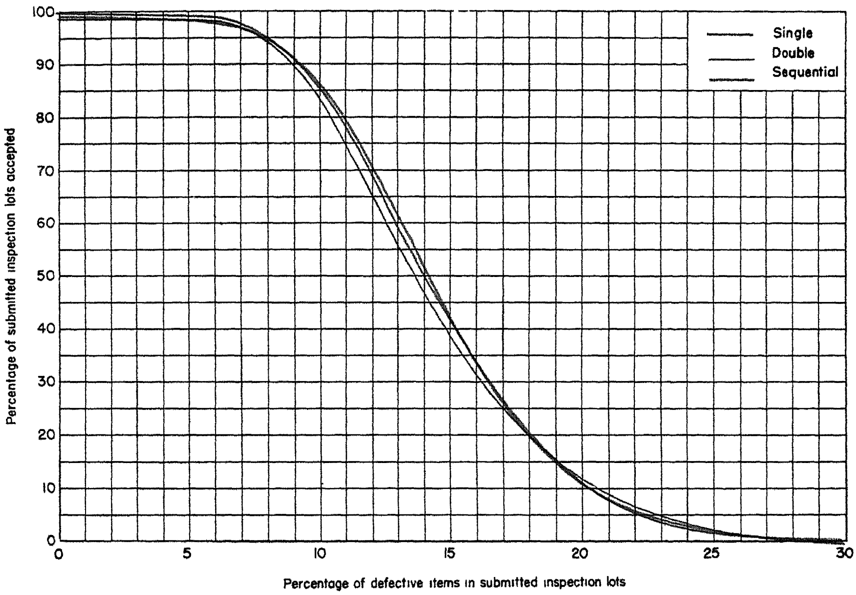


TABLE 4 (*continued*)

I Sample-size letter
0.024–0.035 AQL classes (percent defective)
0.035–0.06
0.06 –0.12
0.12 –0.17

For AQL class	Use sample-size letter
0.024–0.035	J
0.035–0.06	L
0.06 –0.12	K
0.12 –0.17	J

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

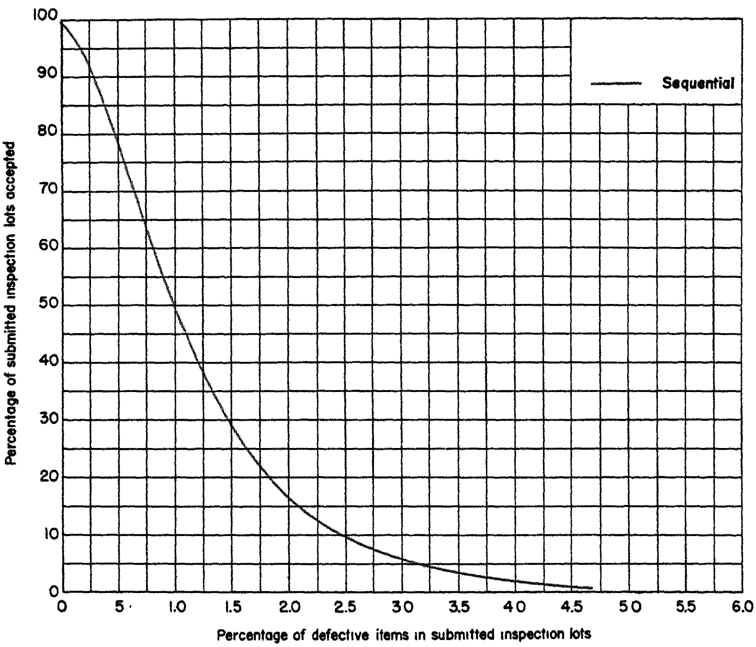
Sample-size letter **I**
AQL class (percent defective) **0.17–0.22**
AOQL class (percent defective) **0.50–0.90**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	For single sampling, use sample-size letter K			
Double	First Second	For double sampling, use sample-size letter J			
Sequential	First	30	30	*	2
	Second	30	60	*	2
	Third	30	90	*	2
	Fourth	30	120	0	2
	Fifth	30	150	0	2
	Sixth	30	180	1	3
	Seventh	30	210	2	3

* Acceptance not permitted until four samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVE



I Sample-size letter
0.22–0.32 AQL class (percent defective)
0.50–0.90 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....	For single sampling, use sample-size letter J			
Double	First....................	75	75	0	2
	Second.................	150	225	1	2
Sequential	First....................	30	30	*	2
	Second...	30	60	*	2
	Third.....	30	90	0	2
	Fourth.	30	120	0	2
	Fifth.	30	150	1	3
	Sixth	30	180	1	3
	Seventh.................	30	210	2	3

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

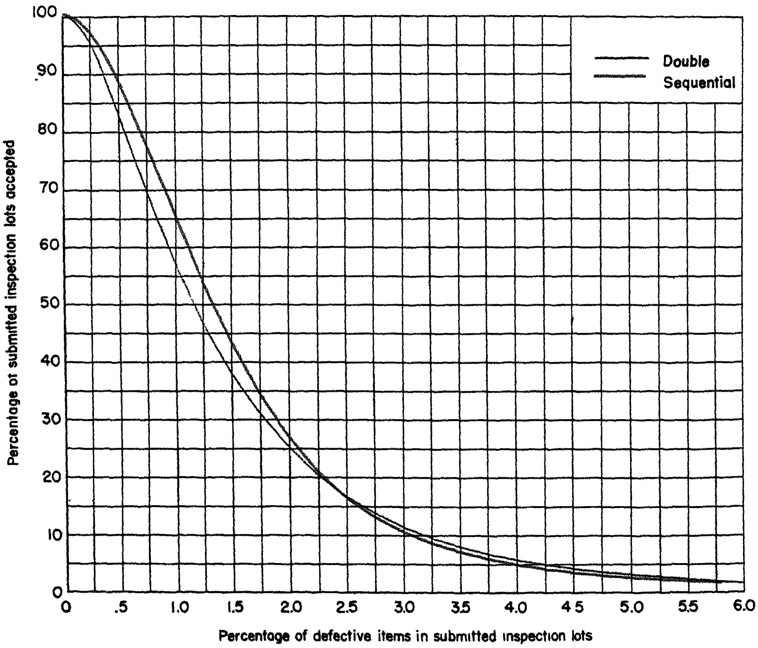


TABLE 4 (*continued*)

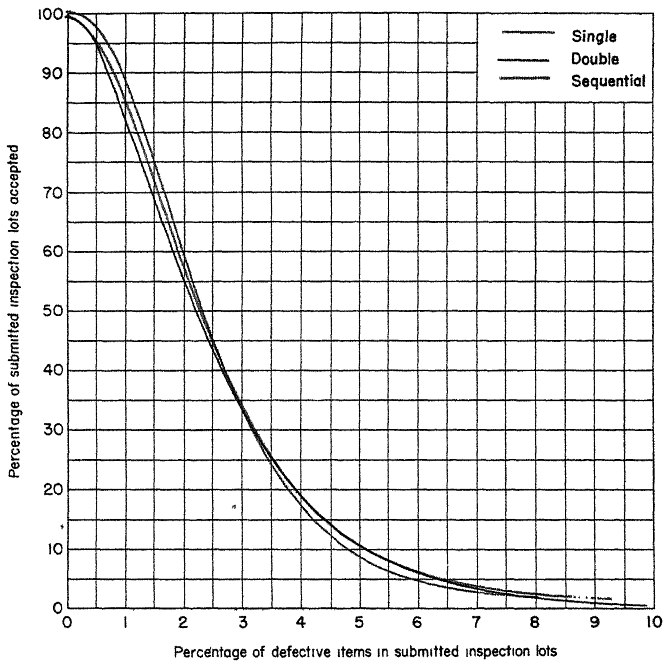
Sample-size letter **I**
AQL class (percent defective) **0.32–0.65**
AOQL class (percent defective) **0.90–1.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	115	115	2	3
Double	First	75	75	1	3
	Second	150	225	2	3
Sequential	First	30	30	*	2
	Second	30	60	0	2
	Third	30	90	1	3
	Fourth	30	120	2	4
	Fifth	30	150	3	5
	Sixth	30	180	3	5
	Seventh	30	210	4	5

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



I Sample-size letter
0.65–1.2 AQL class (percent defective)
1.5 –2.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	115	115	3	4
Double	First	75	75	1	6
	Second	150	225	5	6
Sequential	First	30	30	*	3
	Second	30	60	0	3
	Third	30	90	1	4
	Fourth	30	120	2	6
	Fifth	30	150	3	6
	Sixth	30	180	5	7
	Seventh	30	210	6	7

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

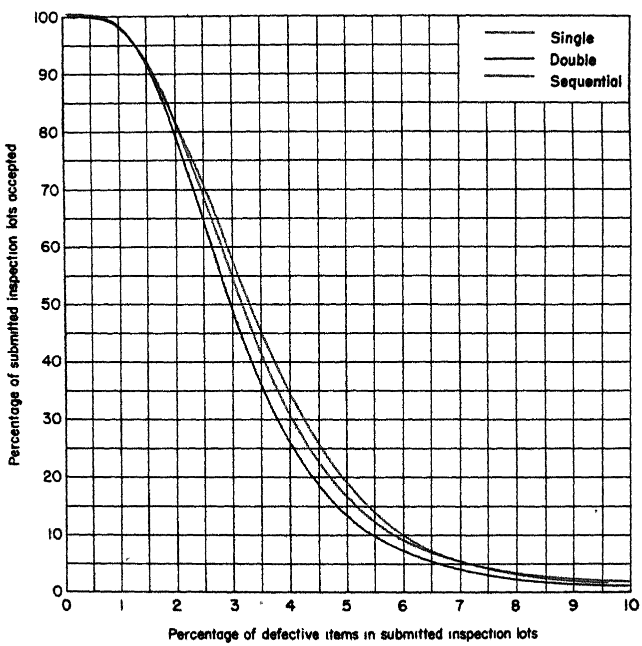


TABLE 4 (*continued*)

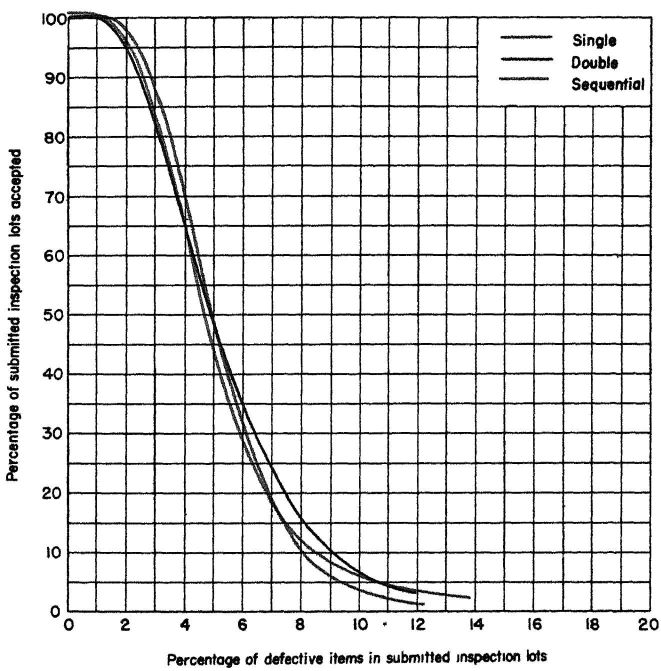
Sample-size letter **I**
AQL class (percent defective) **1.2–2.2**
AOQL class (percent defective) **2.5–3.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	115	115	5	6
Double	First	75	75	3	6
	Second	150	225	5	6
Sequential	First	30	30	*	3
	Second	30	60	1	4
	Third	30	90	3	6
	Fourth	30	120	5	7
	Fifth	30	150	6	8
	Sixth	30	180	7	9
	Seventh	30	210	8	9

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



I Sample-size letter

2.2–3.2 AQL class (percent defective)

2.5–3.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	115	115	6	7
Double	First	75	75	4	9
	Second	150	225	8	9
Sequential	First	30	30	0	4
	Second	30	60	2	5
	Third	30	90	3	8
	Fourth	30	120	4	9
	Fifth	30	150	6	10
	Sixth	30	180	8	12
	Seventh	30	210	11	12

b. Operating-characteristic Curves

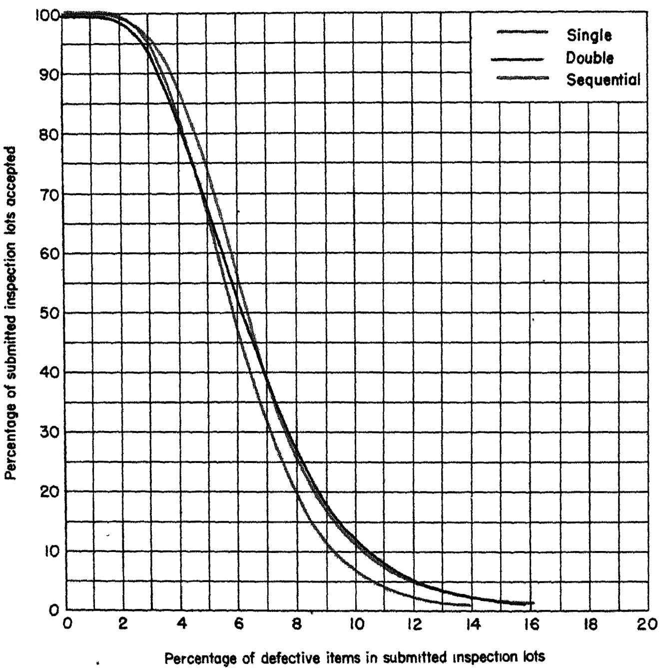


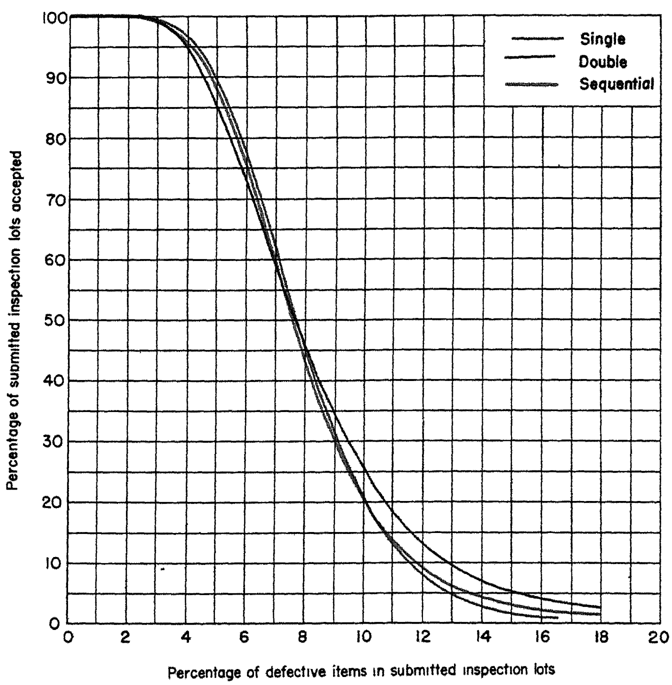
TABLE 4 (*continued*)

Sample-size letter **I**
AQL class (percent defective) **3.2–4.4**
AOQL class (percent defective) **3.5–5.0**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First.	115	115	8	9
Double	First..	75	75	5	12
	Second....	150	225	11	12
Sequential	First....	30	30	0	4
	Second.......	30	60	3	7
	Third	30	90	5	9
	Fourth...	30	120	7	11
	Fifth.....	30	150	9	13
	Sixth..	30	180	12	15
	Seventh..	30	210	14	15

b. OPERATING-CHARACTERISTIC CURVES



I Sample-size letter
4.4–5.3 AQL class (percent defective)
5.0–7.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	115	115	10	11
Double	First	75	75	6	15
	Second	150	225	14	15
Sequential	First	30	30	1	5
	Second	30	60	3	7
	Third	30	90	6	10
	Fourth	30	120	9	13
	Fifth	30	150	11	15
	Sixth	30	180	14	18
	Seventh	30	210	17	18

b. Operating-characteristic Curves

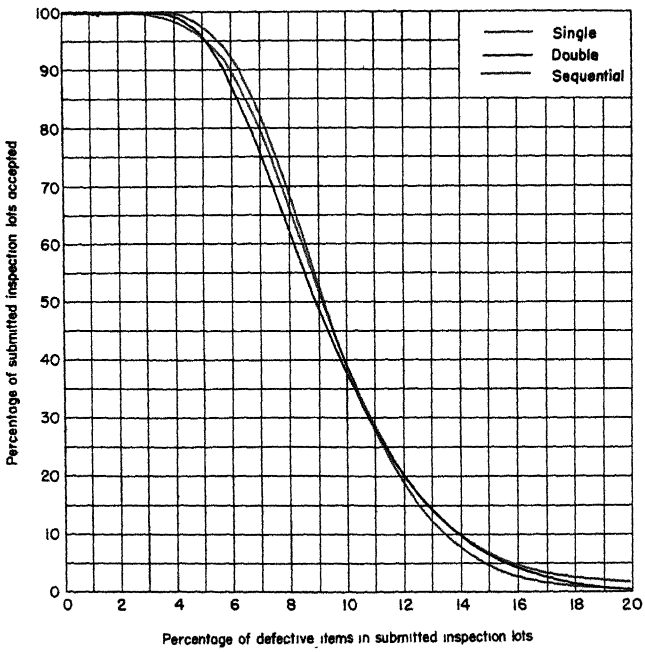


TABLE 4 (*continued*)

Sample-size letter **I**
AQL class (percent defective) **5.3–6.4**
AOQL class (percent defective) **5.0–7.0**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	115	115	11	12
Double	First	75	75	6	19
	Second	150	225	18	19
Sequential	First	30	30	1	5
	Second	30	60	3	9
	Third	30	90	5	12
	Fourth	30	120	8	15
	Fifth	30	150	11	18
	Sixth	30	180	14	20
	Seventh	30	210	19	20

b. OPERATING-CHARACTERISTIC CURVES

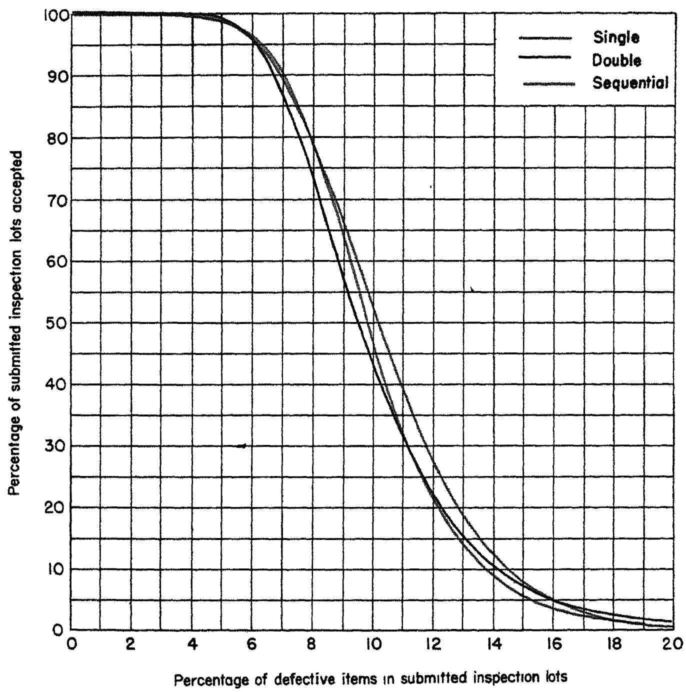


TABLE 4 (*continued*)

I Sample-size letter
6.4– 8.5 AQL class (percent defective)
7.0–11.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First	115	115	14	15
Double	First	75	75	8	24
	Second	150	225	23	24
Sequential	First	30	30	1	7
	Second	30	60	5	10
	Third	30	90	9	13
	Fourth	30	120	13	18
	Fifth	30	150	18	22
	Sixth	30	180	22	25
	Seventh	30	210	25	26

b. Operating-characteristic Curves

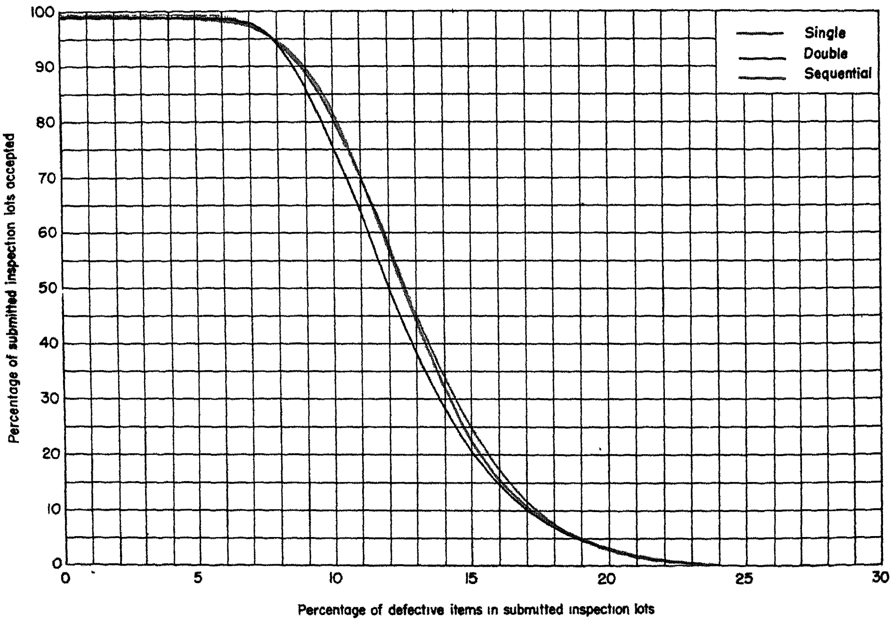


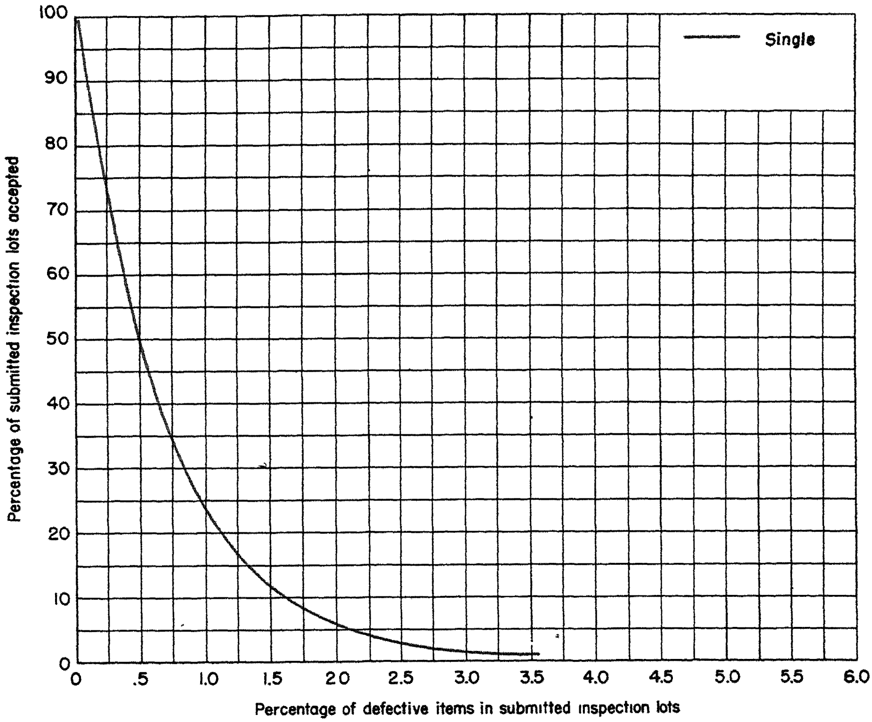
TABLE 4 (*continued*)

Sample-size letter J
AQL class (percent defective) **0.024–0.035**
AOQL class (percent defective) **0.22 –0.30**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	150	150	0	1
Double	Use single sampling				
Sequential	Use single sampling				

b. Operating-characteristic Curve



J Sample-size letter TABLE 4 (*continued*)
0.035–0.06 AQL classes (percent defective)
0.06 –0.12

For AQL class	Use sample-size letter
0 035–0.06	L
0.06 –0.12	K

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

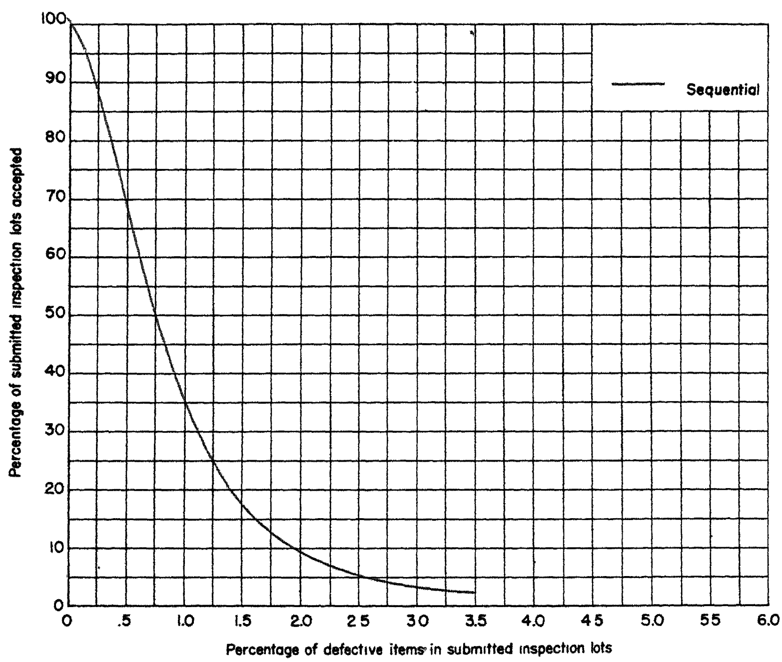
Sample-size letter **J**
AQL class (percent defective) **0.12–0.17**
AOQL class (percent defective) **0.30–0.50**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	For single sampling, use sample-size letter L			
Double	First Second	For double sampling, use sample-size letter K			
Sequential	First	40	40	*	2
	Second	40	80	*	2
	Third	40	120	*	2
	Fourth	40	160	0	2
	Fifth	40	200	0	2
	Sixth	40	240	1	3
	Seventh	40	280	2	3

* Acceptance not permitted until four samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVE



J Sample-size letter TABLE 4 (*continued*)
0.17–0.22 AQL class (percent defective)
0.30–0.50 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	For single sampling, use sample-size letter K			
Double	First....................	100	100	0	2
	Second..................	200	300	1	2
Sequential	First....................	40	40	*	2
	Second..................	40	80	*	2
	Third....................	40	120	0	2
	Fourth..................	40	160	0	2
	Fifth....................	40	200	1	3
	Sixth....................	40	240	1	3
	Seventh................	40	280	2	3

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

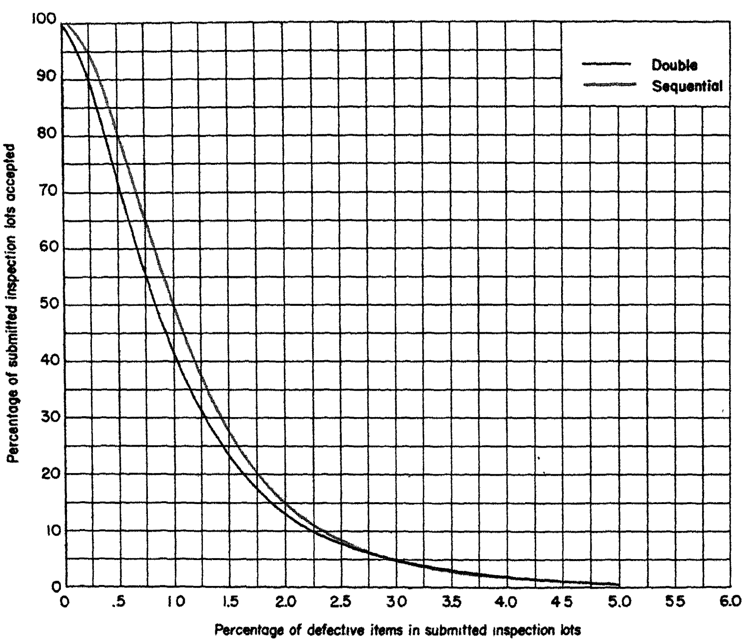


TABLE 4 (*continued*)

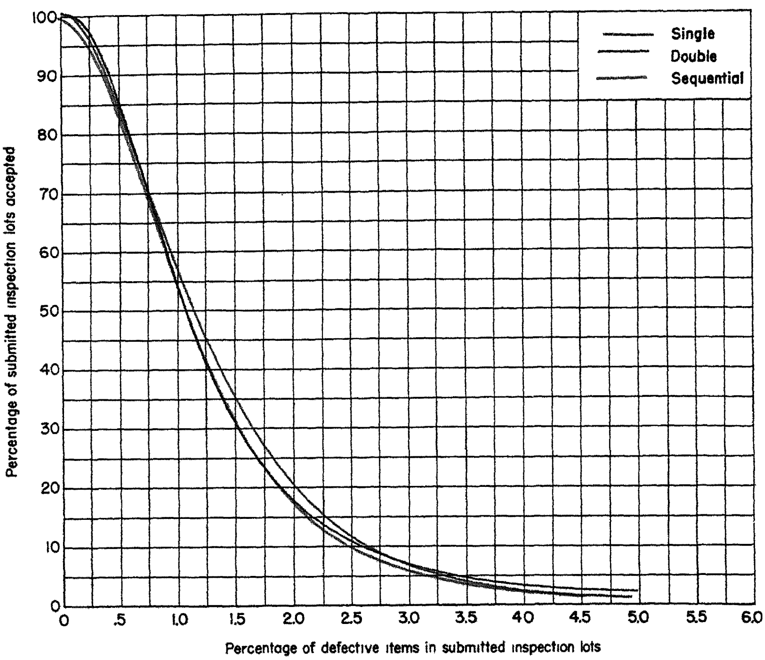
Sample-size letter **J**
AQL class (percent defective) **0.22–0.32**
AOQL class (percent defective) **0.50–0.90**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	150	150	1	2
Double	First	100	100	0	3
	Second	200	300	2	3
Sequential	First	40	40	*	2
	Second	40	80	*	2
	Third	40	120	0	2
	Fourth	40	160	0	3
	Fifth	40	200	1	3
	Sixth	40	240	2	4
	Seventh	40	280	4	5

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves



J Sample-size letter TABLE 4 (*continued*)
0.32–0.65 AQL class (percent defective)
0.90–1.5 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	150	150	2	3
Double	First	100	100	1	4
	Second	200	300	3	4
Sequential	First	40	40	*	2
	Second	40	80	0	3
	Third	40	120	1	3
	Fourth	40	160	3	5
	Fifth	40	200	3	5
	Sixth	40	240	3	5
	Seventh	40	280	4	5

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

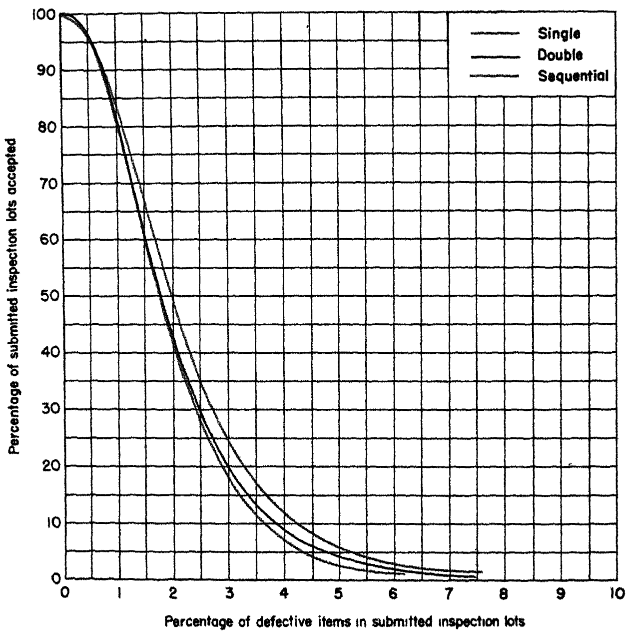


TABLE 4 (*continued*)

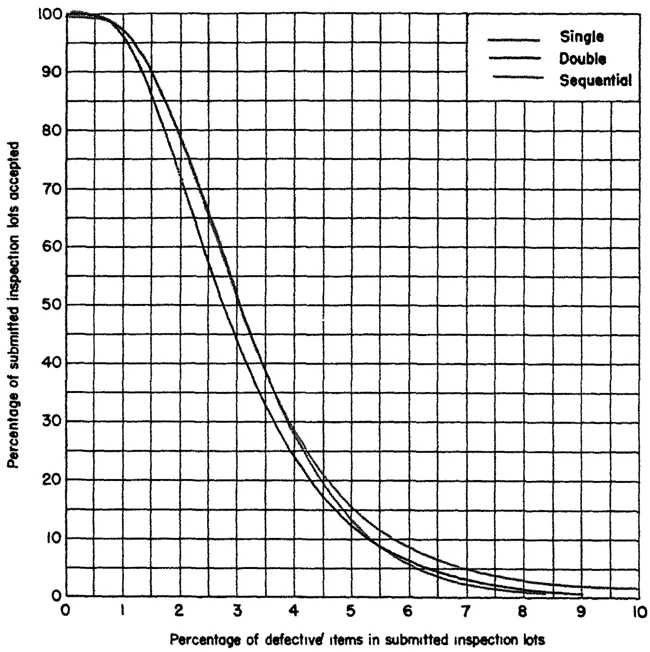
Sample-size letter **J**
AQL class (percent defective) **0.65–1.2**
AOQL class (percent defective) **1.5 –2.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	150	150	4	5
Double	First	100	100	2	6
	Second	200	300	5	6
Sequential	First	40	40	*	3
	Second	40	80	1	4
	Third	40	120	2	4
	Fourth	40	160	4	6
	Fifth	40	200	5	7
	Sixth	40	240	6	8
	Seventh	40	280	8	9

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



J Sample-size letter TABLE 4 (*continued*)
1.2–2.2 AQL class (percent defective)
1.5–2.5 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	150	150	6	7
Double	First	100	100	3	10
	Second	200	300	9	10
Sequential	First	40	40	0	4
	Second	40	80	1	5
	Third	40	120	3	6
	Fourth	40	160	5	8
	Fifth	40	200	6	10
	Sixth	40	240	8	12
	Seventh	40	280	11	12

b. Operating-characteristic Curves

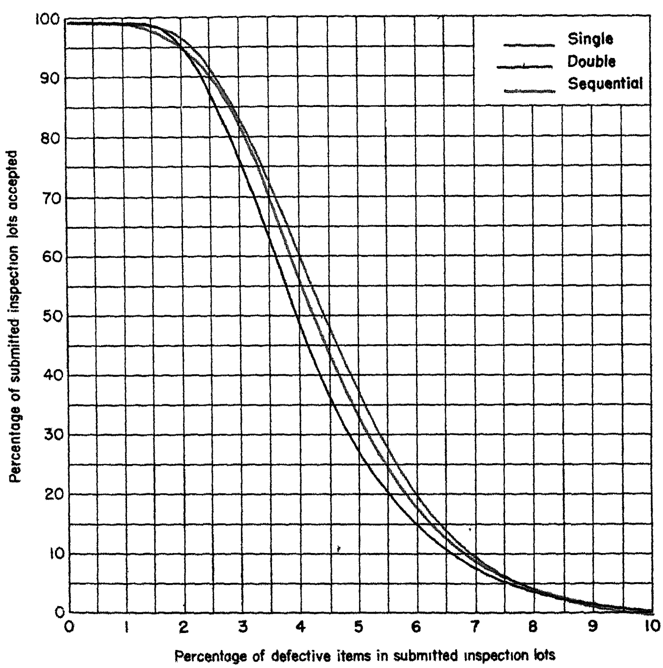


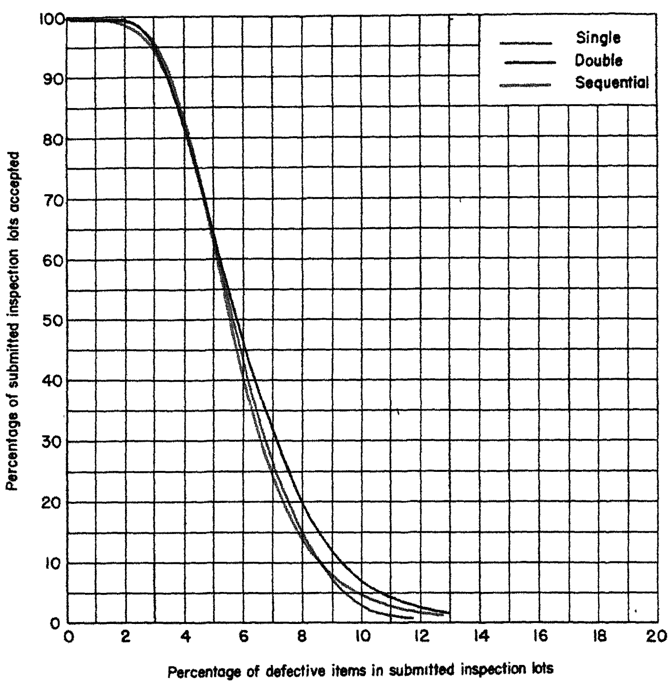
TABLE 4 (*continued*)

Sample-size letter **J**
AQL class (percent defective) **2.2–3.2**
AOQL class (percent defective) **2.5–3.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	150	150	8	9
Double	First	100	100	5	12
	Second	200	300	11	12
Sequential	First	40	40	0	4
	Second	40	80	2	7
	Third	40	120	5	9
	Fourth	40	160	7	11
	Fifth	40	200	9	13
	Sixth	40	240	11	15
	Seventh	40	280	15	16

b. Operating-characteristic Curves



J Sample-size letter

3.2–4.4 AQL class (percent defective)

3.5–5.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	150	150	10	11
Double	First	100	100	6	17
	Second	200	300	16	17
Sequential	First	40	40	1	5
	Second	40	80	2	8
	Third	40	120	5	11
	Fourth	40	160	8	14
	Fifth	40	200	11	17
	Sixth	40	240	14	19
	Seventh	40	280	18	19

b. Operating-characteristic Curves

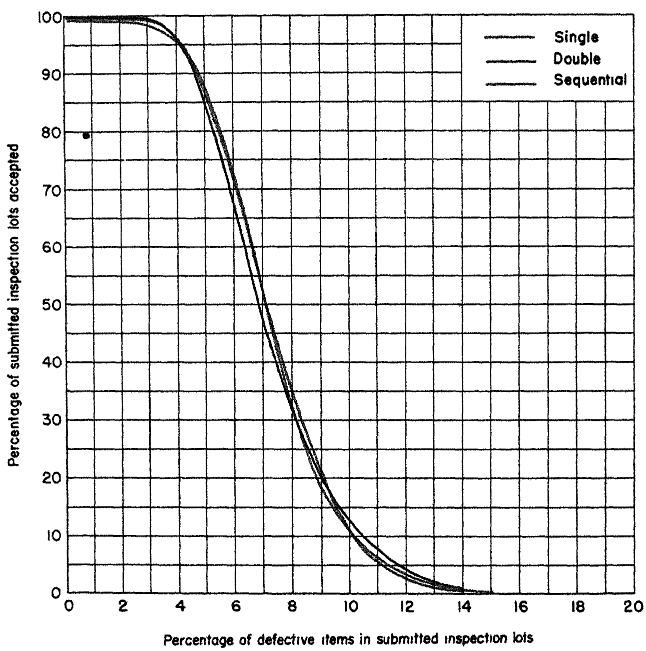


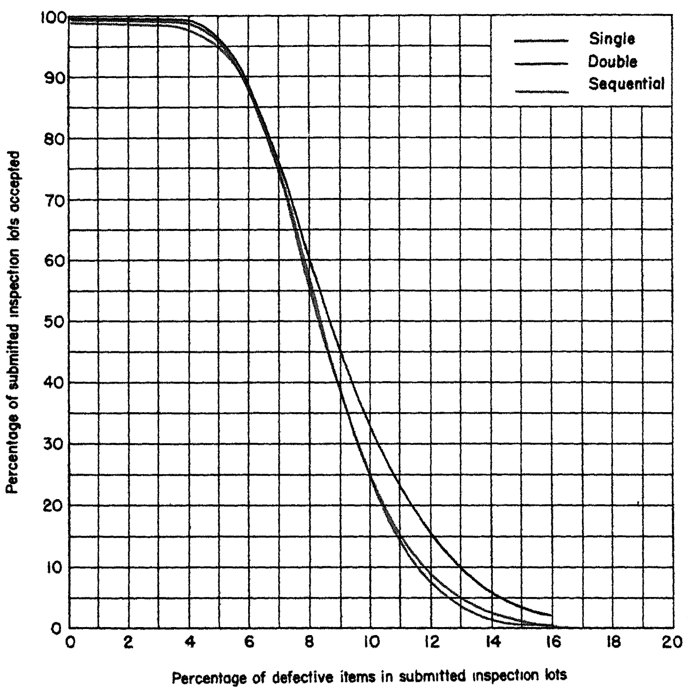
TABLE 4 (*continued*)

Sample-size letter J
AQL class (percent defective) **4.4–5.3**
AOQL class (percent defective) **5.0–7.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	150	150	12	13
Double	First	100	100	8	20
	Second	200	300	19	20
Sequential	First	40	40	1	6
	Second	40	80	5	9
	Third	40	120	7	12
	Fourth	40	160	10	16
	Fifth	40	200	13	19
	Sixth	40	240	17	21
	Seventh	40	280	22	23

b. Operating-characteristic Curves



J Sample-size letter
5.3–6.4 AQL class (percent defective)
5.0–7.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	150	150	14	15
Double	First....................	100	100	8	26
	Second............. . .	200	300	25	26
Sequential	First.........	40	40	2	6
	Second	40	80	5	11
	Third..........	40	120	9	15
	Fourth..	40	160	13	19
	Fifth....	40	200	17	22
	Sixth....	40	240	22	26
	Seventh	40	280	26	27

b. Operating-characteristic Curves

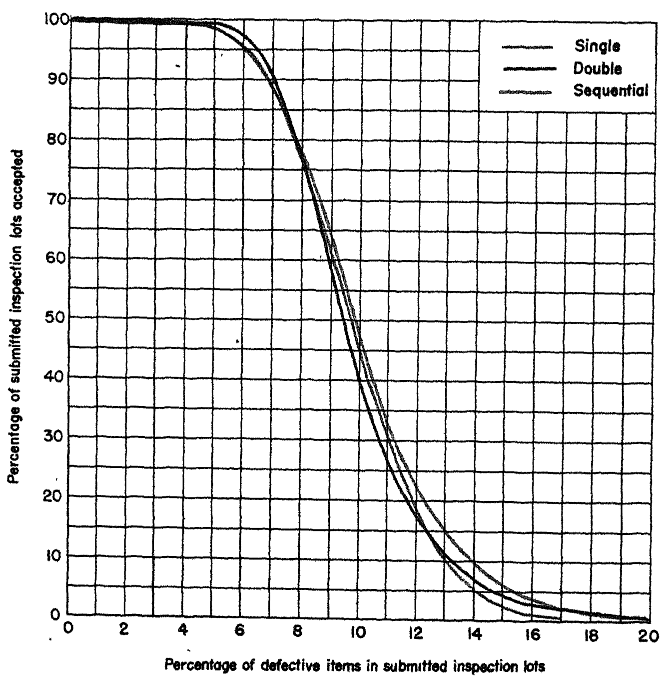


TABLE 4 (*continued*)

Sample-size letter **J**
AQL class (percent defective) **6.4– 8.5**
AOQL class (percent defective) **7.0–11.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	150	150	18	19
Double	First	100	100	11	31
	Second	200	300	30	31
Sequential	First	40	40	2	9
	Second	40	80	7	12
	Third	40	120	13	16
	Fourth	40	160	18	21
	Fifth	40	200	23	26
	Sixth	40	240	28	31
	Seventh	40	280	30	31

b. Operating-characteristic Curves

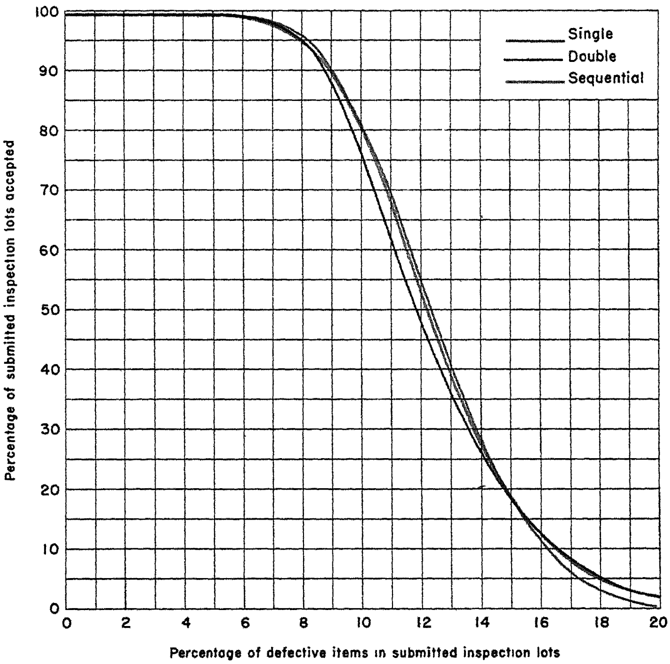


TABLE 4 (*continued*)

J Sample-size letter
8.5–11.0† AQL class (percent defective)
7.0–11.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	150	150	20	21
Double	First	100	100	12	34
	Second	200	300	33	34
Sequential	First	40	40	3	8
	Second	40	80	8	15
	Third	40	120	13	20
	Fourth	40	160	18	25
	Fifth	40	200	24	30
	Sixth	40	240	30	34
	Seventh	40	280	36	37

† The plans on this page are not included in Table 2 because the AQL of each plan is greater than 8.5.

b. Operating-characteristic Curves

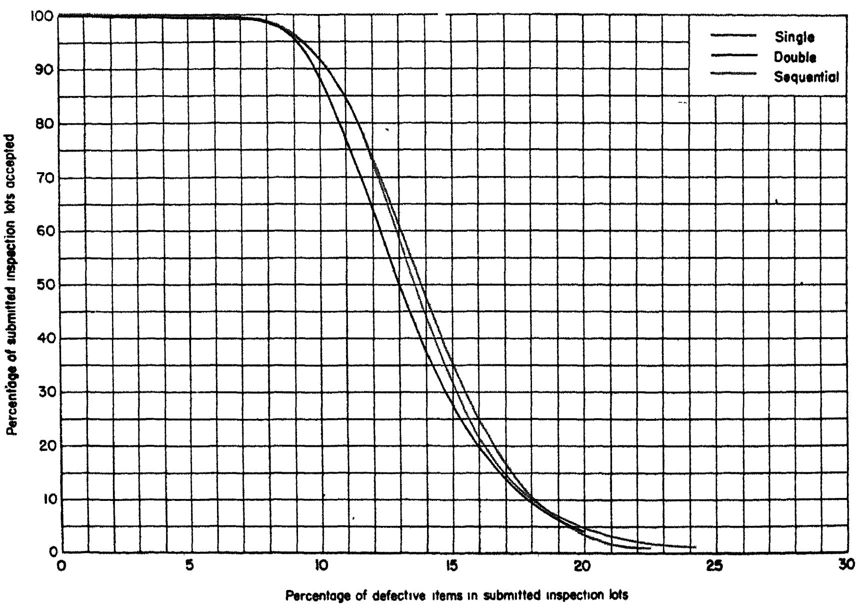


TABLE 4 (*continued*) Sample-size letter **K**
AQL classes (percent defective) **0.024–0.035**
0.035–0.06

For AQL class	Use sample-size letter
0.024–0.035	M
0.035–0.06	L

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

K Sample-size letter
0.06–0.12 AQL class (percent defective)
0.22–0.30 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	For single sampling, use sample-size letter M			
Double	First Second	For double sampling, use sample-size letter L			
Sequential	First	50	50	*	2
	Second	50	100	*	2
	Third	50	150	*	2
	Fourth	50	200	0	2
	Fifth	50	250	0	2
	Sixth	50	300	0	2
	Seventh	50	350	0	2
	Eighth	50	400	1	2

* Acceptance not permitted until four samples have been inspected.

b. Operating-characteristic Curve

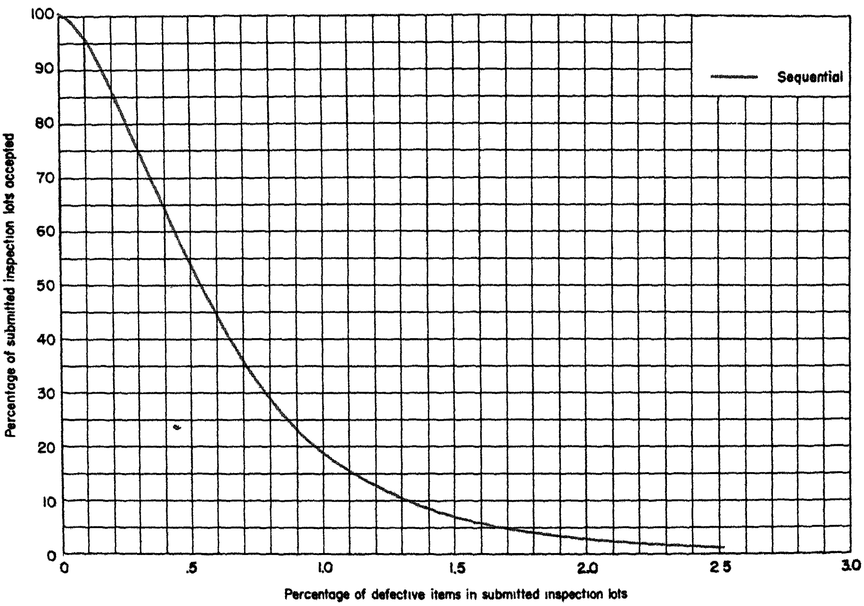


TABLE 4 (*continued*)

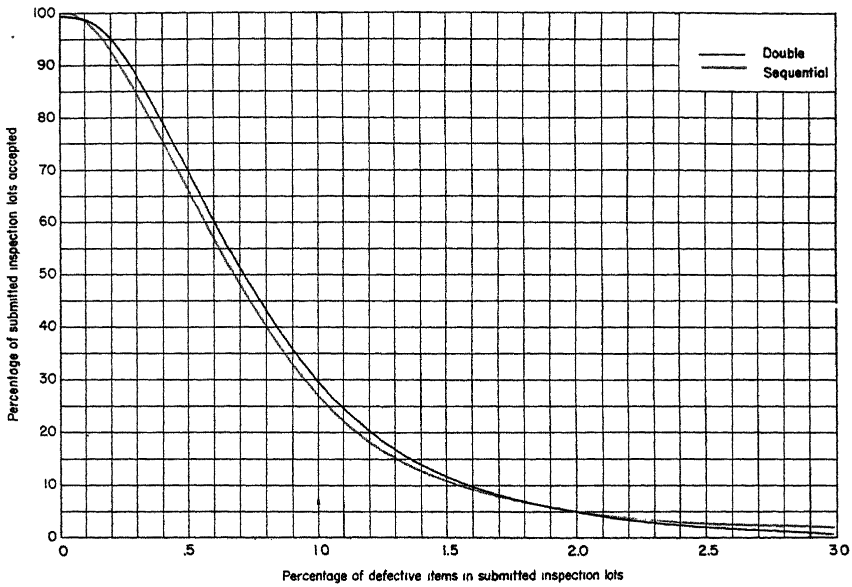
Sample-size letter **K**
AQL class (percent defective) **0.12–0.17**
AOQL class (percent defective) **0.30–0.50**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	For single sampling, use sample-size letter L			
Double	First	150	150	0	3
	Second	300	450	2	3
Sequential	First	50	50	*	2
	Second	50	100	*	2
	Third	50	150	0	2
	Fourth	50	200	0	2
	Fifth	50	250	0	2
	Sixth	50	300	0	3
	Seventh	50	350	0	3
	Eighth	50	400	2	3

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves



K Sample-size letter
0.17–0.22 AQL class (percent defective)
0.30–0.50 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	225	225	1	2
Double	First	150	150	0	3
	Second	300	450	2	3
Sequential	First	50	50	*	2
	Second	50	100	*	2
	Third	50	150	0	2
	Fourth	50	200	0	3
	Fifth	50	250	1	3
	Sixth	50	300	1	3
	Seventh	50	350	2	4
	Eighth	50	400	3	4

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

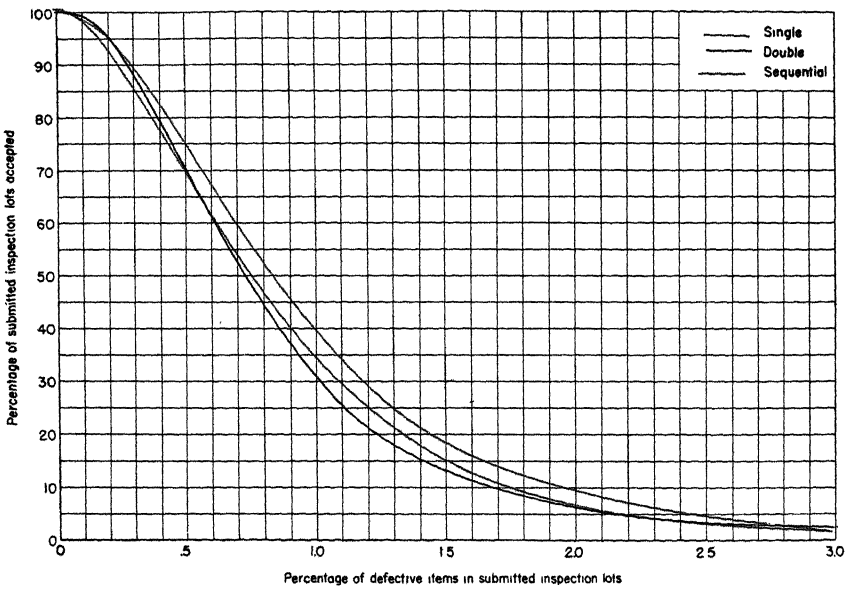


TABLE 4 (*continued*)

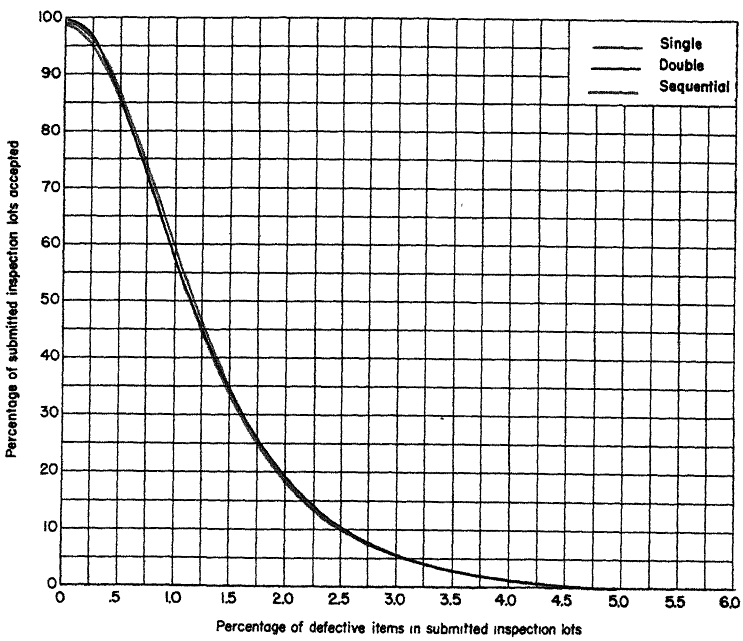
Sample-size letter **K**
AQL class (percent defective) **0.22–0.32**
AOQL class (percent defective) **0.50–0.90**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	225	225	2	3
Double	First	150	150	1	3
	Second	300	450	2	3
Sequential	First	50	50	*	2
	Second	50	100	0	2
	Third	50	150	0	3
	Fourth	50	200	1	3
	Fifth	50	250	2	4
	Sixth	50	300	2	4
	Seventh	50	350	2	4
	Eighth	50	400	3	4

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



K Sample-size letter TABLE 4 (*continued*)
0.32–0.65 AQL class (percent defective)
0.90–1.5 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	225	225	3	4
Double	First	150	150	2	5
	Second	300	450	4	5
Sequential	First	50	50	*	2
	Second	50	100	0	4
	Third	50	150	2	4
	Fourth	50	200	2	5
	Fifth	50	250	3	6
	Sixth	50	300	4	7
	Seventh	50	350	5	7
	Eighth	50	400	6	7

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

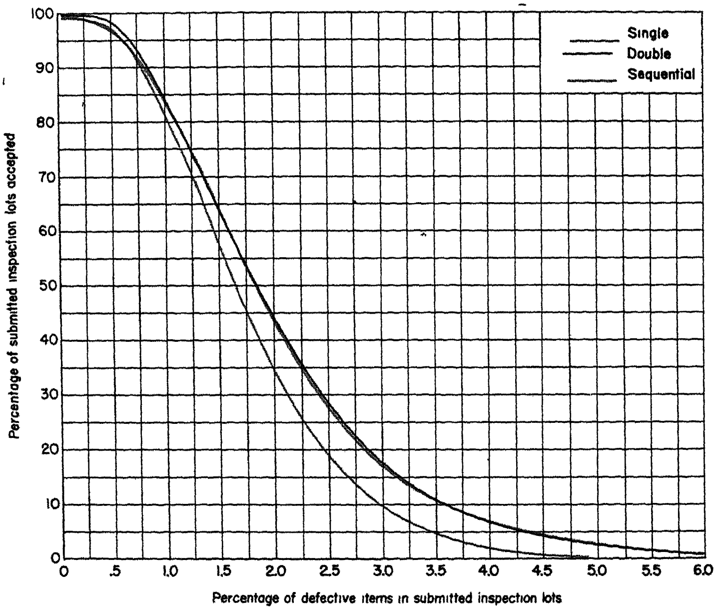


TABLE 4 (*continued*)

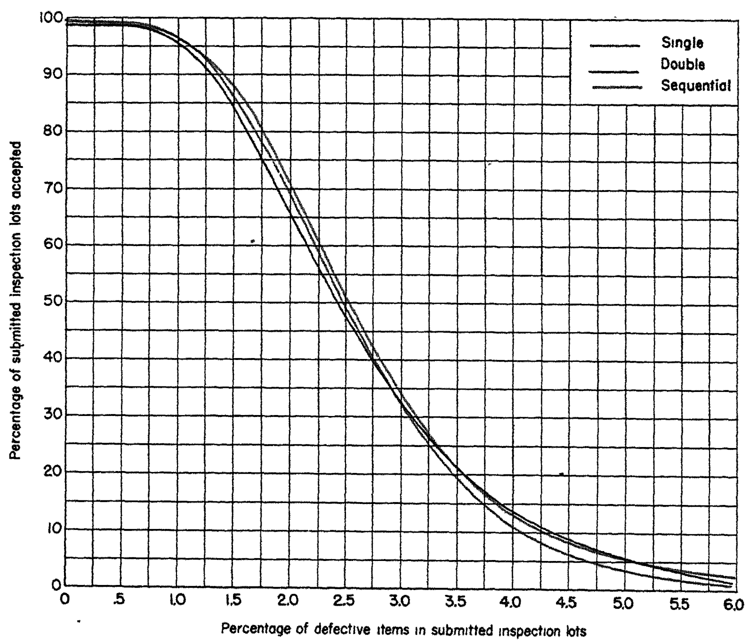
Sample-size letter **K**
AQL class (percent defective) **0.65–1.2**
AOQL class (percent defective) **0.90–1.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	225	225	5	6
Double	First	150	150	3	8
	Second	300	450	7	8
Sequential	First	50	50	*	3
	Second	50	100	1	4
	Third	50	150	2	6
	Fourth	50	200	3	7
	Fifth	50	250	4	8
	Sixth	50	300	5	9
	Seventh	50	350	7	10
	Eighth	50	400	9	10

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



K Sample-size letter

1.2–2.2 AQL class (percent defective)

1.5–2.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First.	225	225	8	9
Double	First.	150	150	5	14
	Second.	300	450	13	14
Sequential	First.	50	50	0	4
	Second.	50	100	2	6
	Third.	50	150	3	8
	Fourth.	50	200	5	10
	Fifth.	50	250	7	12
	Sixth.	50	300	9	14
	Seventh.	50	350	11	16
	Eighth.	50	400	15	16

b. Operating-characteristic Curves

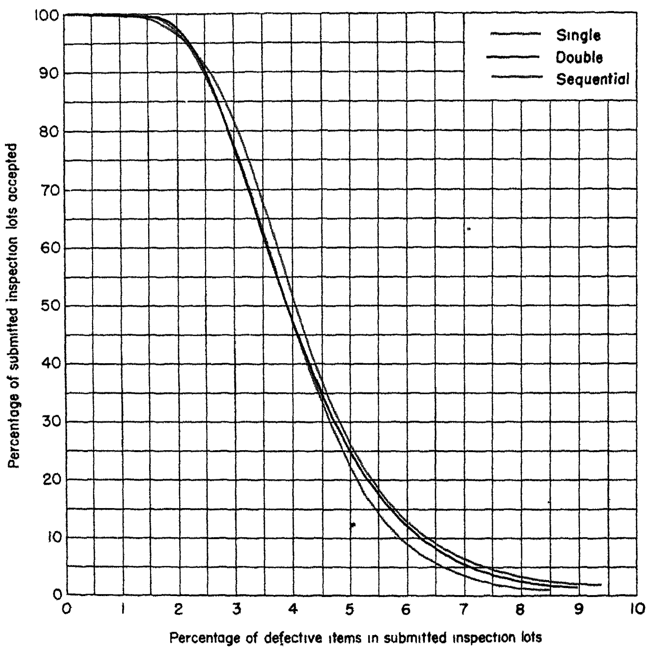


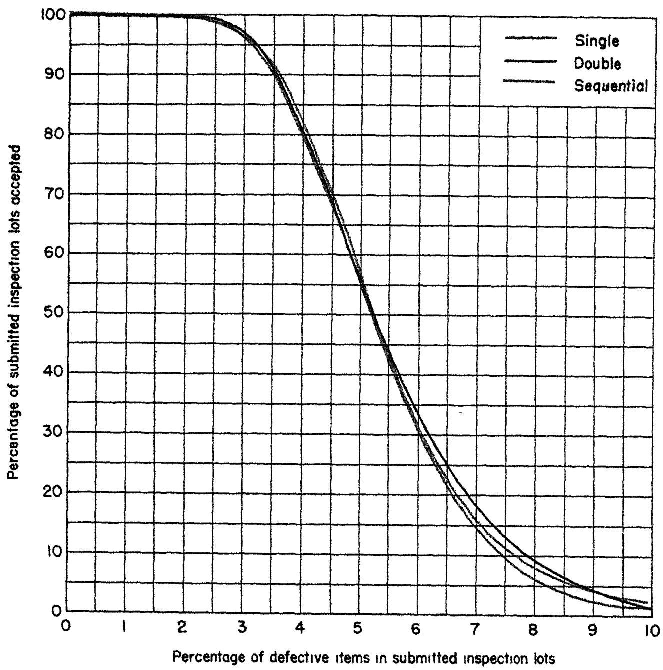
TABLE 4 (*continued*)

Sample-size letter **K**
AQL class (percent defective) **2.2–3.2**
AOQL class (percent defective) **2.5–3.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	225	225	11	12
Double	First	150	150	7	19
	Second	300	450	18	19
Sequential	First	50	50	0	5
	Second	50	100	3	8
	Third	50	150	5	11
	Fourth	50	200	8	13
	Fifth	50	250	10	15
	Sixth	50	300	13	18
	Seventh	50	350	15	20
	Eighth	50	400	19	20

b. Operating-characteristic Curves



K Sample-size letter
3.2–4.4 AQL class (percent defective)
3.5–5.0 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	225	225	14	15
Double	First	150	150	9	24
	Second	300	450	23	24
Sequential	First	50	50	1	6
	Second	50	100	3	9
	Third	50	150	7	13
	Fourth	50	200	10	16
	Fifth	50	250	13	19
	Sixth	50	300	16	22
	Seventh	50	350	19	25
	Eighth	50	400	24	25

b. Operating-characteristic Curves

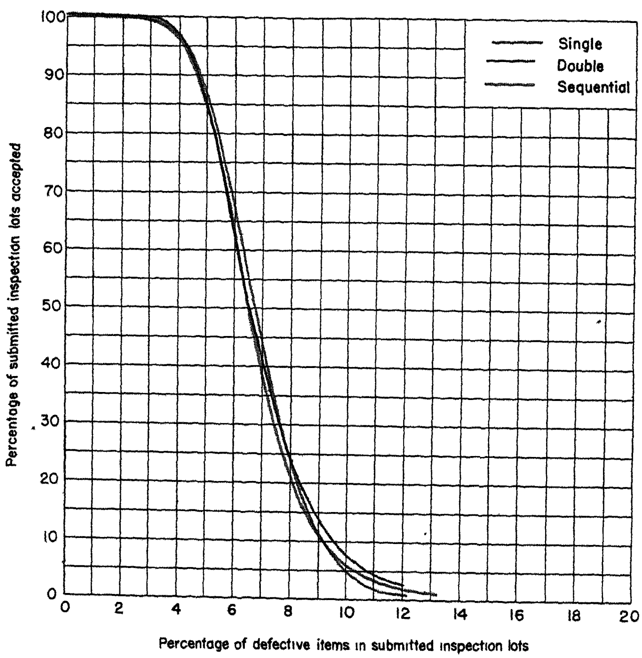


TABLE 4 (*continued*)

Sample-size letter **K**
AQL class (percent defective) **4.4–5.3**
AOQL class (percent defective) **5.0–7.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	225	225	17	18
Double	First	150	150	11	29
	Second	300	450	28	29
Sequential	First	50	50	2	7
	Second	50	100	4	11
	Third	50	150	8	15
	Fourth	50	200	12	19
	Fifth	50	250	16	22
	Sixth	50	300	21	26
	Seventh	50	350	25	29
	Eighth	50	400	28	29

b. Operating-characteristic Curves

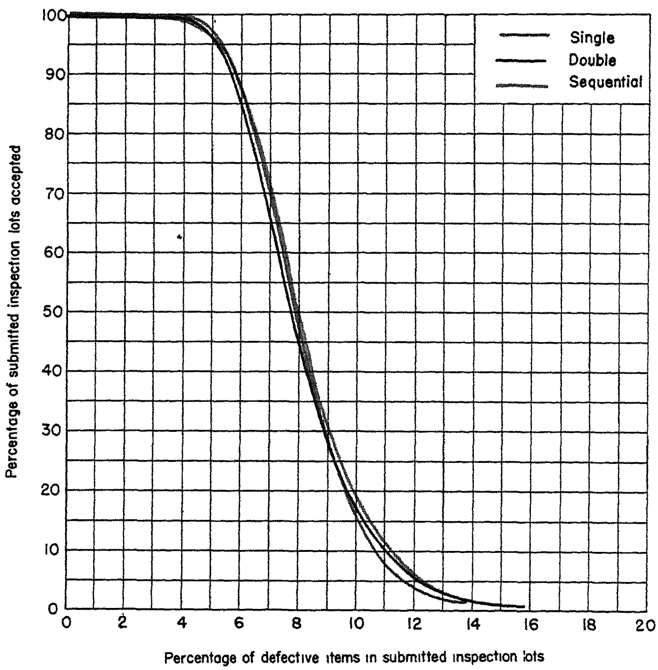


TABLE 4 (*continued*)

K Sample-size letter
5.3–6.4 AQL class (percent defective)
5.0–7.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	225	225	21	22
Double	First	150	150	14	34
	Second	300	450	33	34
Sequential	First	50	50	2	7
	Second	50	100	7	13
	Third	50	150	12	18
	Fourth	50	200	17	22
	Fifth	50	250	21	27
	Sixth	50	300	26	32
	Seventh	50	350	30	36
	Eighth	50	400	36	37

b. Operating-characteristic Curves

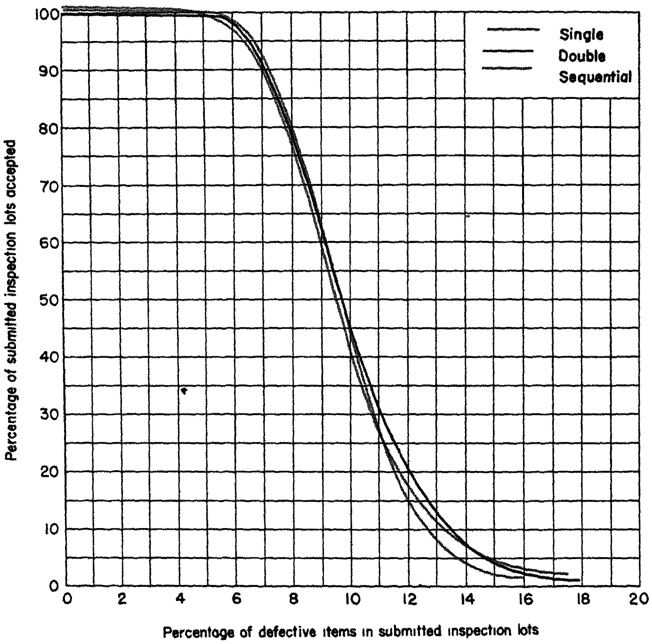


TABLE 4 (*continued*)

Sample-size letter **K**
AQL class (percent defective) **6.4– 8.5**
AOQL class (percent defective) **7.0–11.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	225	225	25	26
Double	First	150	150	16	44
	Second	300	450	43	44
Sequential	First	50	50	3	9
	Second	50	100	8	14
	Third	50	150	14	20
	Fourth	50	200	19	25
	Fifth	50	250	25	31
	Sixth	50	300	30	38
	Seventh	50	350	36	43
	Eighth	50	400	43	44

b. Operating-characteristic Curves

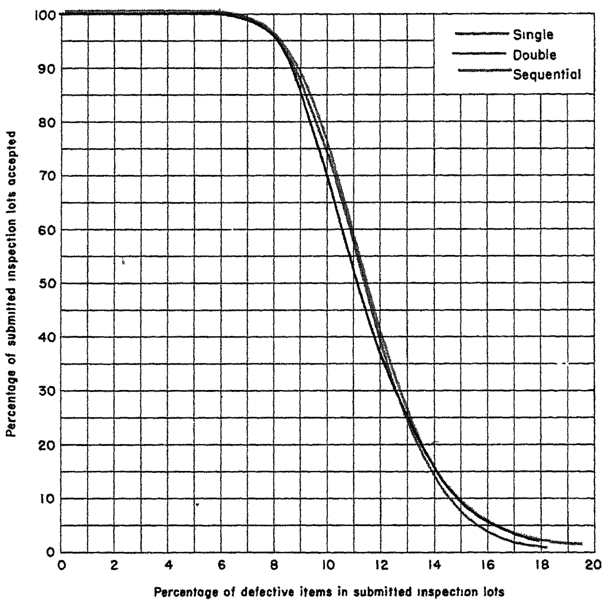


TABLE 4 (*continued*)

K Sample-size letter
8.5–11.0† AQL class (percent defective)
7.0–11.0 AOQL class (percent defective)

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	225	225	29	30
Double	First	150	150	17	55
	Second	300	450	54	55
Sequential	First	50	50	3	10
	Second	50	100	10	17
	Third	50	150	16	24
	Fourth	50	200	21	30
	Fifth	50	250	28	37
	Sixth	50	300	36	43
	Seventh	50	350	44	48
	Eighth	50	400	51	52

† The plans on this page are not included in Table 2 because the AQL of each plan is greater than 8.5.

b. OPERATING-CHARACTERISTIC CURVES

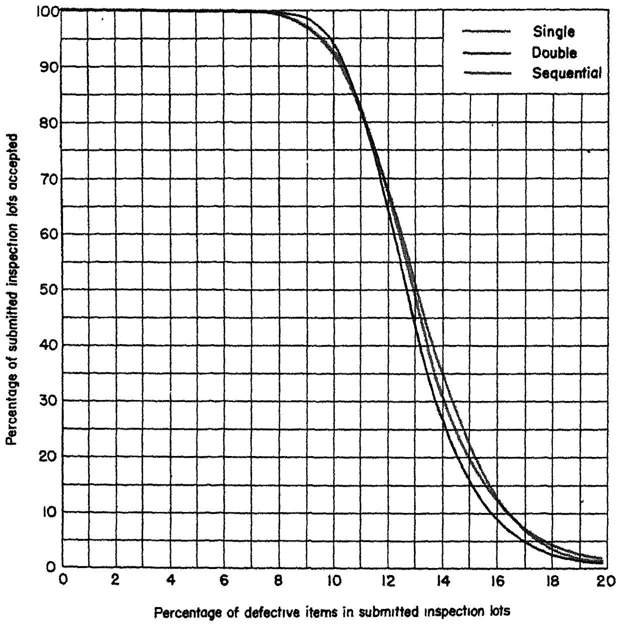


TABLE 4 (*continued*)

Sample-size letter **L**
AQL class (percent defective) **0.024–0.035**

For AQL class	Use sample-size letter
0.024–0.035	M

Note: If sample size exceeds inspection-lot size, 100% inspection must be used or larger inspection lots formed.

TABLE 4 (*continued*)

L Sample-size letter
0.035–0.06 AQL class (percent defective)
0.15 –0.22 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	For single sampling, use sample-size letter N			
Double	First Second	For double sampling, use sample-size letter M			
Sequential	First	75	75	*	1
	Second	75	150	*	2
	Third	75	225	*	2
	Fourth	75	300	0	2
	Fifth	75	375	0	2
	Sixth	75	450	1	3
	Seventh	75	525	2	3

* Acceptance not permitted until four samples have been inspected.

b. Operating-characteristic Curve

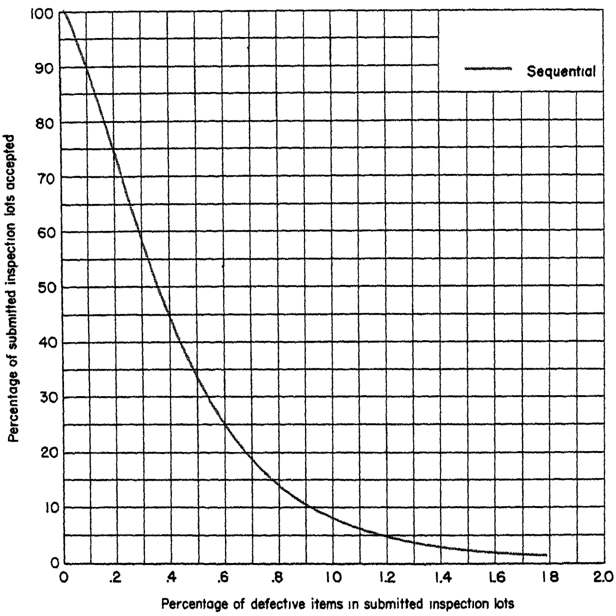


TABLE 4 (*continued*)

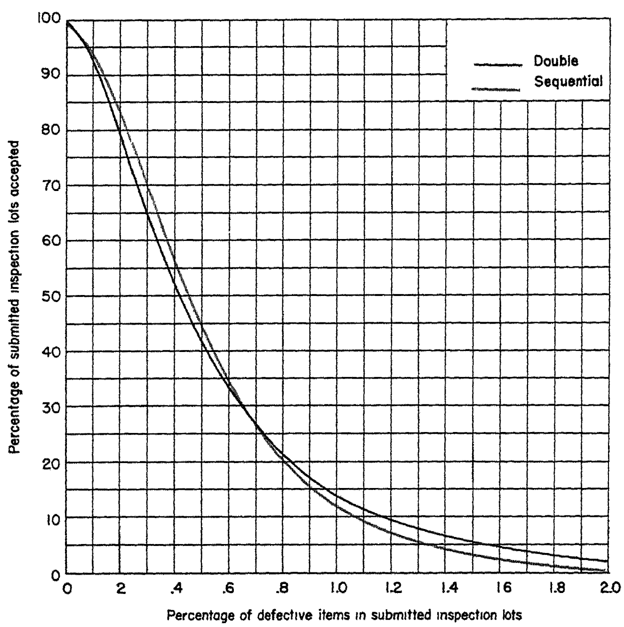
Sample-size letter **L**
AQL class (percent defective) **0.06–0.12**
AOQL class (percent defective) **0.15–0.22**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	For single sampling, use sample-size letter M			
Double	First	200	200	0	2
	Second	400	600	1	2
Sequential	First	75	75	*	2
	Second	75	150	*	2
	Third	75	225	0	2
	Fourth	75	300	0	2
	Fifth	75	375	0	2
	Sixth	75	450	1	3
	Seventh	75	525	2	3

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves



L Sample-size letter
0.12–0.17 AQL class (percent defective)
0.22–0.30 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First	300	300	1	2
Double	First	200	200	0	3
	Second	400	600	2	3
Sequential	First	75	75	*	2
	Second	75	150	*	2
	Third	75	225	0	2
	Fourth	75	300	0	3
	Fifth	75	375	1	4
	Sixth	75	450	2	4
	Seventh	75	525	3	4

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

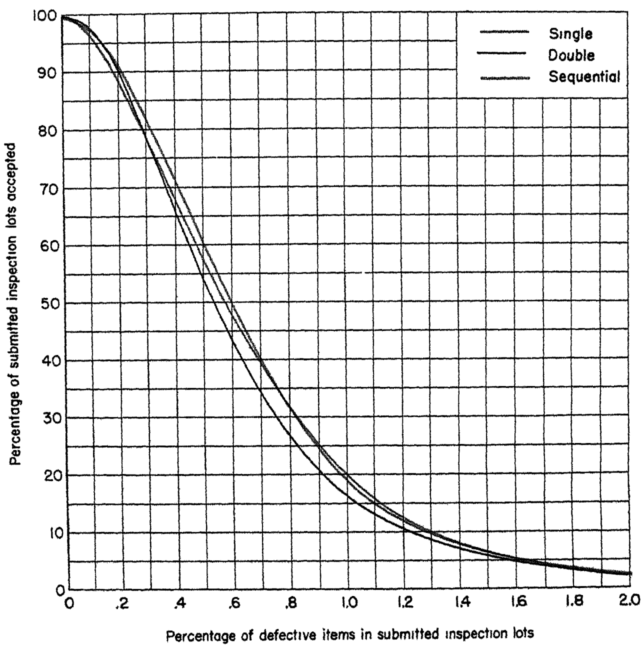


TABLE 4 (*continued*)

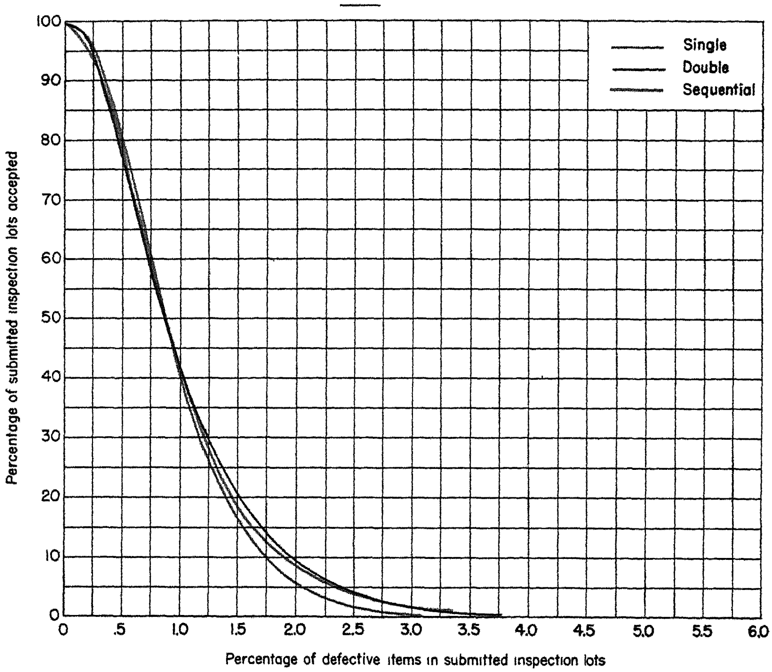
Sample-size letter **L**
AQL class (percent defective) **0.17–0.22**
AOQL class (percent defective) **0.30–0.50**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	300	300	2	3
Double	First........	200	200	1	3
	Second.................	400	600	2	3
Sequential	First....	75	75	*	2
	Second..................	75	150	0	2
	Third..................	75	225	0	4
	Fourth..................	75	300	2	4
	Fifth..................	75	375	2	5
	Sixth..................	75	450	2	5
	Seventh................	75	525	4	5

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES



L Sample-size letter
0.22–0.32 AQL class (percent defective)
0.30–0.50 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	300	300	2	3
Double	First	200	200	1	4
	Second	400	600	3	4
Sequential	First	75	75	*	2
	Second	75	150	0	3
	Third	75	225	0	3
	Fourth	75	300	1	4
	Fifth	75	375	2	5
	Sixth	75	450	2	5
	Seventh	75	525	4	5

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

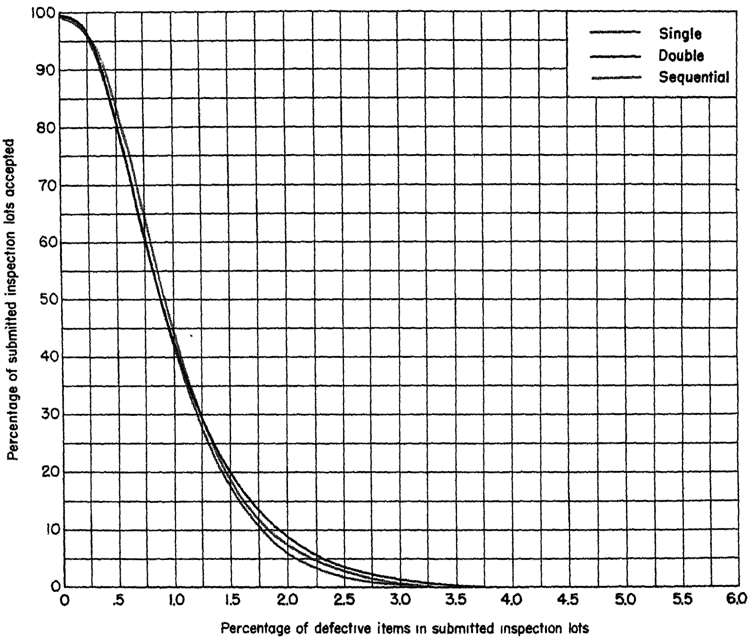


TABLE 4 (*continued*)

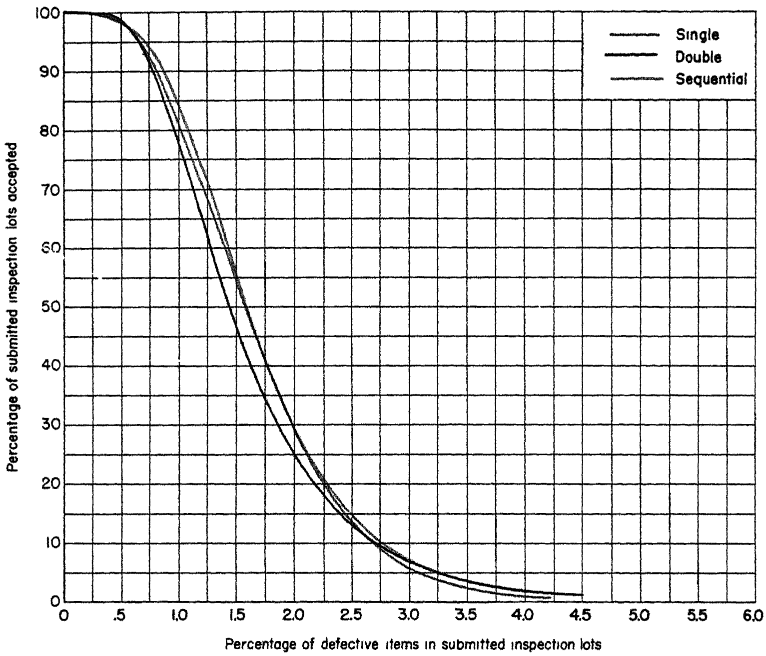
Sample-size letter **L**
AQL class (percent defective) **0.32–0.65**
AOQL class (percent defective) **0.50–0.90**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	300	300	4	5
Double	First	200	200	2	7
	Second	400	600	6	7
Sequential	First	75	75	*	3
	Second	75	150	1	4
	Third	75	225	1	5
	Fourth	75	300	2	6
	Fifth	75	375	3	6
	Sixth	75	450	5	8
	Seventh	75	525	7	8

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



L Sample-size letter
0.65–1.2 AQL class (percent defective)
0.90–1.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	300	300	7	8
Double	First	200	200	4	10
	Second	400	600	9	10
Sequential	First	75	75	0	4
	Second	75	150	2	5
	Third	75	225	3	7
	Fourth	75	300	5	9
	Fifth	75	375	7	10
	Sixth	75	450	9	11
	Seventh	75	525	10	11

b. Operating-characteristic Curves

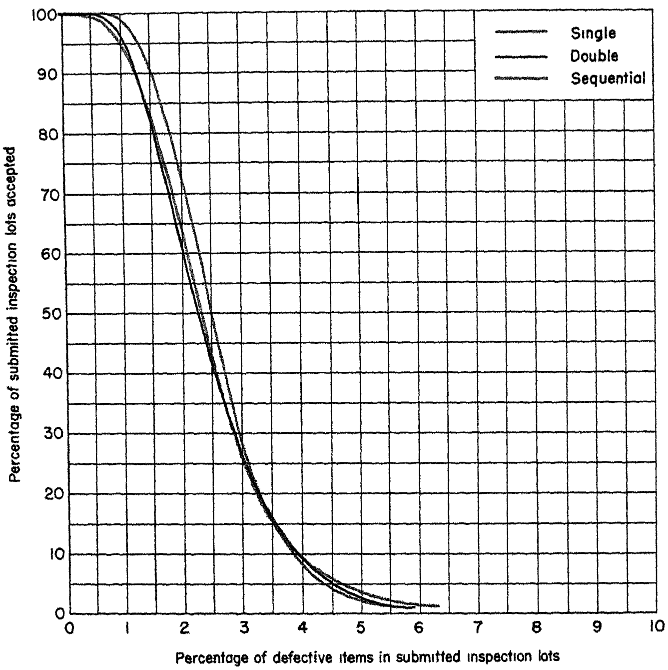


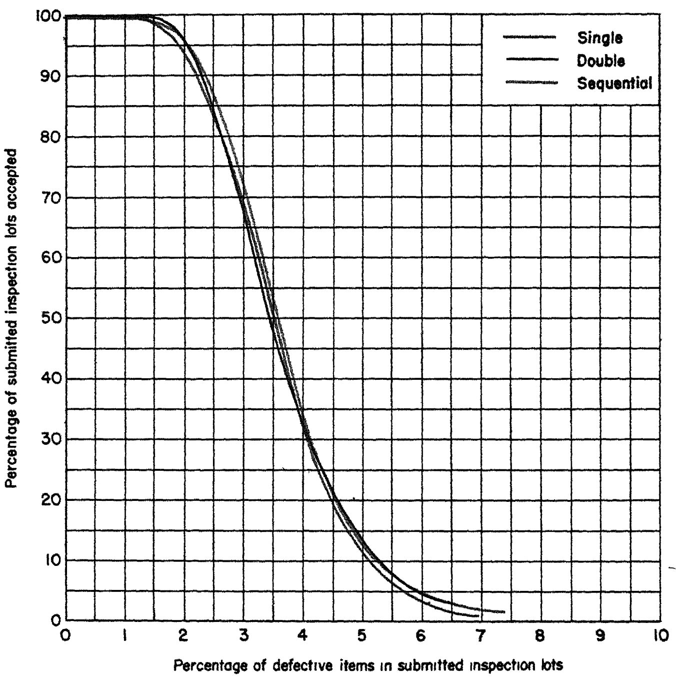
TABLE 4 (*continued*)

Sample-size letter **L**
AQL class (percent defective) **1.2–2.2**
AOQL class (percent defective) **1.5–2.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First..................	300	300	10	11
Double	First..................	200	200	6	17
	Second..................	400	600	16	17
Sequential	First..................	75	75	0	5
	Second..................	75	150	3	8
	Third..................	75	225	6	10
	Fourth..................	75	300	8	13
	Fifth..................	75	375	11	16
	Sixth..................	75	450	13	18
	Seventh..................	75	525	18	19

b. Operating-characteristic Curves



L Sample-size letter
2.2–3.2 AQL class (percent defective)
2.5–3.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First....................	300	300	14	15
Double	First....................	200	200	9	25
	Second..................	400	600	24	25
Sequential	First....................	75	75	1	7
	Second..................	75	150	4	10
	Third....................	75	225	8	14
	Fourth..................	75	300	12	18
	Fifth......................	75	375	15	21
	Sixth....................	75	450	18	25
	Seventh.................	75	525	24	25

b. Operating-characteristic Curves

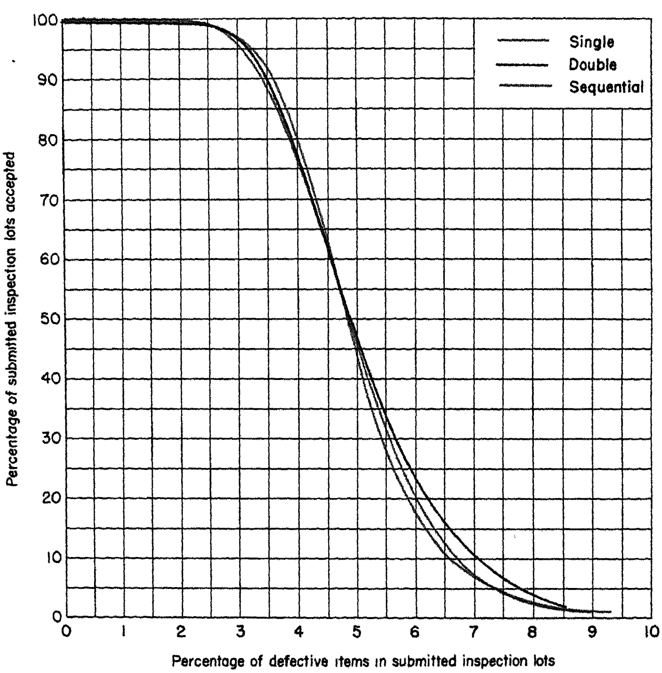


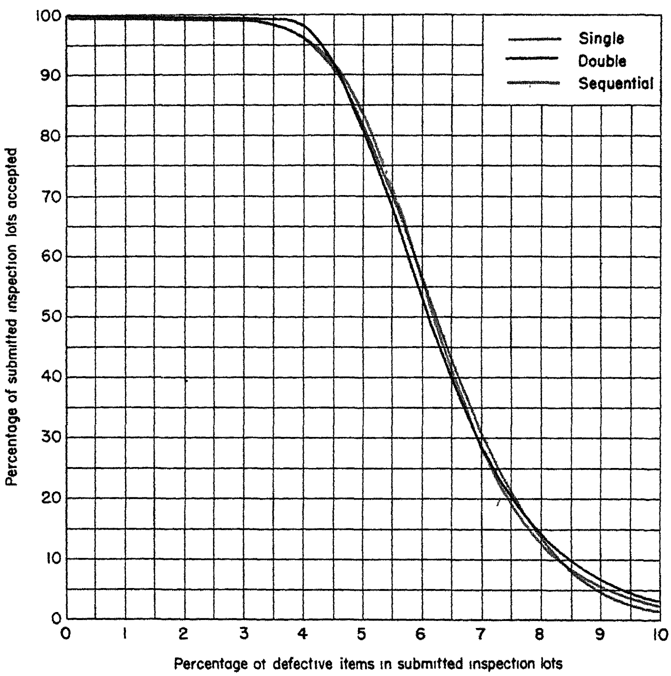
TABLE 4 (*continued*)

Sample-size letter **L**
AQL class (percent defective) **3.2–4.4**
AOQL class (percent defective) **3.5–5.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	300	300	18	19
Double	First	200	200	11	33
	Second	400	600	32	33
Sequential	First	75	75	2	8
	Second	75	150	6	13
	Third	75	225	10	17
	Fourth	75	300	15	22
	Fifth	75	375	19	26
	Sixth	75	450	24	31
	Seventh	75	525	30	31

b. Operating-characteristic Curves



L Sample-size letter TABLE 4 (*continued*)
4.4–5.3 AQL class (percent defective)
5.0–7.0 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First.	300	300	22	23
Double	First.	200	200	14	38
	Second.	400	600	37	38
Sequential	First	75	75	3	9
	Second.	75	150	8	15
	Third.	75	225	13	20
	Fourth.	75	300	19	26
	Fifth.	75	375	25	32
	Sixth.	75	450	30	37
	Seventh.	75	525	36	37

b. Operating-characteristic Curves

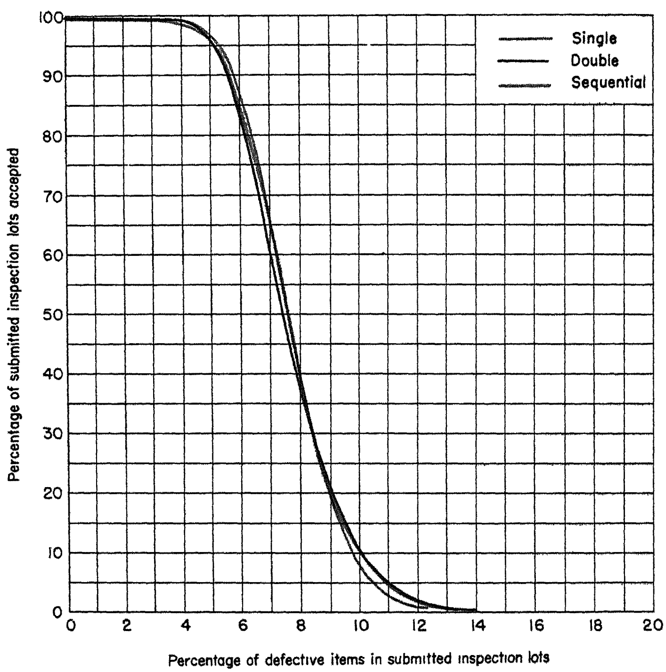


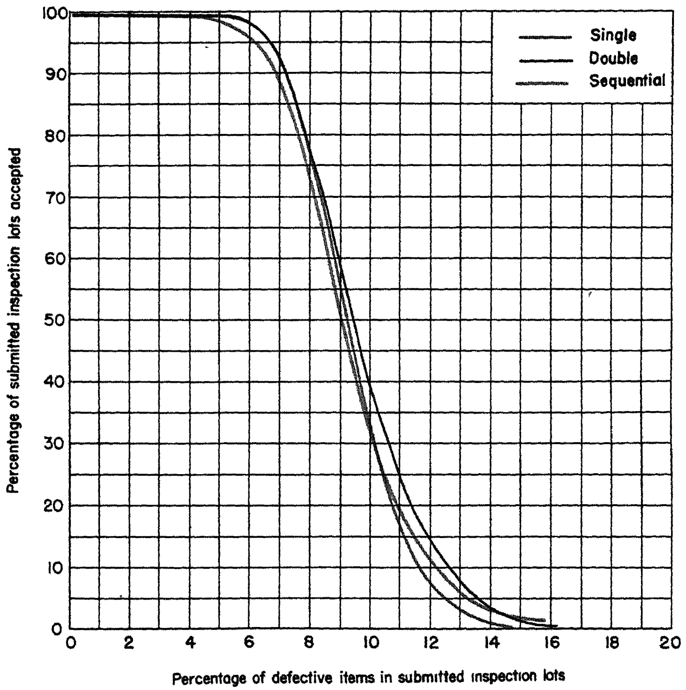
TABLE 4 (*continued*)

Sample-size letter **L**
AQL class (percent defective) **5.3–6.4**
AOQL class (percent defective) **5.0–7.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	300	300	27	28
Double	First	200	200	18	45
	Second	400	600	44	45
Sequential	First	75	75	4	10
	Second	75	150	11	16
	Third	75	225	18	23
	Fourth	75	300	25	30
	Fifth	75	375	32	36
	Sixth	75	450	39	43
	Seventh	75	525	42	43

b. Operating-characteristic Curves



L Sample-size letter
6.4–8.5 AQL class (percent defective)

TABLE 4 (*continued*)

For AQL class	Use sample-size letter
6.4–8.5	K

TABLE 4 (*continued*)

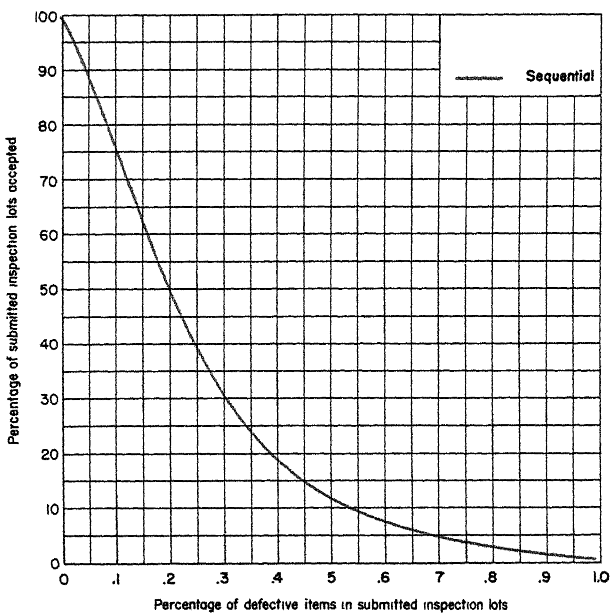
Sample-size letter **M**
AQL class (percent defective) **0.024–0.035**
AOQL class (percent defective) **0.10 –0.15**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First....................	For single sampling, use sample-size letter O			
Double	First.................... Second....................	For double sampling, use sample-size letter N			
Sequential	First....................	100	100	*	1
	Second....................	100	200	*	2
	Third....................	100	300	*	2
	Fourth....................	100	400	*	2
	Fifth....................	100	500	0	2
	Sixth....................	100	600	0	2
	Seventh....................	100	700	0	2
	Eighth....................	100	800	1	2

* Acceptance not permitted until five examples have been inspected.

b. OPERATING-CHARACTERISTIC CURVE



M Sample-size letter TABLE 4 (*continued*)
0.035–0.06 AQL class (percent defective)
0.10 –0.15 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	For single sampling, use sample-size letter N			
Double	First	300	300	0	2
	Second	600	900	1	2
Sequential	First	100	100	*	1
	Second	100	200	*	2
	Third	100	300	*	2
	Fourth	100	400	0	3
	Fifth	100	500	0	3
	Sixth	100	600	1	3
	Seventh	100	700	1	3
	Eighth	100	800	2	3

* Acceptance not permitted until four samples have been inspected.

b. Operating-characteristic Curves

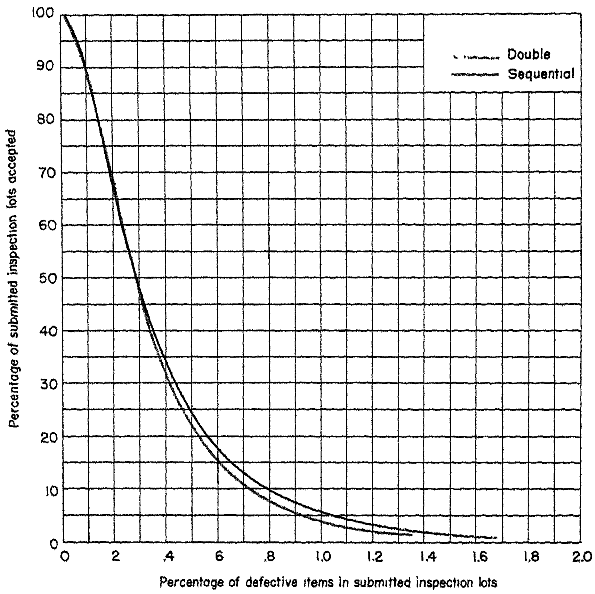


TABLE 4 (*continued*)

Sample-size letter **M**
AQL class (percent defective) **0.06–0.12**
AOQL class (percent defective) **0.15–0.22**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First....................	450	450	1	2
Double	First....................	300	300	0	3
	Second..................	600	900	2	3
Sequential	First....................	100	100	*	2
	Second..................	100	200	*	2
	Third...................	100	300	0	2
	Fourth..................	100	400	0	3
	Fifth...................	100	500	1	3
	Sixth...................	100	600	1	3
	Seventh.................	100	700	1	3
	Eighth..................	100	800	2	3

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

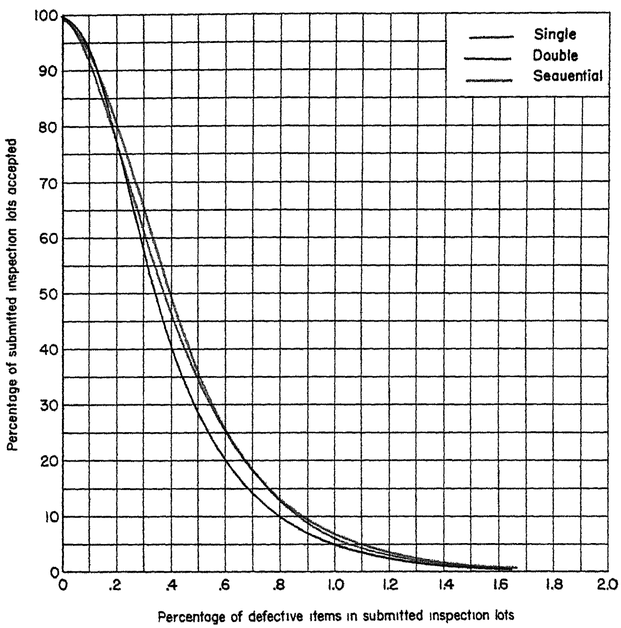


TABLE 4 (*continued*)

M Sample-size letter
0.12–0.17 AQL class (percent defective)
0.22–0.30 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	450	450	2	3
Double	First	300	300	1	3
	Second	600	900	2	3
Sequential	First	100	100	*	2
	Second	100	200	0	3
	Third	100	300	0	3
	Fourth	100	400	1	3
	Fifth	100	500	1	4
	Sixth	100	600	1	4
	Seventh	100	700	2	4
	Eighth	100	800	3	4

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

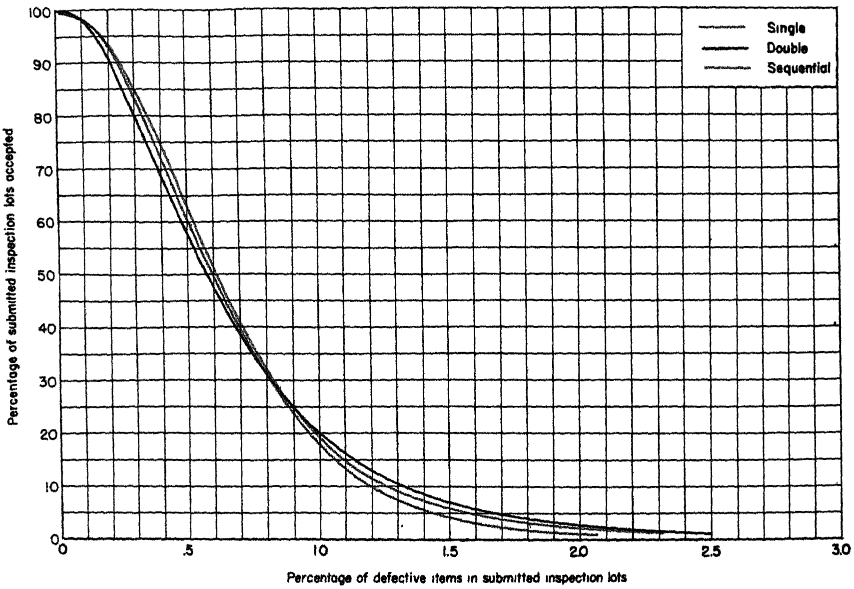


TABLE 4 (*continued*)

Sample-size letter **M**
AQL class (percent defective) **0.17–0.22**
AOQL class (percent defective) **0.30–0.50**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	450	450	2	3
Double	First	300	300	1	4
	Second	600	900	3	4
Sequential	First	100	100	*	2
	Second	100	200	0	3
	Third	100	300	0	3
	Fourth	100	400	1	4
	Fifth	100	500	1	4
	Sixth	100	600	2	4
	Seventh	100	700	3	5
	Eighth	100	800	5	6

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES

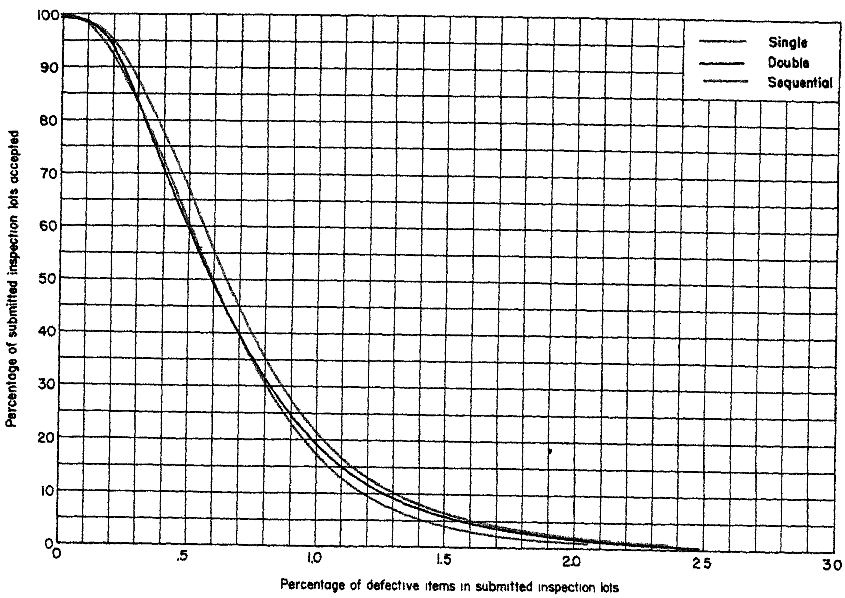


TABLE 4 (*continued*)

M Sample-size letter
0.22–0.32 AQL class (percent defective)
0.30–0.50 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First....................	450	450	3	4
Double	First....................	300	300	1	6
	Second..................	600	900	5	6
Sequential	First....................	100	100	*	3
	Second..................	100	200	0	3
	Third....................	100	300	0	4
	Fourth..................	100	400	1	4
	Fifth....................	100	500	2	5
	Sixth....................	100	600	3	6
	Seventh................	100	700	4	7
	Eighth..................	100	800	6	7

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

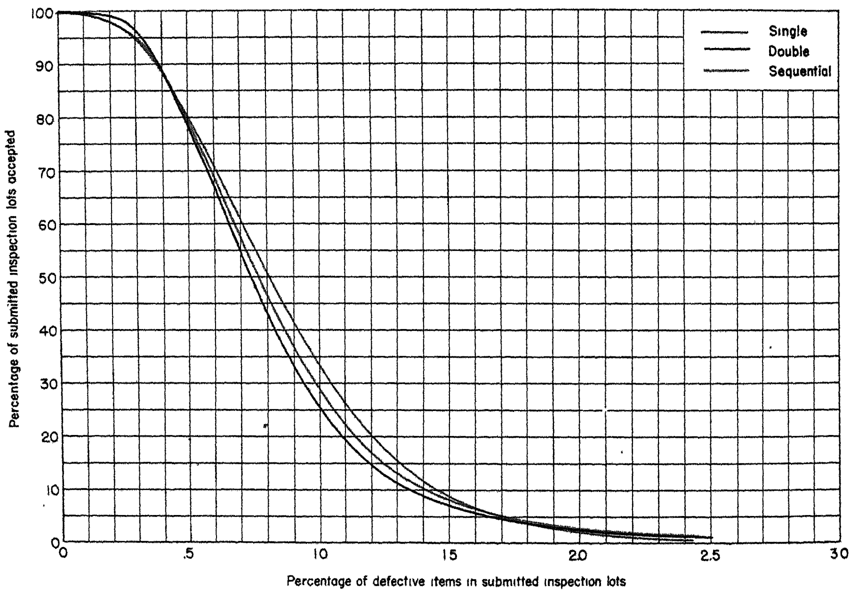


TABLE 4 (*continued*)

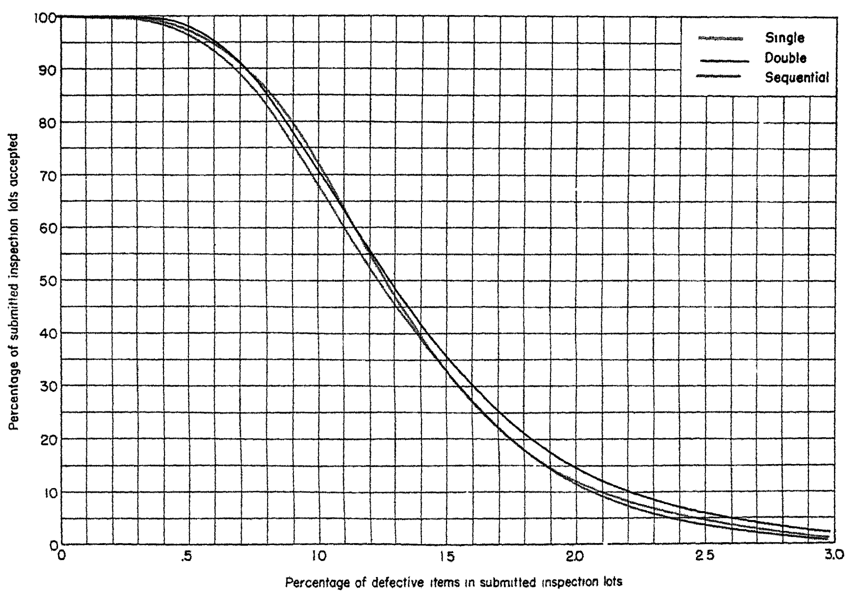
Sample-size letter **M**
AQL class (percent defective) **0.32–0.65**
AOQL class (percent defective) **0.50–0.90**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First....................	450	450	5	6
Double	First....................	300	300	3	9
	Second..	600	900	8	9
Sequential	First................ ...	100	100	*	3
	Second.................	100	200	1	5
	Third.	100	300	1	6
	Fourth.................	100	400	3	7
	Fifth...................	100	500	4	8
	Sixth...................	100	600	5	9
	Seventh................	.100	700	7	10
	Eighth	100	800	9	10

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



M Sample-size letter
0.65–1.2 AQL class (percent defective)
0.90–1.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	450	450	9	10
Double	First	300	300	5	16
	Second	600	900	15	16
Sequential	First	100	100	0	4
	Second	100	200	2	6
	Third	100	300	4	8
	Fourth	100	400	6	11
	Fifth	100	500	8	13
	Sixth	100	600	10	15
	Seventh	100	700	12	17
	Eighth	100	800	16	17

b. Operating-characteristic Curves

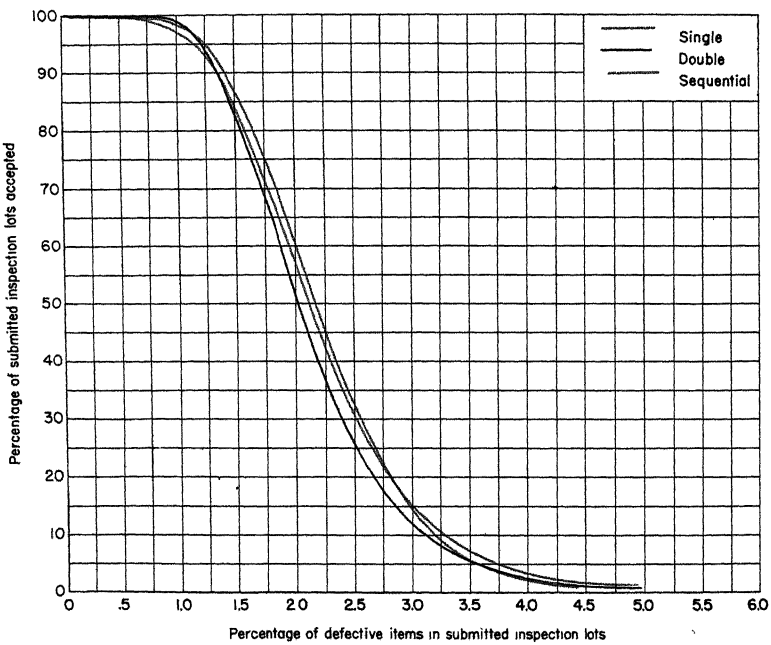


TABLE 4 (*continued*)

Sample-size letter **M**
AQL class (percent defective) **1.2–2.2**
AOQL class (percent defective) **1.5–2.5**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	450	450	14	15
Double	First	300	300	8	26
	Second	600	900	25	26
Sequential	First	100	100	0	7
	Second	100	200	4	9
	Third	100	300	6	13
	Fourth	100	400	9	16
	Fifth	100	500	12	19
	Sixth	100	600	15	22
	Seventh	100	700	19	25
	Eighth	100	800	24	25

b. Operating-characteristic Curves

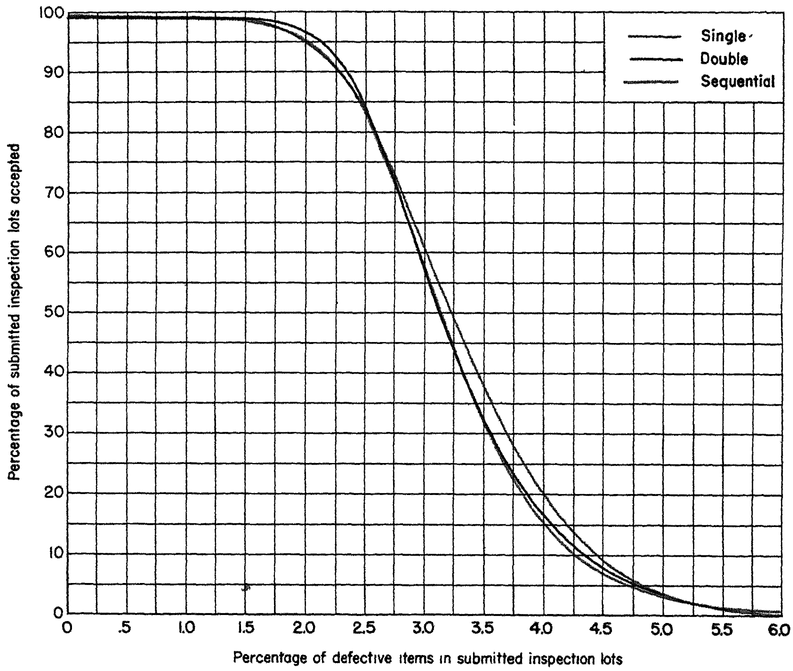


TABLE 4 (*continued*)

M Sample-size letter
2.2–3.2 AQL class (percent defective)
2.5–3.5 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	450	450	20	21
Double	First	300	300	12	36
	Second	600	900	35	36
Sequential	First	100	100	1	8
	Second	100	200	5	13
	Third	100	300	10	18
	Fourth	100	400	14	22
	Fifth	100	500	18	26
	Sixth	100	600	23	31
	Seventh	100	700	27	35
	Eighth	100	800	34	35

b. Operating-characteristic Curves

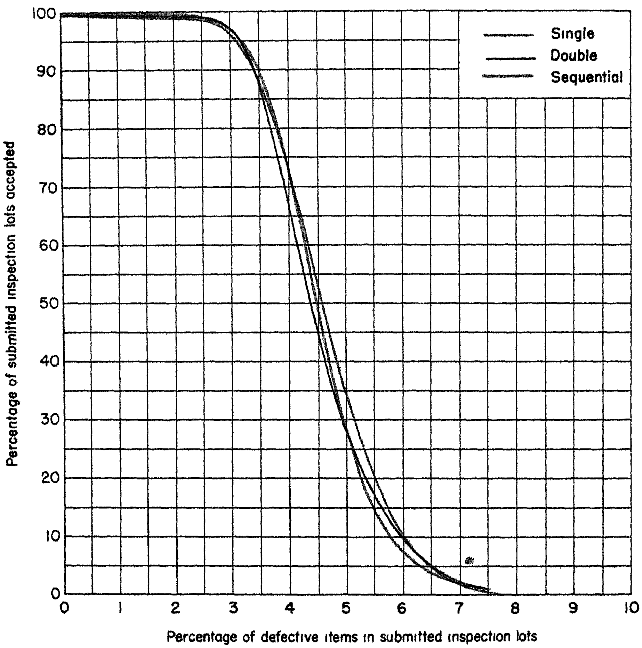


TABLE 4 (*continued*)

Sample-size letter **M**
AQL class (percent defective) **3.2–4.4**
AOQL class (percent defective) **3.5–5.0**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First	450	450	26	27
Double	First	300	300	16	48
	Second	600	900	47	48
Sequential	First	100	100	2	10
	Second	100	200	8	16
	Third	100	300	13	21
	Fourth	100	400	19	27
	Fifth	100	500	24	33
	Sixth	100	600	30	38
	Seventh	100	700	35	44
	Eighth	100	800	43	44

b. Operating-characteristic Curves

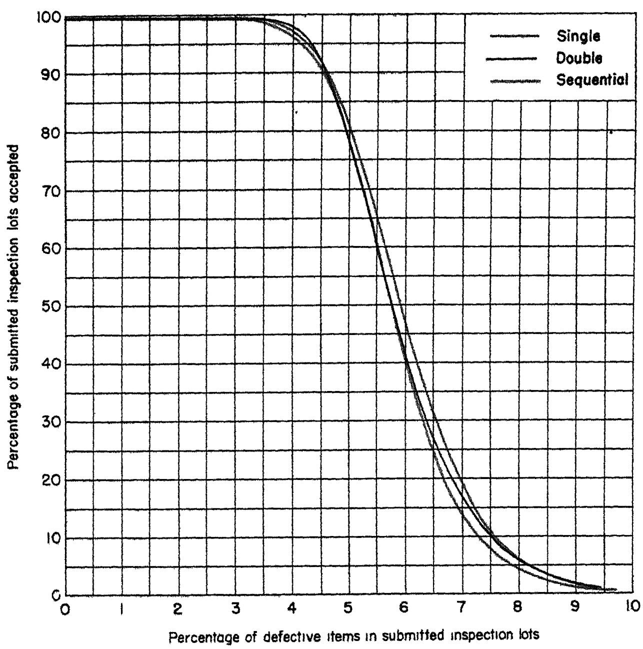


TABLE 4 (*continued*)

M Sample-size letter
4.4–5.3 AQL classes (percent defective)
5.3–6.4
6.4–8.5

For AQL class	Use sample-size letter
4.4–5.3	L
5.3–6.4	L
6.4–8 5	K

TABLE 4 (*continued*)

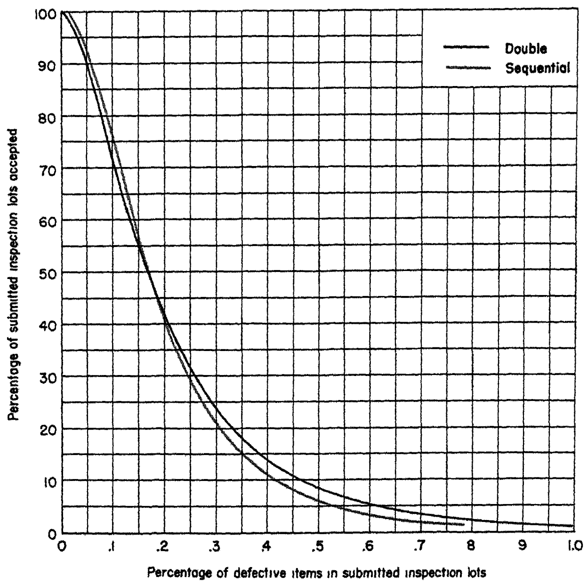
Sample-size letter **N**
AQL class (percent defective) **0.024–0.035**
AOQL class (percent defective) **0.075–0.10**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	For single sampling, use sample-size letter O			
Double	First	500	500	0	2
	Second	1,000	1,500	1	2
Sequential	First	150	150	*	1
	Second	150	300	*	2
	Third	150	450	*	2
	Fourth	150	600	0	2
	Fifth	150	750	0	2
	Sixth	150	900	0	3
	Seventh	150	1,050	1	3
	Eighth	150	1,200	1	3
	Ninth	150	1,350	2	3

* Acceptance not permitted until four samples have been inspected.

b. Operating-characteristic Curves



N Sample-size letter TABLE 4 (*continued*)
0.035–0.06 AQL class (percent defective)
0.10 –0.15 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	750	750	1	2
Double	First	500	500	0	3
	Second	1,000	1,500	2	3
Sequential	First	150	150	*	2
	Second	150	300	*	2
	Third	150	450	0	2
	Fourth	150	600	0	2
	Fifth	150	750	0	2
	Sixth	150	900	0	3
	Seventh	150	1,050	0	3
	Eighth	150	1,200	1	3
	Ninth	150	1,350	2	3

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

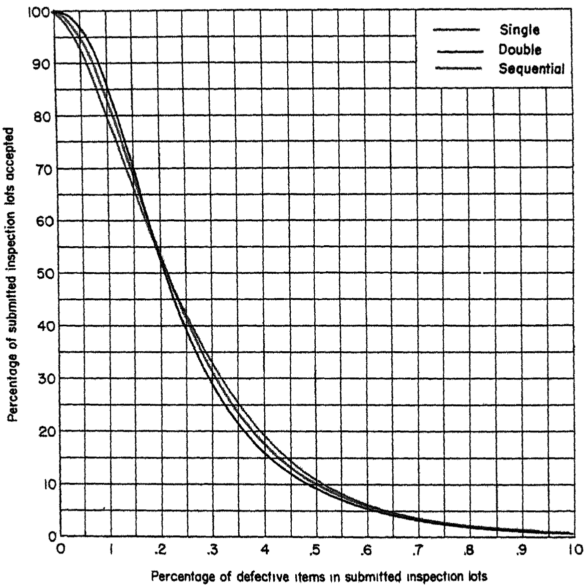


TABLE 4 (*continued*)

Sample-size letter	**N**
AQL class (percent defective)	**0.06–0.12**
AOQL class (percent defective)	**0.15–0.22**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	750	750	2	3
Double	First	500	500	1	4
	Second	1,000	1,500	3	4
Sequential	First	150	150	*	2
	Second	150	300	*	2
	Third	150	450	0	3
	Fourth	150	600	0	3
	Fifth	150	750	1	4
	Sixth	150	900	1	5
	Seventh	150	1,050	2	5
	Eighth	150	1,200	3	5
	Ninth	150	1,350	4	5

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

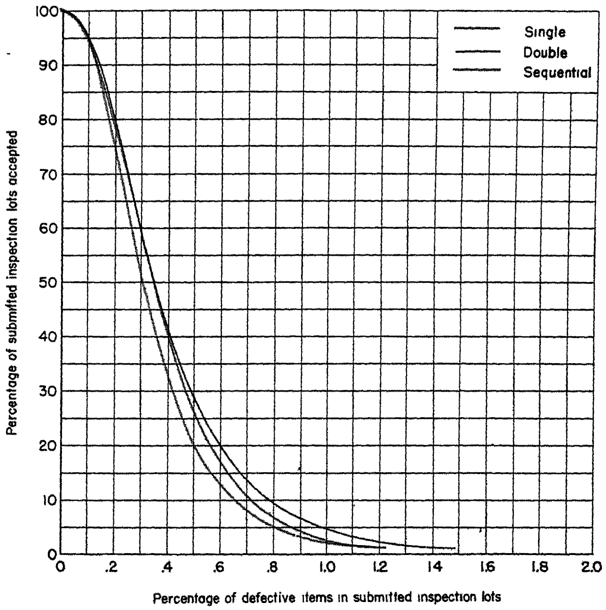


TABLE 4 (*continued*)

N Sample-size letter
0.12–0.17 AQL class (percent defective)
0.22–0.30 AOQL class (percent defective)

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	750	750	3	4
Double	First	500	500	1	6
	Second	1,000	1,500	5	6
Sequential	First	150	150	*	2
	Second	150	300	0	3
	Third	150	450	0	4
	Fourth	150	600	0	4
	Fifth	150	750	1	4
	Sixth	150	900	2	5
	Seventh	150	1,050	3	5
	Eighth	150	1,200	4	6
	Ninth	150	1,350	5	6

* Acceptance not permitted until two samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES

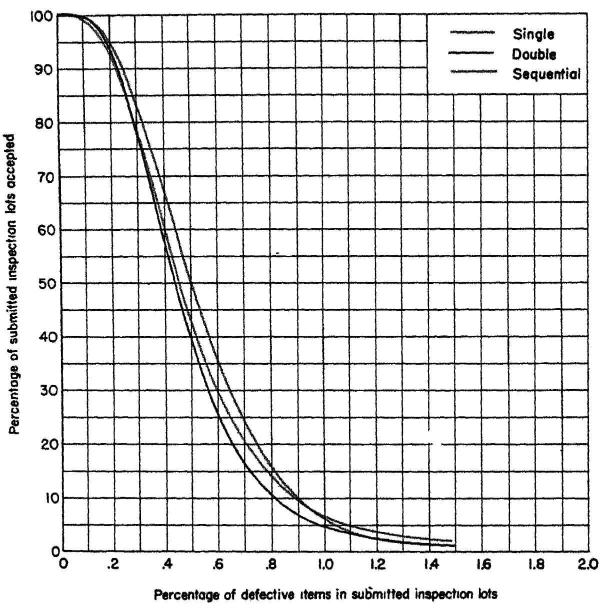


TABLE 4 (*continued*)

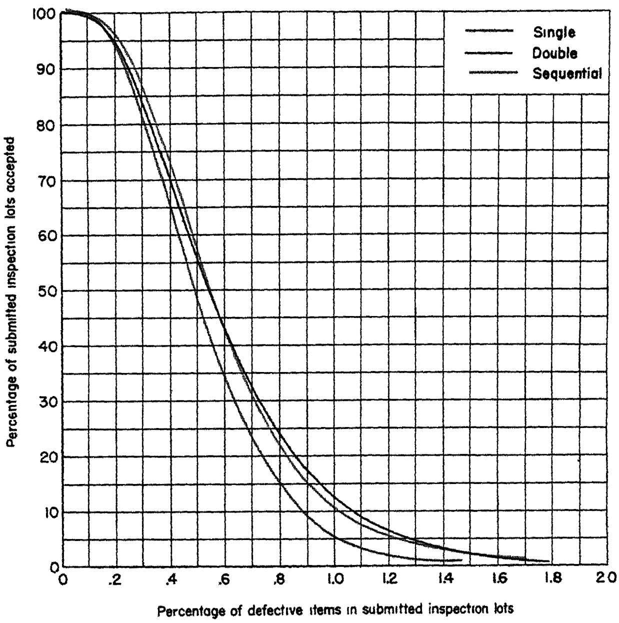
Sample-size letter **N**
AQL class (percent defective) **0.17–0.22**
AOQL class (percent defective) **0.22–0.30**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	750	750	3	4
Double	First	500	500	2	5
	Second	1,000	1,500	4	5
Sequential	First	150	150	*	2
	Second	150	300	0	4
	Third	150	450	1	4
	Fourth	150	600	1	5
	Fifth	150	750	2	6
	Sixth	150	900	3	6
	Seventh	150	1,050	4	7
	Eighth	150	1,200	5	8
	Ninth	150	1,350	7	8

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



N Sample-size letter TABLE 4 (*continued*)
0.22–0.32 AQL class (percent defective)
0.30–0.50 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First	750	750	5	6
Double	First	500	500	3	8
	Second	1,000	1,500	7	8
Sequential	First	150	150	*	3
	Second	150	300	1	4
	Third	150	450	2	4
	Fourth	150	600	3	6
	Fifth	150	750	4	7
	Sixth	150	900	5	8
	Seventh	150	1,050	6	9
	Eighth	150	1,200	7	10
	Ninth	150	1,350	10	11

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

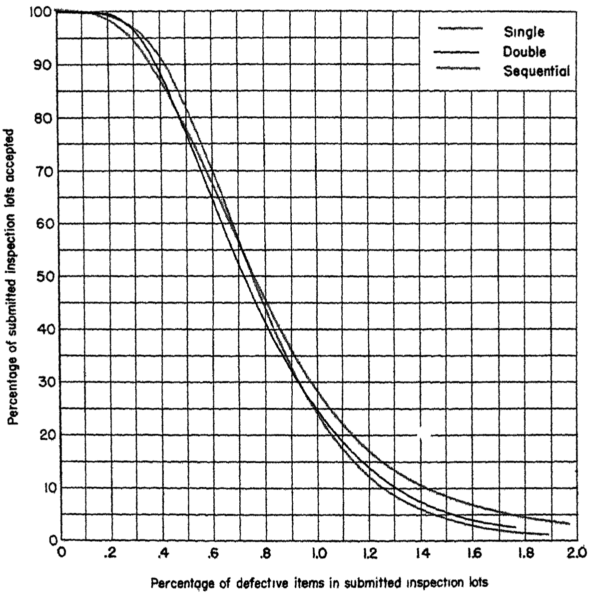


TABLE 4 (*continued*)

Sample-size letter **N**
AQL class (percent defective) **0.32–0.65**
AOQL class (percent defective) **0.50–0.90**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First	750	750	8	9
Double	First	500	500	5	13
	Second	1,000	1,500	12	13
Sequential	First	150	150	0	4
	Second	150	300	1	5
	Third	150	450	2	7
	Fourth	150	600	4	9
	Fifth	150	750	6	11
	Sixth	150	900	8	13
	Seventh	150	1,050	10	14
	Eighth	150	1,200	12	15
	Ninth	150	1,350	15	16

b. Operating-characteristic Curves

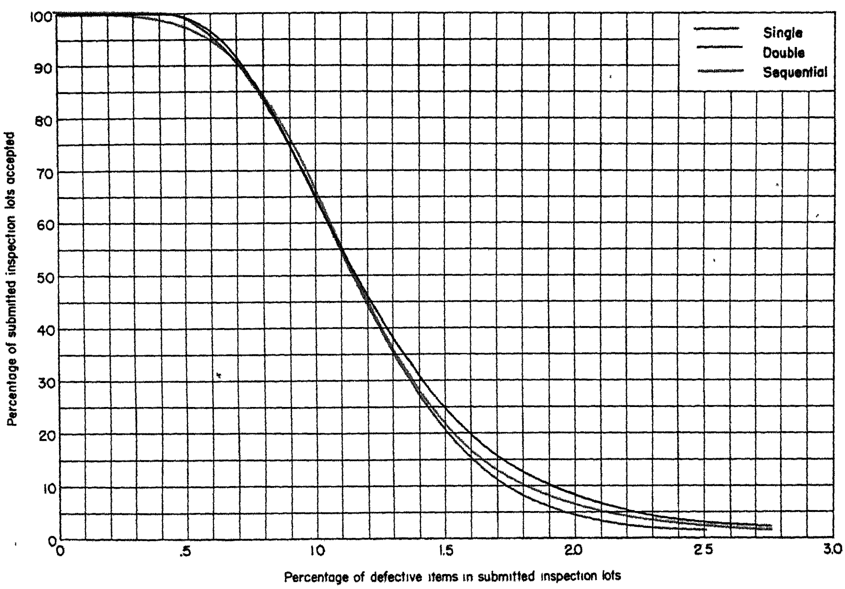


TABLE 4 (*continued*)

N Sample-size letter
0.65–1.2 AQL class (percent defective)
0.90–1.5 AOQL class (percent defective)

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	750	750	13	14
Double	First	500	500	7	24
	Second	1,000	1,500	23	24
Sequential	First	150	150	0	6
	Second	150	300	3	8
	Third	150	450	5	11
	Fourth	150	600	7	13
	Fifth	150	750	10	16
	Sixth	150	900	13	19
	Seventh	150	1,050	15	21
	Eighth	150	1,200	18	24
	Ninth	150	1,350	23	24

b. OPERATING-CHARACTERISTIC CURVES

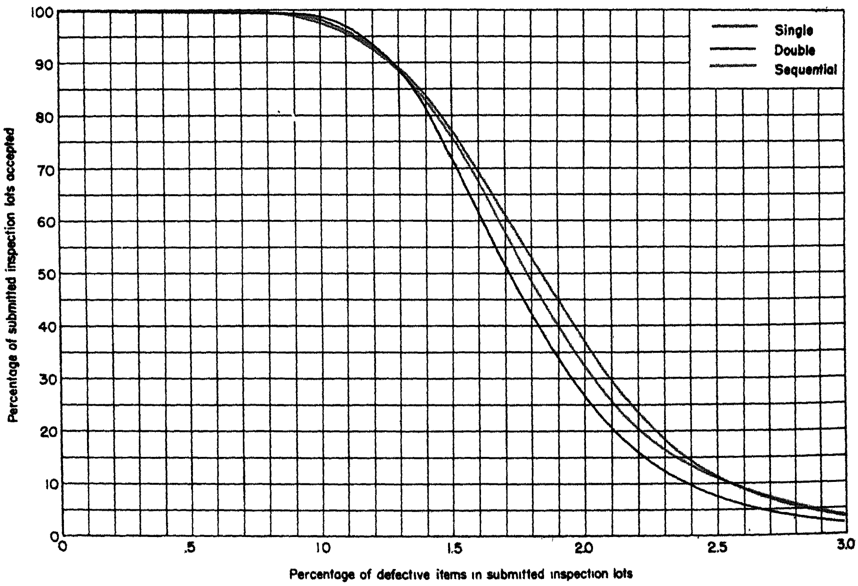


TABLE 4 (*continued*)

Sample-size letter		**N**
AQL class (percent defective)		**1.2–2.2**
AOQL class (percent defective)		**1.5–2.5**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First	750	750	23	24
Double	First	500	500	14	41
	Second	1,000	1,500	40	41
Sequential	First	150	150	1	8
	Second	150	300	6	13
	Third	150	450	10	18
	Fourth	150	600	15	22
	Fifth	150	750	19	27
	Sixth	150	900	24	31
	Seventh	150	1,050	28	36
	Eighth	150	1,200	33	41
	Ninth	150	1,350	41	42

b. OPERATING-CHARACTERISTIC CURVES

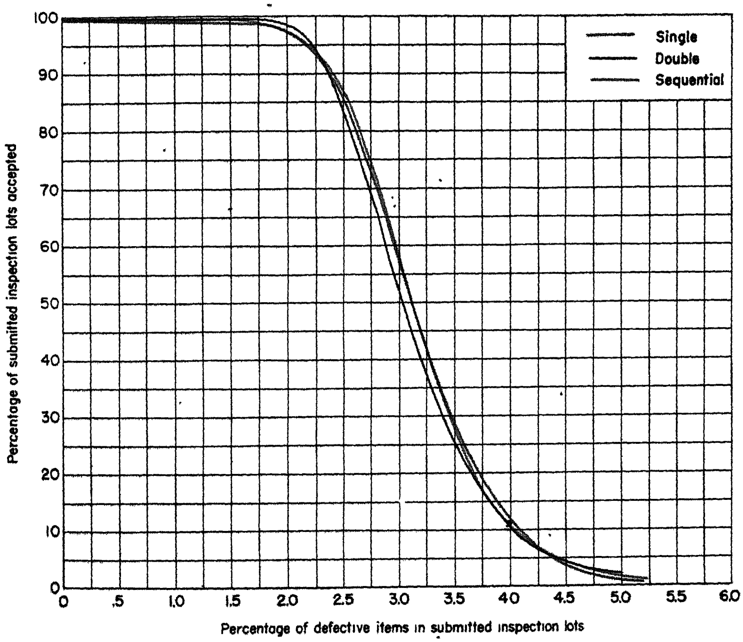


TABLE 4 (*continued*)

N Sample-size letter
2.2–3.2 AQL classes (percent defective)
3.2–4.4
4.4–5.3
5.3–6.4
6.4–8.5

For AQL class	Use sample-size letter
2.2–3.2	M
3.2–4.4	M
4.4–5.3	L
5.3–6.4	L
6.4–8.5	K

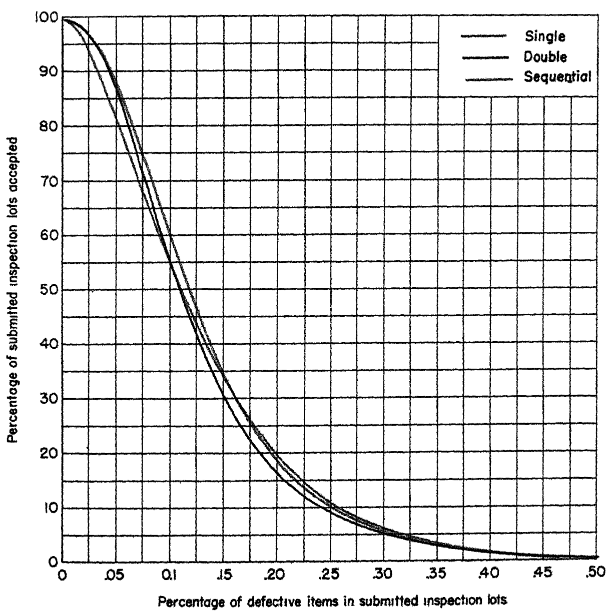
TABLE 4 (*continued*) Sample-size letter **O**
AQL class (percent defective) **0.024–0.035**
AOQL class (percent defective) **0.05 –0.075**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	1,500	1,500	1	2
Double	First	1,000	1,000	0	3
	Second	2,000	3,000	2	3
Sequential	First	300	300	*	2
	Second	300	600	*	2
	Third	300	900	0	2
	Fourth	300	1,200	0	3
	Fifth	300	1,500	0	3
	Sixth	300	1,800	0	3
	Seventh	300	2,100	0	3
	Eighth	300	2,400	1	3
	Ninth	300	2,700	2	3

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves



O Sample-size letter TABLE 4 (*continued*)
0.035–0.06 AQL class (percent defective)
0.075–0.10 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	1,500	1,500	2	3
Double	First	1,000	1,000	1	4
	Second	2,000	3,000	3	4
Sequential	First	300	300	*	2
	Second	300	600	*	2
	Third	300	900	0	3
	Fourth	300	1,200	1	4
	Fifth	300	1,500	2	5
	Sixth	300	1,800	2	5
	Seventh	300	2,100	2	5
	Eighth	300	2,400	3	5
	Ninth	300	2,700	4	5

* Acceptance not permitted until three samples have been inspected.

b. Operating-characteristic Curves

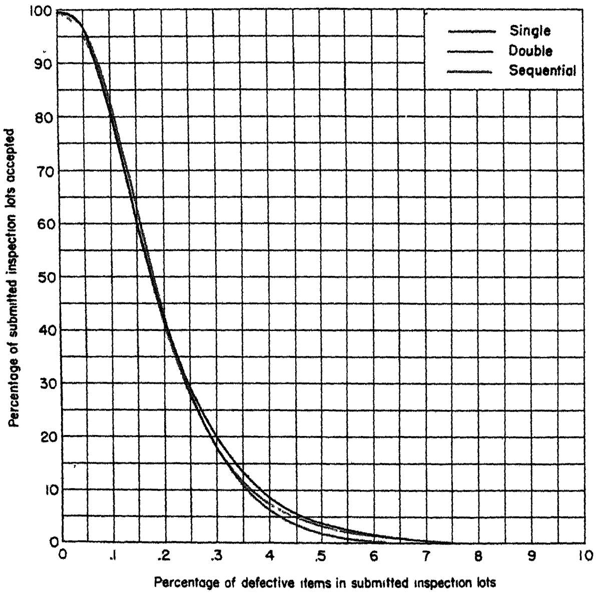


TABLE 4 (*continued*)

Sample-size letter **O**
AQL class (percent defective) **0.06–0.12**
AOQL class (percent defective) **0.10–0.15**

a. SAMPLING PLANS

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	1,500	1,500	3	4
Double	First	1,000	1,000	1	6
	Second	2,000	3,000	5	6
Sequential	First	300	300	*	3
	Second	300	600	*	3
	Third	300	900	0	4
	Fourth	300	1,200	1	4
	Fifth	300	1,500	1	5
	Sixth	300	1,800	2	6
	Seventh	300	2,100	3	7
	Eighth	300	2,400	3	7
	Ninth	300	2,700	6	7

* Acceptance not permitted until three samples have been inspected.

b. OPERATING-CHARACTERISTIC CURVES

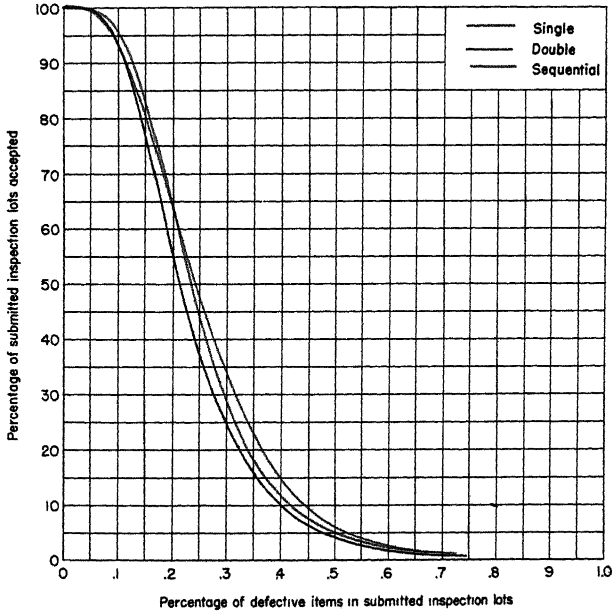


TABLE 4 (*continued*)

O Sample-size letter
0.12–0.17 AQL class (percent defective)
0.15–0.22 AOQL class (percent defective)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples		
			Size	Acceptance number	Rejection number
Single	First	1,500	1,500	4	5
Double	First	1,000	1,000	2	8
	Second	2,000	3,000	7	8
Sequential	First	300	300	*	3
	Second	300	600	0	4
	Third	300	900	1	5
	Fourth	300	1,200	2	6
	Fifth	300	1,500	3	7
	Sixth	300	1,800	4	7
	Seventh	300	2,100	5	8
	Eighth	300	2,400	6	9
	Ninth	300	2,700	8	9

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves

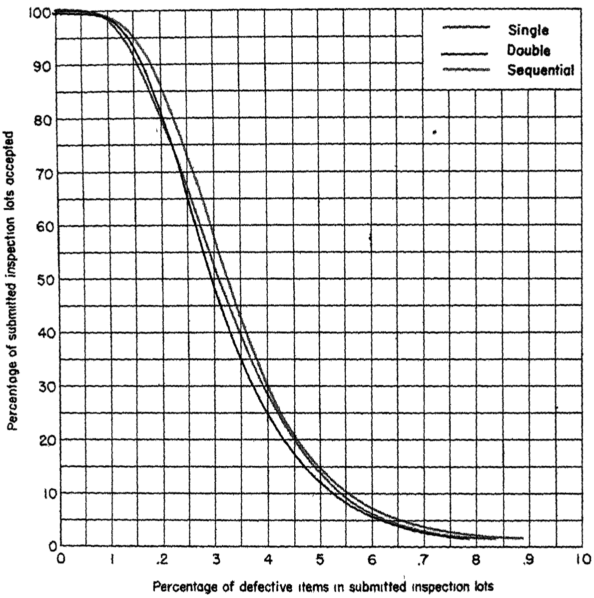


TABLE 4 (*continued*)

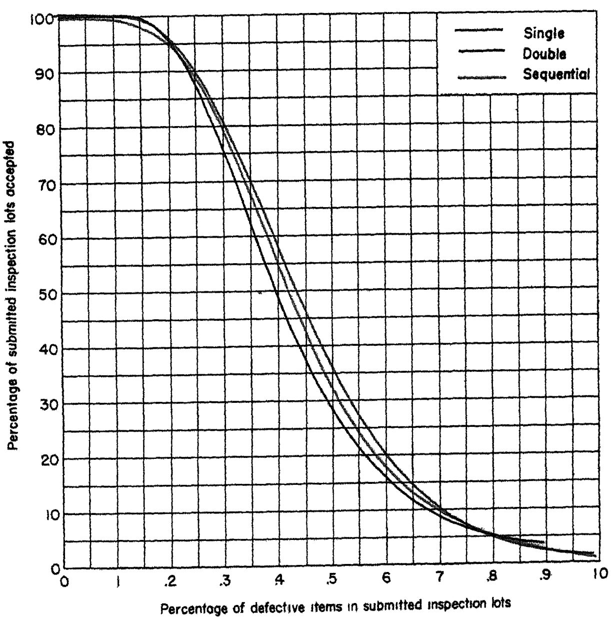
Sample-size letter **O**
AQL class (percent defective) **0.17–0.22**
AOQL class (percent defective) **0.22–0.30**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Combined samples: Acceptance number	Combined samples: Rejection number
Single	First....................	1,500	1,500	6	7
Double	First....................	1,000	1,000	3	10
	Second....................	2,000	3,000	9	10
Sequential	First....................	300	300	*	3
	Second....................	300	600	1	5
	Third....................	300	900	1	6
	Fourth....................	300	1,200	3	7
	Fifth....................	300	1,500	4	8
	Sixth....................	300	1,800	5	9
	Seventh....................	300	2,100	7	10
	Eighth....................	300	2,400	8	11
	Ninth....................	300	2,700	10	11

* Acceptance not permitted until two samples have been inspected.

b. Operating-characteristic Curves



O Sample-size letter
0.22–0.32 AQL class (percent defective)
0.30–0.50 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	1,500	1,500	8	9
Double	First	1,000	1,000	5	13
	Second	2,000	3,000	12	13
Sequential	First	300	300	0	4
	Second	300	600	1	5
	Third	300	900	2	7
	Fourth	300	1,200	4	9
	Fifth	300	1,500	6	11
	Sixth	300	1,800	8	13
	Seventh	300	2,100	10	14
	Eighth	300	2,400	12	15
	Ninth	300	2,700	15	16

b. Operating-characteristic Curves

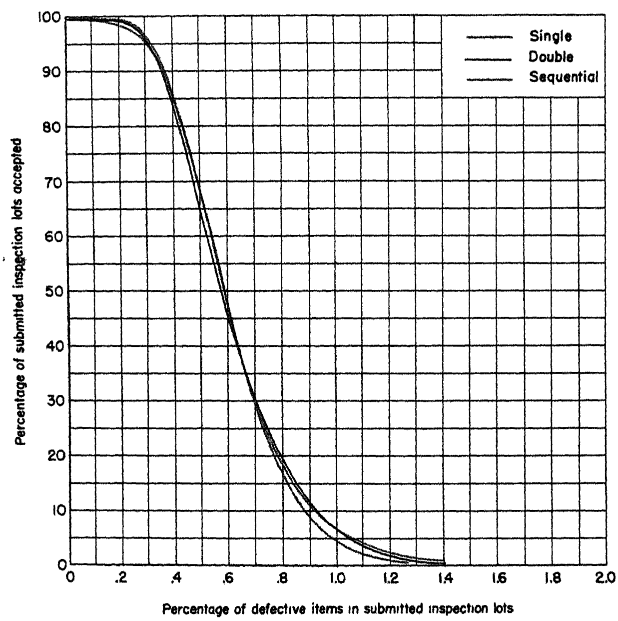


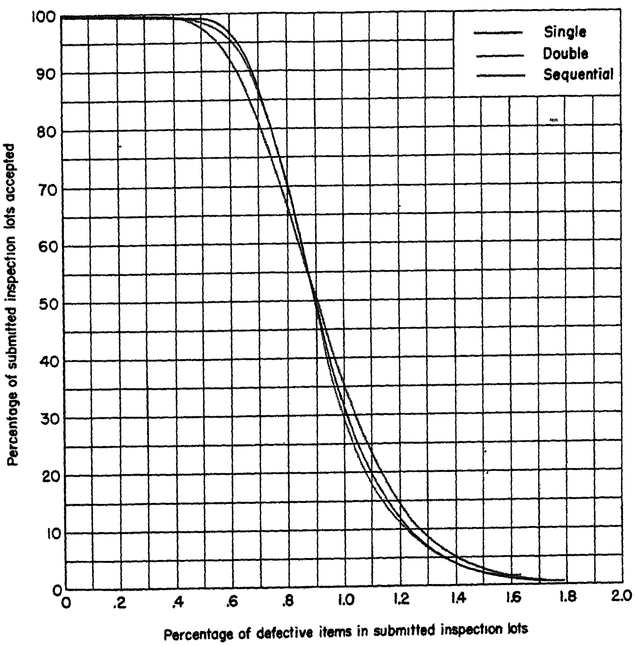
TABLE 4 (*continued*)

Sample-size letter **O**
AQL class (percent defective) **0.32–0.65**
AOQL class (percent defective) **0.50–0.90**

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	1,500	1,500	13	14
Double	First	1,000	1,000	7	26
	Second	2,000	3,000	25	26
Sequential	First	300	300	0	6
	Second	300	600	2	9
	Third	300	900	4	11
	Fourth	300	1,200	7	14
	Fifth	300	1,500	10	17
	Sixth	300	1,800	13	20
	Seventh	300	2,100	15	23
	Eighth	300	2,400	18	24
	Ninth	300	2,700	23	24

b. Operating-characteristic Curves



O Sample-size letter

0.65–1.2 AQL class (percent defective)

0.90–1.5 AOQL class (percent defective)

TABLE 4 (*continued*)

a. Sampling Plans

Type of sampling	Sample	Sample size	Combined samples: Size	Acceptance number	Rejection number
Single	First	1,500	1,500	24	25
Double	First	1,000	1,000	14	44
	Second	2,000	3,000	43	44
Sequential	First	300	300	1	8
	Second	300	600	6	13
	Third	300	900	10	18
	Fourth	300	1,200	15	22
	Fifth	300	1,500	19	27
	Sixth	300	1,800	24	31
	Seventh	300	2,100	28	36
	Eighth	300	2,400	33	41
	Ninth	300	2,700	41	42

b. Operating-characteristic Curves

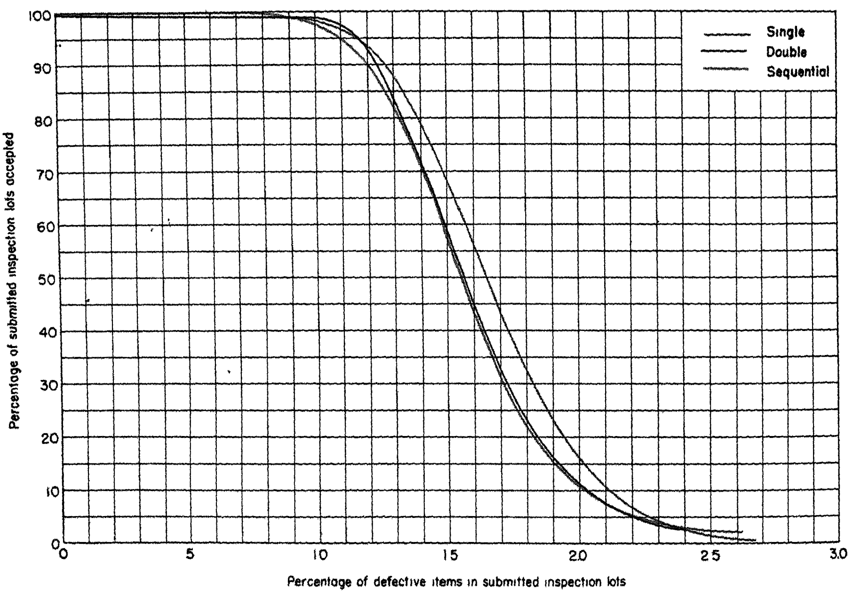


TABLE 4 (*continued*)

Sample-size letter	**O**
AQL classes (percent defective)	**1.2–2.2**
	2.2–3.2
	3.2–4.4
	4.4–5.3
	5.3–6.4
	6.4–8.5

For AQL class	Use sample-size letter
1.2–2.2	N
2.2–3.2	M
3.2–4.4	M
4.4–5.3	L
5.3–6.4	L
6.4–8.5	K

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GLOSSARY

acceptable-quality level (AQL). Percentage of defective items in an inspection lot such that the sampling plan will result in the acceptance of 95 per cent of submitted inspection lots containing that percentage of defective items.

acceptable-quality level, major (AQL, major). The acceptable-quality level of the sampling plan used for major defects.

acceptable-quality level, minor (AQL, minor). The acceptable-quality level of the sampling plan used for minor defects.

acceptance number. A number associated with each sample of a sampling plan; an inspection lot is passed for a class of defects after inspection of a particular sample if the total number of defectives of that class found in that and all preceding samples combined is equal to or less than the acceptance number associated with that sample.

assignable cause. Cause, other than chance, of variation in quality.

attributes, inspection by. Inspection in which an item is classified as either defective or nondefective.

average amount of inspection. Average number of items inspected per inspection lot under a particular sampling plan.

average outgoing quality (AOQ). Average percentage of defective items in product finally accepted. Product finally accepted includes both inspection lots accepted on the basis of the sampling plan and inspection lots originally rejected on the basis of the sampling plan and finally accepted after removal by the supplier of all defective items.

average outgoing-quality limit (AOQL). Maximum average percentage of defective items in product finally accepted. Product finally accepted includes both inspection lots accepted on the basis of the sampling plan and inspection lots originally rejected on the basis of the sampling plan and finally accepted after removal by the supplier of all defective items.

blind sampling. Drawing items without regard to their quality.

control (as applied to a process). A situation in which variation in quality is attributable to chance causes only.

control chart for number of defectives. A chart on which is plotted the number of defectives in each first sample and the control limits within which this number should fall in the absence of assignable causes.

control limits. Limits within which the number of defectives in a sample usually falls, if the process is in control.

curtailed sampling. Termination of inspection as soon as a decision to accept or reject the submitted inspection lot can be reached.

defect. Any deviation from the requirements of the specification, drawing, contract, or order.

defective. An item containing one or more defects.

defective material. Rejected inspection lots and defective items found in the sample or samples inspected.

double sampling. A type of sampling in which a decision to accept or reject an inspection lot may be reached after one sample from that inspection lot has been

inspected and will always be reached after not more than two samples from that inspection lot have been inspected.

first sample. Initial sample inspected from an inspection lot; for single sampling, the first sample is the only sample inspected from each inspection lot.

homogeneity (as applied to an inspection lot or sublot). A situation in which quality variations within the inspection lot or sublot are attributable to chance causes only.

inspection level. A term used in this book to designate the relative amount of inspection given a product.

inspection lot. A collection of items accepted or rejected as a whole on the basis of a sampling plan.

inspection-lot size. Number of items in the inspection lot.

irregularity. Any defect, other than a major or minor defect.

item. Single member of an inspection lot, usually though not necessarily a single article.

lot tolerance percent defective (LTPD). Percentage of defective items in an inspection lot such that the sampling plan will result in the rejection of 90 percent of submitted inspection lots containing that percentage of defective items.

lower control limit (LCL) of number of defectives. Limit *above* which the number of defectives in a sample usually falls, in the absence of assignable causes.

lower limit of process average. Lower limiting value of the process average given in Table 11.5. If the supplier's process averages for all classes of defectives are less than the appropriate limiting values, reduced inspection may be installed (if other conditions are satisfied).

major defect. Defect that will cause a failure of the item to function in accordance with the intent of the design or that will prevent the use of the item.

major defective. An item containing one or more major defects.

minor defect. Defect that will impair the efficiency or shorten the life of an item.

minor defective. An item containing one or more minor defects.

mixed defective. An item with one or more defects of each of two or more classes.

normal inspection. Inspection under a sampling plan for the AQL and inspection level specified for the product.

one hundred percent inspection. Inspection of all items in the inspection lot.

operating-characteristic (OC) curve of a sampling plan. Curve showing the percentage of submitted inspection lots that will be accepted on the basis of the sampling plan for each percentage of defective items in submitted inspection lots.

process average. Average percentage of defective items in first samples from submitted inspection lots.

process average, major. Average percentage of major defectives in first samples from submitted inspection lots.

process average, minor. Average percentage of minor defectives in first samples from submitted inspection lots.

process inspection. Study of process, including spot-checking of product at various stages of production.

proportional sample. A sample that gives representation to each portion of an inspection lot in proportion to the size of that portion.

random sample. A sample drawn in such a manner that the chance that any given item in the inspection lot will be included in the sample is independent of the quality of that item and independent of the quality of other items selected for the sample.

reduced inspection. Inspection under a sampling plan with the same AQL as for normal inspection but requiring a smaller amount of inspection.

rejection number. Number associated with each sample of a sampling plan; an inspection lot is rejected for a class of defectives after inspection of a particular sample if the total number of defectives of that class found in that and all preceding samples combined is equal to or greater than the rejection number associated with that sample.

sample. Collection of items drawn from an inspection lot.

sample size. Number of items in a sample.

sample-size letter. A letter used in this book to stand for a sample size or for a sequence of sample sizes.

sampling inspection. Evaluation of the quality of material by inspecting some of the material.

screening. One hundred percent inspection accompanied by rejection of all defective items.

sequential sampling. A type of sampling in which a decision to accept or reject an inspection lot may be reached after one, two, three, or more samples from that inspection lot have been inspected and will always be reached after not more than a limited number of samples have been inspected.

set of sampling plans. The single-, double-, and sequential-sampling plans on a single page of Table 4.

single sampling. A type of sampling in which a decision to accept or reject an inspection lot is reached after one sample from that inspection lot has been inspected.

standard sampling-inspection procedure. The procedure described in Part III of this book.

tightened inspection. Inspection under a sampling plan with the same sample size or sequence of sample sizes as for normal inspection but with lower acceptance and rejection numbers; thus, very few inspection lots of unsatisfactory and mediocre quality will be accepted under tightened inspection.

upper control limit (UCL) of number of defectives. Limit *below* which the number of defectives in a sample usually falls, in the absence of assignable causes.

upper limit of process average. Upper limiting value of process average given in Table 11.5. If the supplier's process average for a class of defectives is greater than this limiting value, tightened inspection is required for that class of defects.

variables, inspection by. Inspection in which quality characteristics of an item are measured.

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